RADIAL VELOCITY PROFILES FOR ANISOTROPIC SPHERICALLY SYMMETRIC CLUSTERS: AN EXAMPLE

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A 1–parameter family of anisotropic models is presented. They all satisfy the Plummer law in the mass density, but have different velocity dispersions. Moreover, the stars are not confined to a particular subset of the total accessible phase space. This family is mathematically simple enough to be explored analytically in detail. The family is rich enough though to allow for a 3–parameter generalization which illustrates that even when both the mass density and the velocity dispersion profiles are required to be the same, a degeneracy in the possible distribution functions persists. The observational consequences of the degeneracy can be studied by calculating the observable radial velocity line profiles obtained with different distribution functions. It turns out that line profiles are relatively sensitive to changes in the distribution function. They therefore can be considered to be more natural observables when a determination of the distribution function is desired.

Be E the binding energy of a single star and L its total angular momentum. The fundamental integral equation that relates the distribution function $F(E,L)$ to the mass density reads

$$\rho(\psi,r) = 2\pi\,\mathrm{M}\int_0^{\psi} dE \int_0^{2(\psi-E)} \frac{F(E,L)}{\sqrt{2(\psi-E)-L^2/r^2}}\, d(L^2/r^2). \qquad (1)$$

The double integration over the distribution function gives the mass density as a function of the potential ψ and the spherical radius r. It is immediately apparent that different $F(E,L)$ usually lead to different $\rho(\psi,r)$ but not necessarily to different $\rho(r) = \rho(\psi(r),r)$. Essentially, the problem comes to the necessity of knowing the full $\rho(\psi,r)$, while we have only the cut $\rho(\psi(r),r)$, provided by the observation of the mass density. We instead need infinitely many cuts, at least in principle. This means that the distribution function $F(E,L)$ is not uniquely determined by the mass density, and not even by the mass density together with the velocity dispersions.

It follows from (1) that anisotropic models can be constructed by expression of ρ as a function of both ψ and r. The simple form for the potential and mass density of the Plummer model (1911) suggests the following choice for the function $\rho(\psi,r)$

$$\rho_q(\psi,r) = \frac{3}{4\pi}\psi^{5-q}(1+r^2)^{-q/2}, \qquad (2)$$

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with one parameter q, but leading for all q to the same mass density. The distribution function $F_q(E, L)$ that is consistent with $\rho_q(\psi, r)$ can be calculated analytically (see Dejonghe 1986a). It is expressed conveniently in terms of the hypergeometric function. Positiveness of $F_q(E, L)$ restricts the possible parameter values to $q \leq 2$. When $q < 2$, the distribution functions are nonnegative over the full region in phase space which is in principle accessible, and are zero only for $E = 0$. This is an interesting property in view of stability requirements. The limiting case $q = 2$ gives rise to a very simple distribution function (see Osipkov (1979)). It is also identical to one of the Merritt (1985) models (with anisotropy parameter $r_a = 1$).

All the spatial moments of the distribution function for general q, the marginal distributions and all the moments of the line of sight velocity distributions can be calculated analytically (Dejonghe 1986b). As an application, we can infer from the expression of the velocity dispersions that q has the same sign as Binney's anisotropy parameter β. We call a cluster with $q > 0$ a radial cluster and a cluster with $q < 0$ tangential. The well-known isotropic case has $q = 0$.

A line profile $\mathrm{lp}_{\mathrm{r_p}}(v_p)$ at a particular projected radius r_p is defined as the distribution of the velocities v_p projected along the line of sight (and which are therefore the observed radial velocities). In order to calculate them, we need for every projected radius r_p and every projected velocity v_p to perform a triple integration: two of them integrate over three-dimensional velocity space, reducing its dimensionality to one, and the last integrates through the cluster along the line of sight. This is a somewhat tricky calculation, and it is worthwhile to try to avoid it by using the expansion

$$\mathrm{lp}_{\mathrm{r_p}}(v_p)\, dv_p = (1 - {v_p}^2)^{5-\alpha} \left[c_0 + \sum_{i \geq 1} c_i (1 - {v_p}^2)^i \right] dv_p. \tag{3}$$

The first term follows from the asymptotical form of the tails of $\mathrm{lp}_{\mathrm{r_p}}$. The additional terms in c_i, $i \geq 1$ are correction terms: the coefficients c_i can be determined by the known moments of the line of sight velocity distribution.

At the centre, tangential clusters have a more peaked projected velocity distribution relative to radial clusters. At core radius though, the different types are virtually indistinguishable. At large radius, we see even a qualitative difference. Tangential clusters show bimodal lines, arising from the nearly circular orbits, with stars populating them in both senses in equal amount (no net streaming). Radial clusters persist in the old unimodal type of profile. Therefore, the conclusions to be drawn from the line profiles depend essentially on the region one is looking at.

As was already pointed out above, the indeterminacy in $F(E, L)$ comes essentially from the degeneracy $\rho(\psi, r) \equiv \rho(r)$. It is only natural to exploit this fact to construct distribution functions that all give rise to the same spatial mass density $\rho(r)$ and the same spatial velocity dispersions. Details about this construction are very technical and can be found in Dejonghe (1986b, preprint).

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