THE INVESTIGATION OF LUNAR LIMB STRUCTURE BY MEANS OF STELLAR OCCULTATIONS

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It has long been recognized that the analysis of occultation traces from point source stars might provide a means of investigating the structure of the lunar limb on a remarkably small scale, certainly of tens of meters, possibly on a scale of meters.

The routine process of analysis of such an occultation trace produces a curve fitted to the standard model for a point source, in which the observed rate of fringe passage is matched to that computed from the rate and position angle of the relative motion of the moon with respect to the star background and the position angle of the point at which the occultation occurs. If θ_v is the position angle towards which the relative motion of the lunar center takes place, θ that at which the occultation occurs, and $\psi = \theta_v - \theta$, then the predicted rate of the lunar limb perpendicular to itself at this point is

$$R_P = V \cos \psi$$

where V is the velocity of the lunar center.

The rate derived from the trace is R_D and is not, in general, equal to R_P. We can attribute the difference to the presence of a slope inclined at an angle φ' to the horizontal, given by

$$R_D = V \cos (\psi - \varphi')$$

where we adopt the convention that φ' is positive if R_D is numerically greater than R_P. This means that the slope is turned in such a direction as to make the relevant section of lunar limb more nearly perpendicular to the direction of motion relative to the star background than it would be if the limb were level.

The determination of R_D is naturally less precise the more noisy the observed trace, and ceases to be meaningful if, for any reason, the observed trace is not a good fit to the model. The goodness of fit depends not only on the noise, but also on the possible presence of close companions or on the occurrence of a sensible angular diameter. Its sensitivity to the adopted color temperature of the star has been tested and appears to be slight.

Since the investigation is essentially statistical the results may be invalidated by the consideration of too small a number of cases. The present discussion is intended to point the way to more complete investigations which may become possible when a more adequate body of data is available. From our file of results 35 traces have been selected which have rms errors of noise and rms errors of fit always less than 10% for at least one of the two, and in most cases for both. The median for each is about

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7%. Details of these traces will be published elsewhere. They represent the best and largest body of slope determinations so far available.

The slopes determined from the equations given above are plotted in Figure 1. One missing point, at $\psi = 3\overset{\circ}{.}7$, gave $R_D/R_P = 1.0079$, no doubt owing to observational error, so that in that case there is no solution for φ'. It will be noticed that for very small values of ψ, the values of φ', with the exception noted above, are all negative, and the scatter is very large. For values of ψ between 10° and 41° the scatter is large, the values of φ' mostly negative, with a trend upwards. As it happens, no observations were made with ψ between 41° and 55°, and this may turn out to be a very interesting

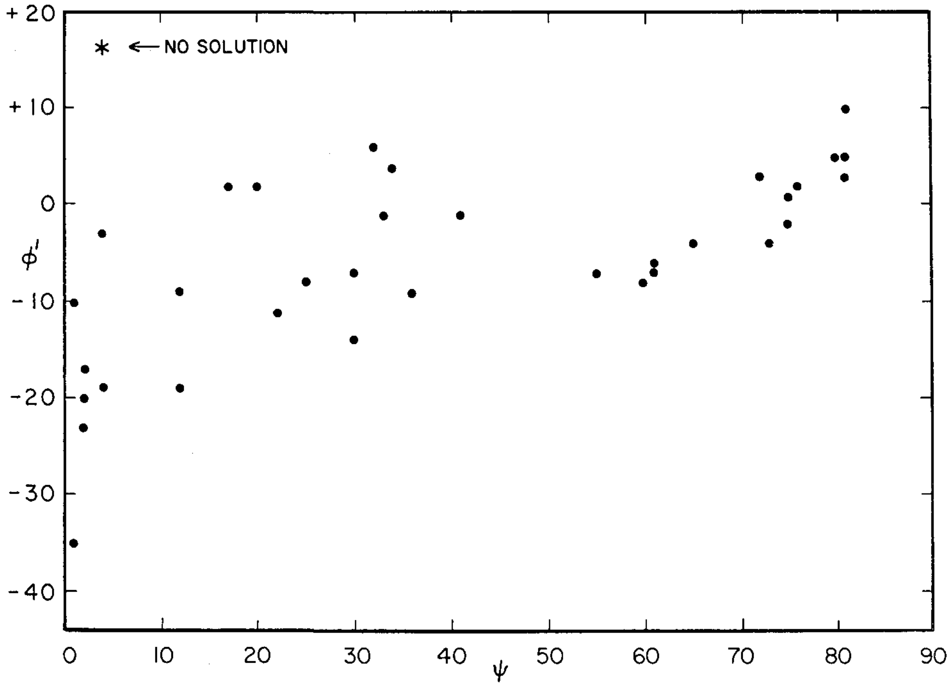

Fig. 1.

range. Then on to 81° there is a very much reduced scatter about the general trend, which is markedly upwards towards positive values of φ'.

Our purpose is to explain these features which may be valuable for the investigation of the lunar limb structure. At first sight what we see seems repugnant to good sense, since the diagram is supposed to be a statement about the properties of the lunar limb, and we ought to expect that what we find out about the lunar limb should not be dependent on the angle of incidence of the star. The results might seem to indicate that we have not been successful in separating effects associated with the limb from the particular circumstances of occultation. These fears do not seem to be justified.

Let us construct a model of the lunar limb, highly simplified it may be granted,

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which consists of a certain proportion of level limb, and a certain proportion of slopes all having a certain inclination φ. The dimensions of these slopes are important and we discuss this point later. Suffice it to say that for observations in visual light we are thinking of slopes with lengths of 20 m or more. The assumption of a constant angle of slope may not be entirely artificial: many terrestrial landscapes have this property, and the angle in question may be the angle of repose of material of which talus slopes are formed.

Now consider the slope values φ' which will be inferred from occultations taking place at various angles ψ on this model lunar limb.

First consider normal incidence. In the absence of observational error only two values of φ' are possible – zero and $-\varphi$. The first is produced when occultations occur on the level portion of the limb. All occultations on slopes cause the derived rate to be slower than the predicted rate. No positive slope values can be found until ψ exceeds φ. As ψ increases and covers the range from φ to $90°-\varphi$ we find that the inferred values of φ', in the absence of observational error are confined to the three choices, $+\varphi$, zero, and $-\varphi$. The proportion of zeros depends on the proportion of the limb which is level. The proportions in which $+\varphi$ and $-\varphi$ occur are in the ratio $\cos(\varphi-\psi):\cos(\varphi+\psi)$, that is, positive values of slope are statistically more frequent, because these correspond to an open face slope turned towards the line of incidence of the occulted star. The diagonal lengths of the slopes must be equal since there will be no asymmetry on the Moon – indeed, with the variation of circumstances, a left slope in a particular region may be the open face for one occultation, and, some time later, the closed face for another. As ψ increases throughout this range the occurrence of values of $+\varphi$ as compared with $-\varphi$ will increase. At $\psi=90°-\varphi$ and beyond, the only possible values will be $+\varphi$ and zero. An error of one per cent in the derived rate will reduce the derived slope by 8° or eliminate the solution, for $\psi=0°$, while for $\psi=30°$ the resulting uncertainty for the same proportional error is only 1°. Thus observational scatter will decrease with increasing ψ. Many of the features of Figure 1 are thus explicable, which is encouraging. There are however, features which are not entirely accounted for. Consider Figure 2b which shows the expected numbers of occultations at various values of ψ, for the Moon travelling through a star field of constant density, for the limb character shown in Figure 2a, namely, 50% level, and 50% occupied by slopes of 10 deg.

Although all the theoretical values are constant the occupancy of various parts of the diagram might give the impression of a trend. The peak positive value for large ψ is numerically equal to the minimum for small ψ. Except for small ψ, occultations on positive slopes are always more numerous than those on negative ones. These features are absent from Figure 1.

We have so far discussed slopes on the lunar limb, using this to denote features of dimensions greater than about 20 m. The lunar limb is probably also rough in the sense that each level or slope may have on it a series of features with dimensions of the order of 1–5 m, which I have got into the habit of calling crinkles.

Now from calculations made by others and myself (Evans, 1970) we know that

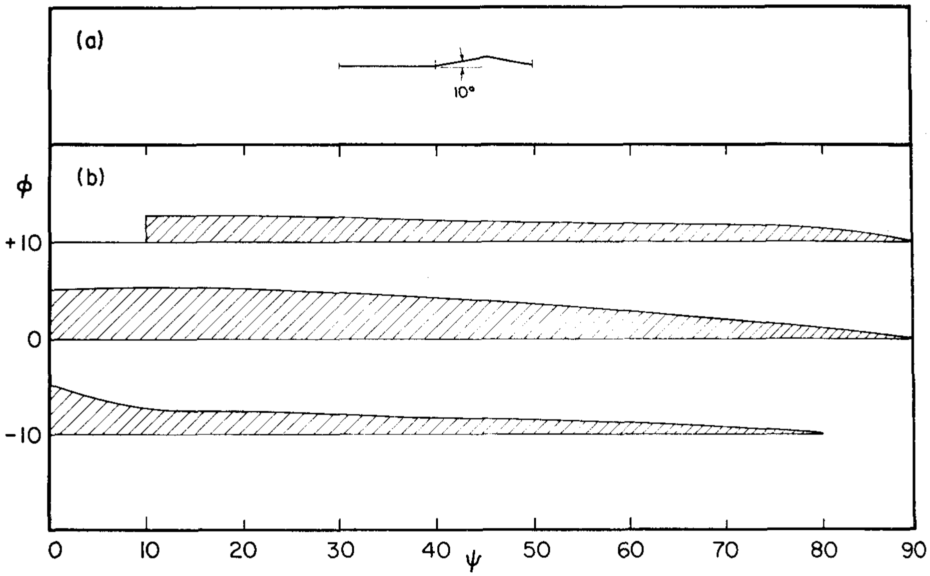

Fig. 2.

crinkles can alter the spacing between the diffraction bands, which is used to determine R_D. One can think of the shadow of the Moon cast by a star, with the diffraction bands running along the geometrical outline like the depth contours off-shore on a marine chart. What we call a slope is a deviation from circular form which is followed by the diffraction contours and this is true for dimensions larger than, say, 20 m. The depth contours follow the larger bays and capes but do not reproduce every tiny irregularity of the shore line. In the same way the crinkles on the limb of dimensions from, say, 1 m to 5 m are not reproduced in the shape of the diffraction contours. There seems to be a tendency for the presence of crinkles to expand the spacing between the fringes, which we can understand in the following general way. The The presence of an elevation on the limb puts a contribution into the diffraction integrals which is advanced (in a disappearance) with respect to the rest. Putting in a depression next to this introduces a delayed portion; the downward crinkle does not cancel the upward one, it reinforces it. Now when we compute R_D we are in effect saying how long it took to pass from, say, the second to the first maximum in the pattern, and we equate this to so many meters assuming that the spacing is normal. But if the spacing is enlarged, R_D ought to be replaced by a number KR_D where K is a number a little larger than unity, and to compute the true slope φ we should use the formula

$$\varphi = \psi - \cos^{-1}\left[\frac{KR_D}{R_P}\cos\psi\right]$$

where R_D is the value computed on the assumption of standard spacing. The deviation of the number K from unity will be a measure of the small scale roughness of the limb.

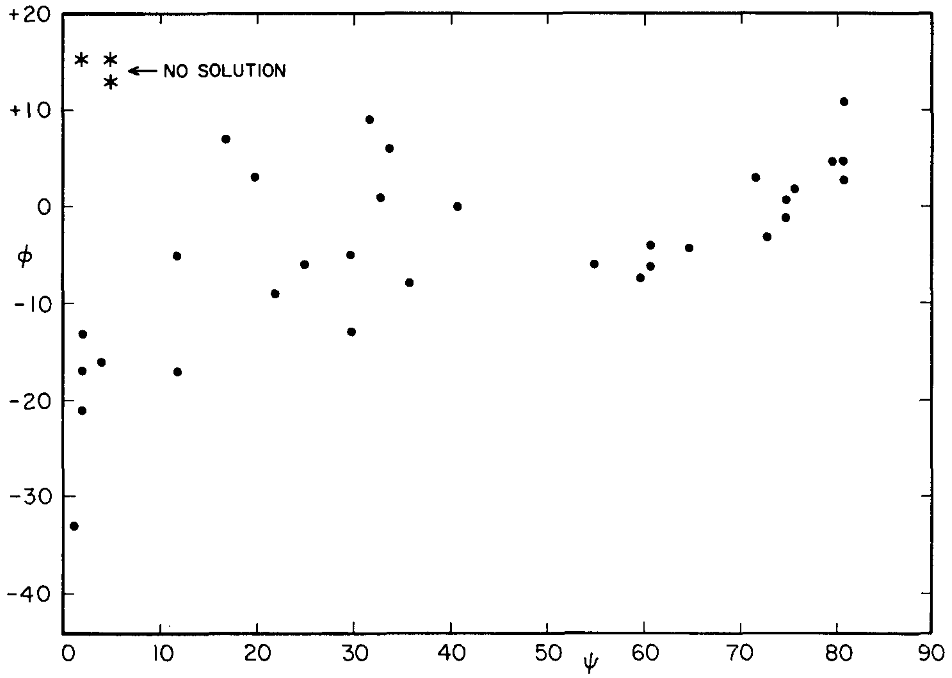

Fig. 3.

In a purely experimental way we try introducing $K=1.02$ into our body of slope data. We lose two more points at $\psi=3\overset{\circ}{.}9$ and $1\overset{\circ}{.}1$ because then there is no geometrical solution, since $(KR_D/R_P)\cos\psi$ has respectively the values 1.0118 and 1.0015, but there is no problem in attributing the small excesses over unity to experimental error. We thus arrive at Figure 3. This now has the expected further properties. There is no longer a large preponderance of negative values of φ. The maximum values for ψ near 20° are the same as those for very large ψ. No positive values of φ occur for values of ψ less than the maximum values of φ obtained.

If we plot a histogram (Figure 4) of position angles at which occultations occurred

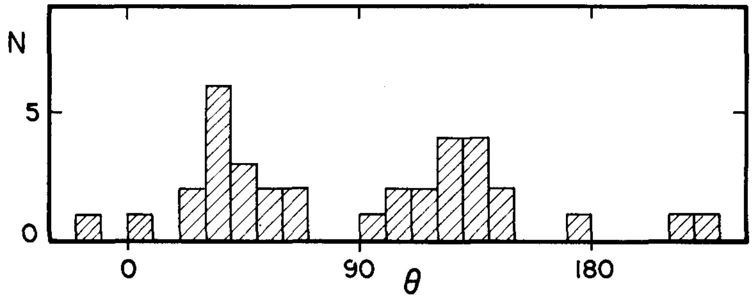

Fig. 4.

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we find that all but a few took place between 20° and 70° and between 90° and 150°. The tentative conclusion from Figure 3 is thus that the maximum slope encountered on these two sections of lunar limb was about 10° with about one third of the limb level and a roughness parameter of 1.02.

Reference

Evans, David S.: 1970, *Astrophys. J.* **75**, 589.