PERIODIC AND APERIODIC BEHAVIOUR IN STELLAR DYNAMOS

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Abstract: We have constructed a simple parametrized mean field dynamo model that includes the dynamical interaction between the magnetic field and differential rotation. This system of seven coupled nonlinear ordinary differential equations has finite amplitude oscillatory solutions (corresponding to Parker's dynamo waves) when the dynamo number D>1. We have studied two regimes. In the first, the velocity shear is reduced by the Lorentz force and there are stable periodic solutions for all D>1. In the second there is a transition from strictly periodic oscillations to aperiodic (chaotic) behaviour as D is increased. This simple example shows that nonlinear hydromagnetic dynamos can produce aperiodic cycles, with Maunder minima, as observed in the sun and other late-type stars.

The solar cycle provides a convincing example of a strange attractor (Ruzmaikin 1981). There are several routes from regularity to chaos (Eckmann 1981) and in order to discover which is relevant to the sun it is natural to investigate the simplest model that captures the essential physics of a nonlinear stellar dynamo. We have therefore constructed a system of equations that describes a parametrized $\alpha\omega$-dynamo in one space dimension, including the dynamical effect of the Lorentz force, which acts to reduce differential rotation. The solutions are nonlinear dynamo waves (Parker 1979).

This seventh order system is given, in dimensionless form, by the equations

$$\ddot{A} = 2DB - A \tag{1}$$
$$\dot{B} = i(1+\omega_0)A - \tfrac{1}{2}iA^*\omega - B, \tag{2}$$
$$\dot{\omega}_0 = \tfrac{1}{2}i(A^*B-AB^*) - \nu_0\omega_0, \tag{3}$$
$$\dot{\omega} = -iAB - \nu\omega. \tag{4}$$

The poloidal flux function A and the toroidal field B are complex, so that $B=B_1+iB_2$ etc., and the change in the velocity shear is separated into a real, spatially uniform component ω_0 and a complex component with twice the spatial frequency of A and B; ν and ν_0 are real constants.

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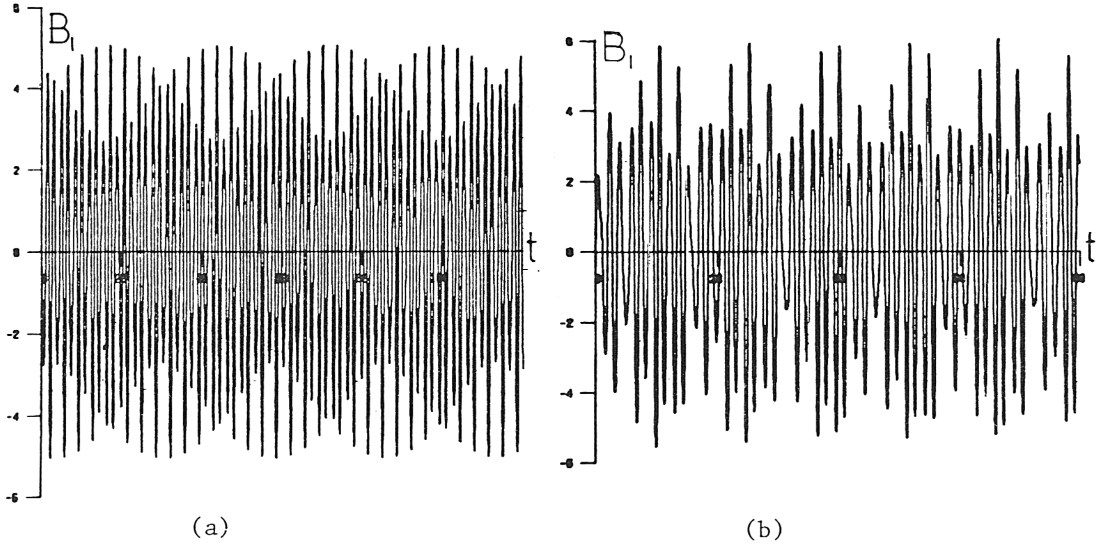

Figure 1: Nonlinear dynamo waves: the toroidal field B as a function of time for the sixth order system with ν=0.5. (a) Doubly periodic behaviour for D=3.0. (b) Triply periodic motion for D=3.5.

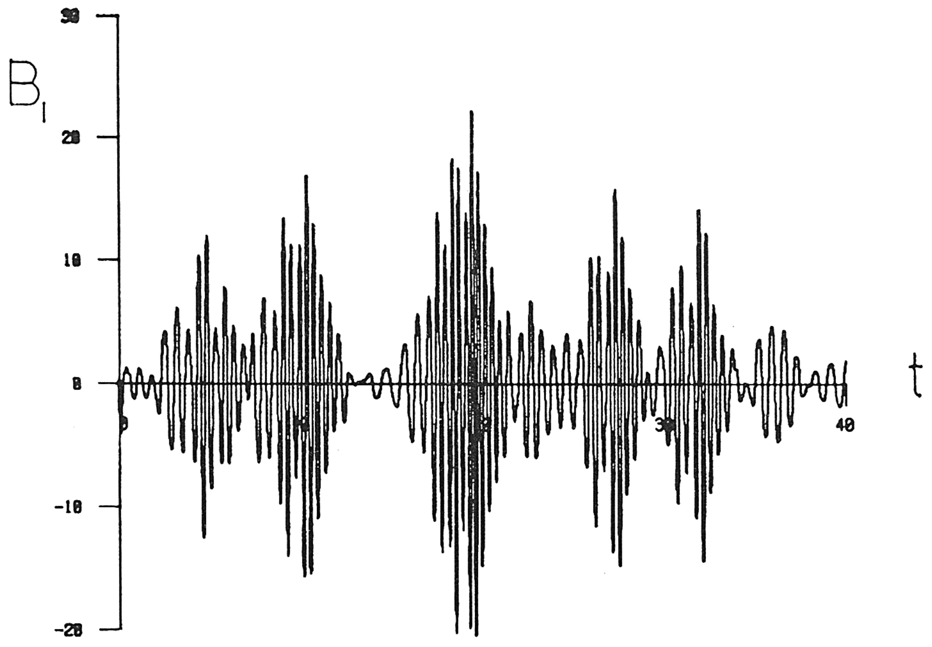

Figure 2: Chaotic behaviour with episodes of reduced activity. As Fig.1 but with D=16.

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The dynamo number D is a stability parameter for this problem and there is a Hopf bifurcation from the trivial solution $A=B=\omega=0$ at $D=1$(Parker 1979).

We have studied two cases. In the limit $\nu\to\infty$, $\omega\to 0$ equations (1)-(3) yield a fifth order system with an exact periodic solution: A and B vary sinusoidally and ω_0 is a (negative) constant. This solution is stable for all D. Thus the Lorentz force reduces the velocity shear, giving a lower effective dynamo number, as in Gilman's (1982) numerical experiments.

When $\nu_0\to\infty, \omega_0\to 0$, equations (1),(2) and (4) give rise to a sixth order system which exhibits more interesting behaviour (cf. Jones 1982). Again there is an exact periodic solution for $D>1$, with A, B, ω varying sinusoidally in time. We have located successive bifurcations from this solution as D is increased for $\nu=0.5$. The exact solution loses stability at $D\approx 2.07$ and Fig.1(a) shows $B_1(t)$ for a doubly periodic solution at $D=3.0$. The next bifurcation leads to triply periodic solutions, shown in Fig.1(b) for $D=3.5$. There follows a transition to chaotic behaviour around $D\approx 3.84$. Fig.2 shows a typical solution for $D=16$. The cycles are aperiodic and are separated by intervals of stasis, with drastically reduced activity. (This pattern is typical of all runs made for $D\geq 4$.) Mathematically, there is a bifurcation to a 2-torus at $D\approx 2.07$, followed by a bifurcation to quasiperiodic motion on a 3-torus at $D\approx 3.47$. Two of the three frequencies subsequently lock and, after a cascade of period doubling bifurcations, the flow eventually becomes chaotic.

The variable ω describes variations in differential rotation with twice the frequency, in space and in time, of the mean magnetic fields. Corresponding behaviour has been found by Howard and LaBonte (1980,1982) on the sun. Dynamical coupling with ω is sufficient to produce recurrent episodes of reduced activity (Maunder minima) even in our simple model. We expect that similar behaviour can be found in a variety of models (cf. Childress and Spiegel 1981) and also in computations like those of Gilman (1982). Provided that the initial bifurcation leads to an oscillatory magnetic field (as in $\alpha\omega$-dynamos) and that the dynamics are sufficiently complex to allow the development of a chaotic regime, this pattern of behaviour is likely to occur. Thus Maunder minima are a characteristic feature of a wide class of nonlinear dynamos.

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DISCUSSION

GILMAN: (1) I am confused as to the role of torsional oscillation in your model. (2) Could you simulate bifurcations and strange attractor behaviour with respect to magnetic field symmetry? My own model calculations show examples of symmetry persistence over several cycles, followed by occasional switchovers to the opposite symmetry.

WEISS: (1) In our model the torsional oscillations are driven by the Lorentz force, and the variations in differential rotation then affect the generation of the toroidal magnetic field. This feedback is sufficient to produce chaotic behaviour (though the amplitude of the torsional oscillations is then embarrassingly large). (2) We have not yet attempted to model bifurcations leading to *spatial* asymmetry, but it is certainly possible to do so.

ROSNER: I would like to reemphasize that, in my view, so-called ad hoc non-linear dynamo models *are* perfectly appropriate for exploring dynamo behaviour, with appropriate approximations (viz. truncations), which retain the essentials of some physical process in question. However, I argue that it is not meaningful to ask such models to *quantitatively* account for observed correlations between, for example, activity diagnostics and rotation.

WEISS: Of course it is also important that the qualitative predictions of such models should be checked by comparison with further calculations, like those that Gilman has described.

SPRUIT: Are your aperiodic solutions always of the Maunder-minimum type, and what features of the model are responsible for producing that particular form of aperiodicity?

WEISS: No, we have obviously looked for chaotic behaviour, and we only found it for the sixth-order system in a restricted parameter range ($\nu < 1$). However, once the solutions become chaotic they exhibit Maunder minima. What is needed for this pattern is irregular behaviour such that trajectories in phase space approach the neighbourhood of the origin and are attracted towards the stable manifold of the origin before being flung out on its unstable manifold. This is likely to be found in a wide variety of models.

GIOVANELLI: (1) Are the Maunder-minimum type solutions fairly periodic, e.g. do they have similar periods between successive minima, and similar harmonics? (2) Can you give any idea as to the way the Maunder-minimum type solutions may vary with solar or stellar evolution?

WEISS: (1) Between the "Maunder minima" the model exhibits oscillations that are fairly regular but nevertheless aperiodic. The average period is well defined, at least in the limit of large dynamo number, but the phase is not preserved through the "Maunder minima". (2) As a star evolves it is spun down, and the dynamo number D decreases. In the model, this leads to more regular behaviour, eventually becoming periodic as D approaches unity. One might expect a rapidly rotating star to vary in a more chaotic manner, but I doubt whether the model can explain the lack of cyclic behaviour in younger, more rapidly rotating stars.