AFTER CORE COLLAPSE. WHAT?

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ABSTRACT. As our understanding of core collapse in globular clusters has improved through detailed computer simulations, attention has naturally turned to dynamical evolution of globular clusters after core collapse. The results of recent simulations of post-collapse cluster evolution are reviewed. An assessment is given of progress towards the goal of developing astrophysically realistic models that cover all phases of globular cluster evolution. A focus of this review is the stability of the post-collapse expansion phase to the large amplitude core oscillations first observed in the simulations of Sugimoto and Bettwieser and now confirmed by several other studies. The implications of core oscillations for the observation of post-collapse clusters are discussed.

1. INTRODUCTION

Detailed computer simulations of globular cluster dynamical evolution in the past 15 years have firmly established the inevitability of the core collapse process (Inagaki, this volume) which drives cluster core radii to very small values ($< 10^{-2}$ pc) while the central density increases to extremely high values ($> 10^8 M_\odot$ pc^{-3}). At the same time, the central surface brightness cusps expected to result from core collapse have been convincingly detected in a substantial number of clusters (Djorgovski and King 1986; Djorgovski, this volume; Lugger *et al.* 1987a and this volume). It is also becoming clear that core collapse has important implications for stellar evolution in globular clusters (Bailyn *et al.*, this volume; Lee 1986a). Following up a line of inquiry initiated by Hénon 25 years ago, a number of investigations of post-collapse evolution have been carried out in the past three years. Under the dense conditions that result from core collapse, rates of binary formation by two-star tidal interactions and three-star interactions are substantially enhanced. It is generally thought that core collapse is halted and cluster cores undergo a post-collapse expansion due to energy release from hard binaries that interact with

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single stars and each other (Hut 1985). Since it seems likely that as many as 25% of all Galactic globular cluster have already undergone core collapse based on theoretical (Lightman 1982, Cohn and Hut 1984) and observational (Djorgovski and King 1986) grounds, the dynamical evolution of globular clusters in this phase is of great interest.

Heggie (1985) presented an excellent summary of the status of studies of dynamical evolution of globular clusters after core collapse at IAU Symposium No. 113, two years ago. The major uncertainty at that time was the *stability* of the post-collapse phase. Sugimoto and Bettweiser (1983) had observed the development of large-amplitude nonlinear core oscillations in their fluid dynamical simulations of cluster evolution that had not yet been seen in any other study. Two important developments have since occurred concerning these 'gravothermal' oscillations. First, core oscillations of quite similar character have now been observed in direct Fokker-Planck simulations of cluster evolution (Cohn and Hut 1985; Cohn *et al.* 1986; Cohn, Hut, and Wise 1987) and fluid dynamical simulations based on the Fokker-Planck equation (Heggie and Ramamani 1986). Second, Goodman (1987) has performed a stability analysis which convincingly demonstrates that the core oscillation instability is intrinsic to the Fokker-Planck model for large-N systems with three-body binary heating.

Core oscillations greatly complicate the study of cluster evolution since the computational time steps that must be used to track oscillations are $\sim 10^4 - 10^5$ times smaller than would otherwise be required. Consequently, there are many uncertainties and unanswered questions concerning core oscillations. This paper will review recent progress towards understanding post-collapse evolution and the major areas in which more work is needed.

2. BINARIES AND CORE COLLAPSE

The pioneering work of Hénon (1961, 1965, 1975) indicated that a central energy source could halt core collapse and drive a subsequent expansion. Two possible energy sources have been suggested: a massive central black hole (Shapiro 1977) and a centrally concentrated population of hard binaries (Heggie 1975, Hills 1975). The presence of massive black holes at the centers of globular clusters appeared, at one time, to present an attractive explanation of observed X-ray emission from clusters (Silk and Arons 1975, Bahcall and Ostriker 1975). However, since there is now strong evidence that the 10 known high luminosity globular cluster X-ray sources are neutron star binaries (Grindlay 1981), there is little evidence for massive black holes in clusters. Instead the X-ray data provide direct evidence for the presence of binaries (Grindlay, this volume).

Three possible mechanisms have been considered for formation of binaries in clusters: primordial formation, two-body tidal capture (Fabian, Pringle, and Rees 1975; Press and Teukolsky 1977), and

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three-body interactions (see Hut 1985). While most simulations of cluster evolution with binaries have concentrated on only one of these mechanisms, the comprehensive, though highly idealized, simulations of Stodółkiewicz (1985) included binary formation by both tidal capture and three-body interactions. The investigations of the evolution of clusters of *identical main sequence* stars by Ostriker (1985) and Hut and Inagaki (1985) suggest that binaries produced by tidal capture should reverse core collapse before the core density becomes high enough to form binaries by three-body interactions. However Stodółkiewicz (1985) and Lee (1986b and this volume) find that three-body binary formation will occur at significant levels when degenerate remnants (either white dwarfs or neutron stars) are present. In the latter study, which included a primordial degenerate component, three-body interactions are found to be the *dominant* binary formation mechanism when the degenerate population is sufficiently large. Thus it is important to carry out detailed simulations that consider both tidal capture and three-body binary formation.

3. RECENT RESULTS

3.1 Goodman's Stability Analysis

Goodman (1987) has performed a stability analysis of an idealized model of a cluster of identical stars undergoing core expansion due to energy input by binaries produced by three-body interactions. He finds that for total star number $N < 7\times10^3$, the model is stable, while for $N > 4\times10^4$, the model is unstable. In between these two limits, the model is overstable. Since N substantially exceeds 4×10^4 for globular clusters, they are predicted to be unstable. Goodman's results support Bettwieser's (1985) interpretation of the oscillations as being a manifestation of the same 'gravothermal' instability that accounts for the original core collapse event. Even with a central energy source, clusters with too high a degree of central concentration are unstable.

Goodman's (1987) work, which is essentially analytic and quite rigorous in nature, lays to rest any lingering suspicions that the core oscillations found in cluster evolution simulations might merely be due to instabilities in the *numerical methods* used. A more fundamental question — and thus even more difficult to answer — is whether the fluid dynamical and direct Fokker-Planck models, for which the core oscillation instability is intrinsic, accurately represent real globular clusters. This issue is returned to in §3.3 below.

3.2 Direct Fokker Planck Calculations

3.2.1 The Method

As discussed at IAU Symposium No. 113, the direct Fokker-Planck method is well suited for simulating post-collapse dynamical evolution (Cohn 1985; Ostriker 1985). This method works with continuous stellar distribution functions in energy-space and thus does not treat the discreteness of individual stars. Binary formation must be explicitly included; three-body binary formation does not naturally occur as in a direct N-body simulation (c.f. McMillan and Lightman 1984a,b). The direct Fokker-Planck method has been extended to include three-body binaries by Cohn, Hut, and Wise (1987) and to include tidal-capture binaries by Statler, Ostriker, and Cohn (1986 and this volume). Three-body binaries are treated by adding a heating term to the Fokker-Planck equation which represents the energy input due to superelastic scatterings of singles by hard binaries. An instantaneous binary 'burning' approximation is adopted, i.e. binary formation is assumed to be immediately followed by energy input into the cluster and ejection of the binary from the cluster. The treatment of tidal capture binaries is much more complex and includes a separately tracked binary component, detailed tidal capture binary formation rates, and binary-single and binary-binary interaction rates.

3.2.2 Time Step Size and Oscillations

Initially, the codes were run with a time step selection algorithm that limited the fractional change in central density per time step to a small value (e.g. 5%). This resulted in time steps nearly 10^5 times longer than the central relaxation time t_{ro}, in the simulations of cluster evolution with three-body binaries reported by Cohn (1985). Smooth, monotonic reexpansion of the cluster core was found for both three-body binaries (Cohn 1985) and tidal capture binaries (Ostriker 1985). Following a suggestion by E. Bettweiser, the three-body and tidal capture codes were run with time steps limited in size to t_{ro}. In both cases, large amplitude core oscillations are observed (Cohn et al. 1986; Cohn, Hut, and Wise 1987; Statler, Ostriker, and Cohn 1986). Some representative results from these studies are included here.

3.2.3 Results for Three-Body Binaries

Figure 1 illustrates the time evolution of the central density from a simulation of a cluster of identical stars including three-body binary heating (Cohn, Hut, and Wise 1987). Time is measured in units of the initial half-mass relaxation time, t_{rh}, which is of order 10^8 - 10^9 yr for the more centrally concentrated Galactic globular clusters. Central density is measured in units which correspond to approximately 10^4 $M_\odot$ pc^{-3}. Since three-body binaries only form when extremely high central densities ($\sim 10^{10}$ $M_\odot$ pc^{-3}) are achieved, the pre-collapse evolution is the same as that of a cluster not containing binaries. Energy input from the binaries halts the core collapse at $t = 15.7$ t_{rh} and a brief expansion phase begins. However the expansion is highly unstable, as predicted by Goodman's (1987) analysis and nonlinear oscillations rapidly develop (Fig. 1a,b). The oscillations are nonsinusoidal in character with more of the time spent near the density

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minima than the maxima. The expansion phases are considerably faster than is the case when the instability is artificially damped using large time steps (Cohn 1985). For example, the density minimum which occurs near $t = 18\ t_{rh}$ is over a factor of 100 lower than the corresponding value in the large time step run. During the last oscillation cycle of the simulation, the cluster spends most of the time near the density minimum. Although the core repeatedly undergoes brief collapse phases during which the central density returns the large value achieved during the first core collapse, the fraction of time spent at these high densities decreases in time. Thus on a time-averaged basis, the *effective* core expansion rate is greatly enhanced over the artificially smoothed result for the large time step run.

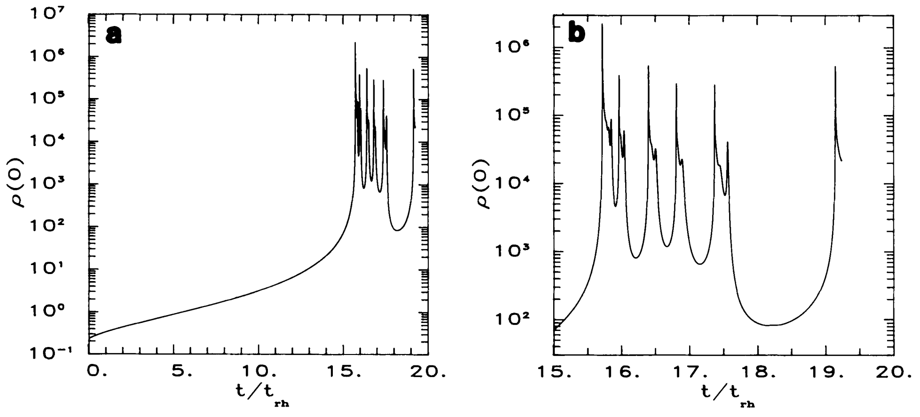

Fig. 1. Evolution of central density from a simulation including binaries produced by three-body interactions. (a) The entire simulation. (b) The core oscillation phase.

Some additional insight into the nature of the oscillations is provided by replacing physical time by a timelike parameter τ that measures the elapsed number of central relaxation times since the start of the simulation, $\tau \equiv \int_0^t dt'/t_{ro}(t')$. The parameter τ is analogous to optical depth in radiative transfer theory. Figure 2 illustrates the evolution of central density as a function of τ. The oscillation appears very much more regular and symmetric with respect to the mean density than when plotted using physical time. This regularity reflects the fact that the time scale of the oscillation is determined by the instantaneous value of the central relaxation time t_{ro}. Since t_{ro} varies inversely with central density, the fraction of an oscillation cycle during which the cluster hovers near the density minimum increases as the amplitude of the oscillation increases as can be seen from Figure 1b.

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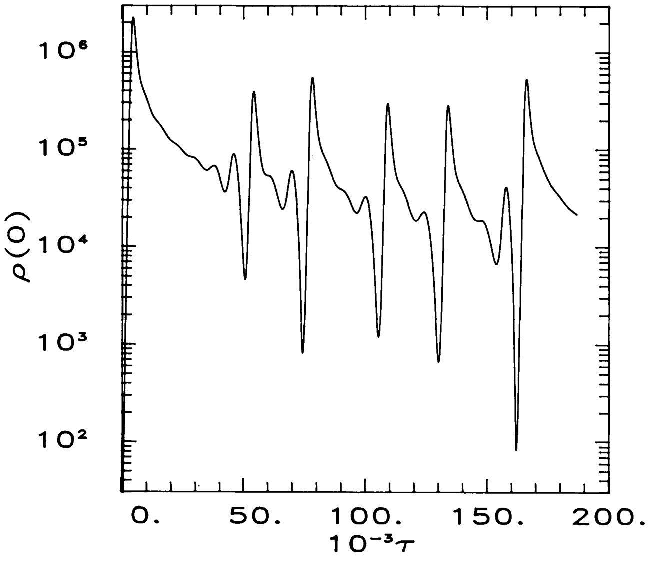

Fig. 2. Regularized core oscillations for the same simulation as in Fig. 1. Central density is plotted versus elapsed number of central relaxation times, τ.

Figure 3 illustrates the time evolution of the core radius in addition to radii containing fixed fractions of the cluster mass. The unit of radius is approximately 1 pc. The outermost radius shown, which contains 10% of the cluster mass, is clearly affected by core oscillations. In contrast, the half-mass radius increases smoothly and monotonically during this period.

Goodman (1987) has predicted that core oscillations will saturate when the core contains 1-2% of the total cluster mass at maximum expansion with a ratio of core radius to half-mass radius of 0.02. Indeed, at maximum expansion during the final oscillation shown in Figure 3, the core radius contains exactly 1% of the cluster mass and $r_c/r_h = 0.02$. While further integrations are necessary to determine whether the core oscillation amplitude has in fact saturated, the apparent agreement between the direct Fokker-Planck calculations and Goodman's (1987) analytic analysis, which is based on a fluid dynamical model, is strikingly good. Thus, there is a high degree of confidence that the oscillations, observed in both direct Fokker-Planck and fluid dynamical calculations of cluster evolution with three-body binary formation, are intrinsic to the models and not a numerical artifact.

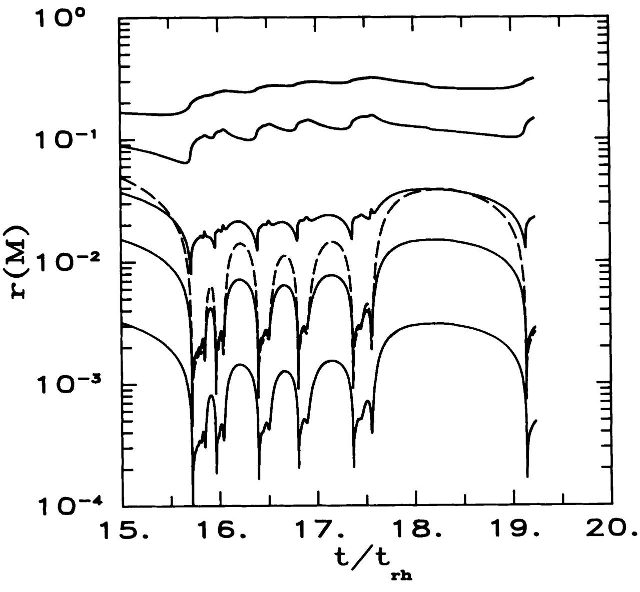

Fig. 3. Evolution of core radius (dashed line) and radii containing 0.001%, 0.1%, 1%, 5%, and 10% of the total cluster mass (solid lines), for the same simulation as in Fig. 1.

3.2.4 Results for Tidal Capture Binaries

While Goodman (1987) only explicitly considered the case of binary formation by three-body interactions (for technical reasons) he argues that his results do not depend on the specific energy input mechanism but instead on the structure of the cluster. He predicts that any expanding equilibrium with $r_c/r_h \lesssim 10^{-2}$ will be unstable to core oscillations. Figure 4 illustrates the development of core oscillations in a cluster simulation including binaries formed by the two-body tidal capture process, bearing out Goodman's (1987) prediction. Statler, Ostriker, and Cohn (1986 and this volume) have reported the results of large time step runs ($\gg t_{ro}$) with this code; the results shown here are for small time steps ($\sim t_{ro}$) which permit core oscillations to develop. There is an initial damping of the instability immediately after the reversal of core collapse, but this is followed by a linear growth phase leading to nonlinear oscillations. During the immediate post-collapse period, the ratio of core radius to half-mass radius hovers near the critical value of 10^{-2}. As can be seen in Fig. 1 of Statler, Ostriker, and Cohn (this volume) r_c/r_h *decreases* following core collapse in the tidal capture binary formation case, so that even if a cluster starts the post-collapse phase on the stable side of the stability criterion, it is expected to cross over to the unstable side. As for the three-body binary formation case, the density hovers near the minimum value for most of an oscillation cycle. Thus, the effective expansion rate is again faster than for smoothed reexpansion.

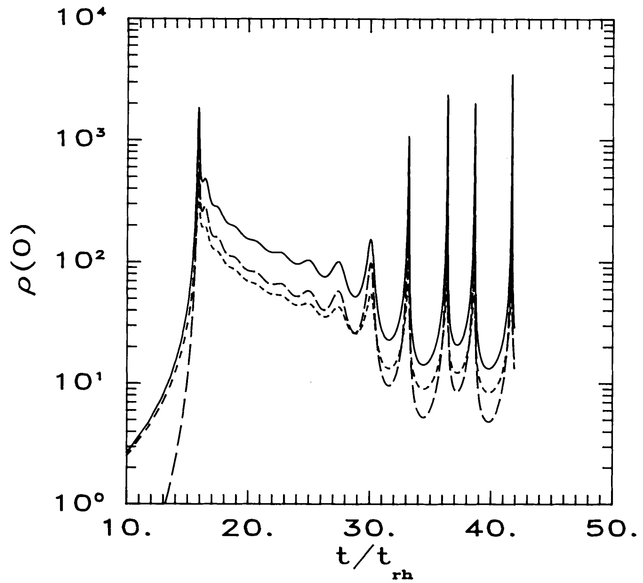

Fig. 4. Evolution of central density from a simulation including binaries formed by tidal capture. Results are shown for total mass density (solid curve), single stars (short dashes), and binaries (long dashes).

3.3 Other Recent Results

Heggie and Ramamani (1986 and private communication) have recently obtained results using a fluid dynamical code that are quite similar in character to the large amplitude oscillations obtained with the direct Fokker-Planck method (Fig. 1a,b). While the the agreement is not exact, it is extremely good. This provides another indication that at least for clusters of identical stars, fluid dynamical analogues are able to reproduce the results of the more exact (and thus more computationally expensive) direct Fokker-Planck method. A common property of these two approaches is a lack of the discreteness present in both real globular clusters and direct N-body simulations.

Neither Inagaki's (1986) direct N-body simulations of cluster evolution nor McMillan's (1986) hybrid N-body/Fokker-Planck simulations

4.2 Observation of Post-Collapse Clusters

As discussed in §3.2, gravothermal oscillations accelerate the *effective* expansion rate following core collapse. In the case of binaries produced by three-body interactions, the core radius reaches a value of nearly 0.04 pc at maximum expansion (Fig. 3) which subtends an angle of 0.8" for a typical cluster near the Galactic center and is thus marginally resolvable by ground-based observations. Djorgovski and King (1986) find that NGC 6397, one of the nearest globular clusters, has a collapsed core. At the distance of this cluster (2.2 kpc) 0.04 pc subtends an angle of 3.8" and is easily resolvable. In fact, Lugger *et al.* (1987b) find that the central power law profile of this cluster flattens out at a radius of order 2". Observations of the central structure of globular clusters, with the highest angular resolution possible are needed to test the predicted effective size of post-collapse cluster cores. While additional ground-based studies — particularly those that take advantage of sites with excellent seeing — are needed, Hubble Space Telescope observations of cluster cores will be of critical importance.

If the most massive stars in a cluster are nonluminous remnants while most of the luminosity in due to less massive giants or horizontal branch stars, then the slope of the central surface brightness cusp in a post-collapse cluster can be significantly flatter than the value of $d \ln S/d \ln r = -1$ expected in a cluster of identical stars. This point is illustrated by the simulations of Lee (1986b), for which a ratio of the mass of a nonluminous star to that of a luminous star of 1.9 produces a slope of -0.6. This slope is quite close to that measured for the central surface brightness cusp in M15 by Lugger *et al.* (1987a and this volume). There is considerable potential for useful interaction between realistic dynamical models and the improved data base of cluster structure observations becoming available.

4.3 Prospects for Future Work

Ideally, an evolving, dynamical model for a globular cluster would include all of the following physical properties and mechanisms:

- realistic stellar mass spectrum
- stellar evolution and mass loss
- tidal capture binary formation
- physical collisions between stars and mergers
- 3-body binary formation
- binary-single and binary-binary interactions
- velocity distribution anisotropy
- cluster rotation
- large angle gravitational scattering
- discreteness of the stellar distribution
- tidal cutoff
- tidal shocks
- and make sure the time steps are small!

show the large amplitude 'gravothermal' oscillations seen in the direct Fokker-Planck and fluid dynamical simulations. However, small amplitude core oscillations, attributable to particular binary formation and ejection events, are seen. These authors interpret their findings as indicating that the presence of discreteness prevents the development of gravothermal oscillations. Goodman (1987) notes that Inagaki (1986) included at most 3000 stars in his simulations, which is below the critical number for instability, and that McMillan's (1986) hybrid simulations only extend for a short period following core collapse which corresponds to the linear growth phase of the instability. Thus, Goodman argues that large amplitude oscillations would not have been *expected* in either simulation. Recently, Makino, Tanekusa, and Sugimoto (1986) have reported gravothermal oscillations in an N-body simulation. Further investigation of this issue is clearly needed.

As noted in §2 above, Lee (1986b and this volume) has carried out direct Fokker-Planck simulations of globular cluster evolution including a primordial degenerate remnant population. This work may be regarded as a first step towards a simulation of cluster evolution beyond core collapse that includes both an astrophysically realistic stellar mass spectrum and stellar evolution. These simulations use a large time step and thus do not allow for the possibility of gravothermal oscillations. Lee (1986a) has also carried out direct Fokker-Planck simulations in which successive stellar mergers take place, as a result of close encounters, leading to the production of massive stars (up to 22 $M_\odot$). Subsequent stellar evolution mass loss provides an energy input into the cluster which plays a role similar to binary heating. Heating by binaries formed by three-body interactions is also included, and it is energy input from massive binaries that ultimately reverses core collapse. The post-collapse phase is, however, driven by energy input from stellar evolution mass loss.

4. DISCUSSION

4.1 Gravothermal Oscillations

'Gravothermal' oscillations of globular cluster cores undergoing binary driven expansion are now well established by direct Fokker-Planck and fluid dynamical simulations, as well as by a rigorous stability analysis based on the fluid dynamical model. Some lingering uncertainty about the role that discreteness might play in suppressing the instability remains to be cleared up by further investigation. The general convergence of the results from different investigations, since IAU Symposium No. 113, is an encouraging sign that progress is being made towards a full understanding of the post-collapse life of globular clusters.

Each of these items has been considered in a particular cluster simulation and some simulations (most notably that of Stodółkiewicz 1985) have included a good number — but not all — of these. That this statement can be made indicates substantial progress. However, it is often the case in the study of a complex physical system like a star cluster that the joint effect of two physical processes is quite different than that of either individual process. In the present age of supercomputers and large-scale computation it does not seem unreasonable to expect a fully inclusive simulation of globular cluster evolution in the not too distant future.

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DISCUSSION

KING: Some years ago Retterer provided, in the Kolmogorov-Feller equation, a mechanism for dealing with large-angle scattering. Why isn't it used?

COHN: Jeremy Goodman's treatment of large angle scattering in an energy-space formulation can be regarded as a follow-up to Retterer's very important work. I'll let Jeremy comment on this.

GOODMAN: I have done collapse calculations with the KF equation to describe large-angle encounters. I was disappointed to find that my results were very similar to those obtained by Cohn with the Fokker-Planck equation.

LEE, H. M.: Could you make a short comment on the reason why we expect gravothermal oscillations for highly concentrated clusters?

COHN: I believe that it is for very much the same reason that highly concentrated clusters without central energy sources are subject to the gravothermal instability leading to core collapse: the cores of these concentrated clusters have negative specific heat.

DJORGOVSKI: Can you tell us what is the behavior of the profile of an oscillating PCC [post core collapse] cluster? Does it ever look like a very concentrated King model?

COHN: This is something that I will have to check. While we did not save radial structure information for the tidal capture binary runs, we do have it for the three-body binary runs. When I analyzed the profiles for the large time step runs I presented at IAU Symposium No. 113, I found that the late time profiles did resemble very extended King models with $W_o \sim 17$. I suspect that this will also be true for the oscillating case.

STATLER: I'd like to amplify one point that you touched on. It's important that the different calculations that now show oscillations have "zero-order" solutions with quite different physics. In those with 3-body binaries, heat is deposited locally in the core and conducted outward, while in those with tidally captured binaries, much of the energy is deposited in the vicinity of the half-mass point and conducted inward. So the presence of the oscillations is independent not only of the microphysics of the heating, but also on the global characteristics of the background solutions

COHN: To amplify on your amplification, I think that it is important to run simulations for as many different treatments of binaries as possible, using small time steps, in order to determine exactly what conditions lead to oscillations. I suspect that Jeremy Goodman's criterion that oscillations occur when the ratio of core radius to half-mass radius is less than $\sim 10^{-2}$ is quite generally valid.

BLECHA: How do you form the binaries? Is it really necessary to go to a core radius of 10^{-6} pc before the energy input from binaries prevents the further collapse?

COHN: The results that I presented were from two sets of simulations that separately treated binaries formed by three-body interactions and binaries formed by tidal captures. In the three-body case, the core radius drops to less than 10^{-3} pc before core collapse is reversed. In the tidal capture case, core collapse is reversed at a core radius of about 10^{-2} pc.

MCMILLAN: The results of runs with my current hybrid code cannot be compared directly with Jeremy Goodman's criterion for gravothermal oscillations, as N is not defined in my work. There are other possible reasons why I have not yet seen gravothermal oscillations in my runs. If they really do exist, and are not just a consequence of the continuous description of the cluster employed in the fluid and Fokker-Planck approaches, it is possible that I did not run my code for long enough to see them. It may also be that my resolution of the outer regions of the cluster (outside fifty or so core radii) was insufficient to distinguish the effect. Both these possibilities are being investigated.

COHN: Thank you for the clarification. I am looking forward to learning of the results of your current simulations.

NEMEC: Hills and Day (~1978) predict that only a few collisions occur during the lifetime of a loose globular cluster. It follows that the 50 blue stragglers, which are relatively massive and therefore possibly binaries, found in NGC 5466 are primordial in their origin. What is your opinion on the possibility that massive blue stragglers are formed by collisions in low central concentration globular clusters?

COHN: A loose globular cluster does not provide a favorable environment for forming binaries by the tidal capture process.

INAGAKI: Your result shows that the mass fraction inside the core radius is ~ 10^{-4} - 10^{-5} in the case of three-body binaries. This means that the number of stars contained in the core is 10 - 100. Do you think that the Fokker-Planck approximation is still valid?

COHN: Of course the Fokker-Planck approximation, which is based on large-N asumptotics, breaks down for sufficiently small N. Steve McMillan has found that the self-similar core collapse solution given by direct Fokker-Planck calculations is valid until about 30 stars remain in the core. At this point, a three-body interaction results in the formation of a binary in Steve's simulations. We have modified the direct Fokker-Planck approach to model explicitly the energy input into the core due to three-body formation and interaction. Further comparisons of our results with those of direct N-body and hybrid N-body/Fokker-Planck simulations will be necessary to establish the validity of our modified Fokker-Planck approach (in an ensemble-averaged sense) when the number of stars in the core becomes small.