ON THE AGES OF ASTEROID FAMILIES

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Abstract

The ages of asteroid families are estimated by using a second order secular perturbation theory. Their ages are rather young. To explain these results we have some possible interpretations. The first interpretation is that in the theory of secular perturbation we do not include the effects of mutual perturbations of asteroids. The second is that we can regard these values as the minimum ages.

1. Solutions of the second-order secular perturbation theory.

We adopt Poincare's canonical variables:

$$p^*=\sqrt{2(L-G)}\cos(g+h), \qquad q^*=-\sqrt{2(L-G)}\sin(g+h),$$
$$r^*=\sqrt{2(G-H)}\cos h \quad , \qquad s^*=-\sqrt{2(G-H)}\sin h, \tag{1}$$

where L, G, H, g and h are Delauney's variables.
The equations of motion are as follows:

$$dp^*/dt=\partial F/\partial q^* \quad , \qquad dp^*/dt=-\partial F/\partial p^* \quad ,$$
$$dr^*/dt=\partial F/\partial s^* \quad , \qquad ds^*/dt=-\partial F/\partial r^* \quad , \tag{2}$$

where F is the Hamiltonian in which the short periodic terms have been eliminated.
The solutions of this system to the second order are given as follows (Yuasa 1973):

$$p^*=\sqrt{L}[\nu\ \cos((g_0+2/L\partial F_1^{**}/\partial\nu^2)t + \beta) + \sum_{j\ell}\sum \nu j\ell/g_0-g_{j\ell}\cos\ (g_{j\ell}t+\beta_{j\ell})], \tag{3}$$

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$$q^* = -\sqrt{L}[\nu\sin((g_0+2/L\partial F_1^{**}/\partial\nu^2)t + \beta) + \sum_j\sum_\ell \nu_{j\ell}/g_0-g_{j\ell}\sin(g_{j\ell}t+\beta_{j\ell})], (4)$$

$$r^* = \sqrt{L}[\mu\cos((-g_0+2/L\partial F_1^{**}/\partial\mu^2)t + \gamma) + \sum_j\sum_\ell \mu_{j\ell}/g_0+g_{j\ell}\cos(f_{j\ell}t+\gamma_{j\ell})], (5)$$

$$s^* = -\sqrt{L}[\mu\sin((-g_0+2/L\partial F_1^{**}/\partial\mu^2)t + \gamma) + \sum_j\sum_\ell \mu_{j\ell}/g_0+f_{j\ell}\sin(f_{j\ell}t+\gamma_{j\ell})], \quad (6$$

where

$$\partial F_1^{**}/\partial\nu^2 = m_5'[(B_1-A_1/4)\{2\nu^2+4(\xi_{55}^2+\xi_{56}^2)\} + (B_7+A_3/2)(\mu^2+\eta_{55}^2+\eta_{56}^2)$$
$$+(B_{13}+A_5/4)(\eta_{55}\mu_{55}+\eta_{56}\mu_{56})+2(B_{15}-A_6/8)(\xi_{55}\nu_{55}+\xi_{56}\nu_{56})$$
$$+B_2(\nu_{55}^2+\nu_{56}^2)+B_8(\mu_{55}^2+\mu_{56}^2)]+m_5'^2D_1, \quad (7)$$

$$\partial F_1^{**}/\partial\mu^2 = m_5'[(B_7+A_3/2)(\nu^2+\xi_{55}^2+\xi_{56}^2)+(B_4-A_3/4)\{2\mu^2+4(\eta_{55}+\eta_{56})$$
$$+2(B_{11}-A_5/8)(\mu_{55}^2\eta_{55}^2+\mu_{56}^2\eta_{56}^2)+B_6(\mu_{55}^2+\mu_{55}^2)$$
$$+B_9(\nu_{55}^2+\nu_{56}^2)+B_{17}(\xi_{55}^2\nu_{55}^2+\xi_{56}^2\nu_{56}^2)] + m_5'^2D_3. \quad (8)$$

The constants g_0, ξ_{55}, ξ_{56}, η_{55} and η_{56} are determined by the disturbing planets and the semi-major axes of the asteroids. And the constants A_i, B_i, and D_i are determined only by the semi-major axes of asteroids. Furthermore ν, μ, β, γ are the constants of integration and their values depend on the initial conditions of asteroids. Other constants $g_{j\ell}$, $\nu_{j\ell}$, $f_{j\ell}$, $\mu_{j\ell}$, $\beta_{j\ell}$ and $\gamma_{j\ell}$ are determined only by the disturbing planets. And m_5' is the mass of Jupiter.

2. The longitude of the proper perihelion Π_1 and the longitude of the proper node θ_1.

The quatities $(g_0+2/L\partial F_1^{**}/\partial\nu^2)t+\beta$ in the equations (3), (4) and and $(-g_0+2/L\partial F_1^{**}/\partial\mu^2)t+\gamma$ in the equations (5), (6) are called the longitude of the proper perihelion and the longitude of the proper node, respectively. Usually they are indicated by Π_1 and θ_1.

The difference of the second order solution and the first order solution appears in Π_1 and θ_1. In the first order theory, $\Pi_1=g_0t+\beta$ and $\theta_1=-g_0t+\gamma$, therefore

$$\Pi_1+\theta_1=\beta+\gamma. \quad (9)$$

On the other hand in the second order theory

$$\Pi_1+\theta_1=2/L(\partial F_1^{**}/\partial\nu^2+\partial F_1^{**}/\partial\mu^2)t + \beta + \gamma. \quad (10)$$

The asteroids belonging to families have similar initial conditions, so

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that they have similar β and γ. By this reason in the first order theory the quantity $\Pi_1 + \theta_1$ must concentrate around a value in each asteroid family. But the observational data indicates that $\Pi_1 + \theta_1$ does not concentrate but rather equally distributes from 0 to 2π (Table 1. (Brouwer 1950)).

Table 1. Distribution of $(\Pi_1 + \theta_1)/2\pi$

	0.0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1.0
Themis	4	7	3	9	6	5	3	4	5	7
Eos	8	8	10	6	2	1	5	5	2	11
Coronis	2	5	2	2	3	5	3	7	1	3
Maria	4	1	2	3	1	1	0	2	1	2
Phocaea	0	2	2	5	2	5	0	2	3	2
Flora	11	9	8	13	14	12	11	5	8	11

In the second order theory $\Pi_1 + \theta_1$ has the term depending on t. And we can interpret the non-concentration of $\Pi_1 + \theta_1$ in each asteroid family.

3. Ages of asteroid families.

In the solutions of the second order theory, we can see the time dependence of $\Pi_1 + \theta_1$. In order to do so, we plot $\Pi_1 + \theta_1$ against the values $-(\partial F_1^{**}/\partial\mu^2 + \partial F_1^{**}/\partial\mu^2)$ for Coronis, Flora, Eos and Themis families. The results are shown in Figures 1-4. In these figures a remarkable and interesting feature can be found. If we shift vertically the left-side points between 0 and 2π by 2π upwards and right-side points by 2π downwards in necessary case, all the points seem to lie on a straight line. From the slope of these lines we can estimate the ages of asteroid families (Kozai and Yuasa 1974). Their values are 3.7×10^6 years, 9.98×10^5 years, 6.3×10^5 years and 4.6×10^5 years for Coronis, Flora, Eos and Themis families respectively.

4. Discussion.

The ages of asteroid families obtained in the preceding section are rather young. To explain this result, we have some possible interpretations. The first interpretation is that in the theory of secular perturbation we do not include the effects of mutual perturbations of asteroids. If the mutual perturbations of asteroids had been considerably large, there would be a possibility that old families have been destroyed and only young families have remained. The second interpretation is that we can regard the values obtained in the preceding section as the minimum ages of the asteroid families. In fact we can shift the points in Figures 1-4, not 2π but $2n\pi$. Then the slope of the straight lines can become more steep and we may be able to get more large values of the ages for the asteroid famlies. But to execute this, we need to construct a more

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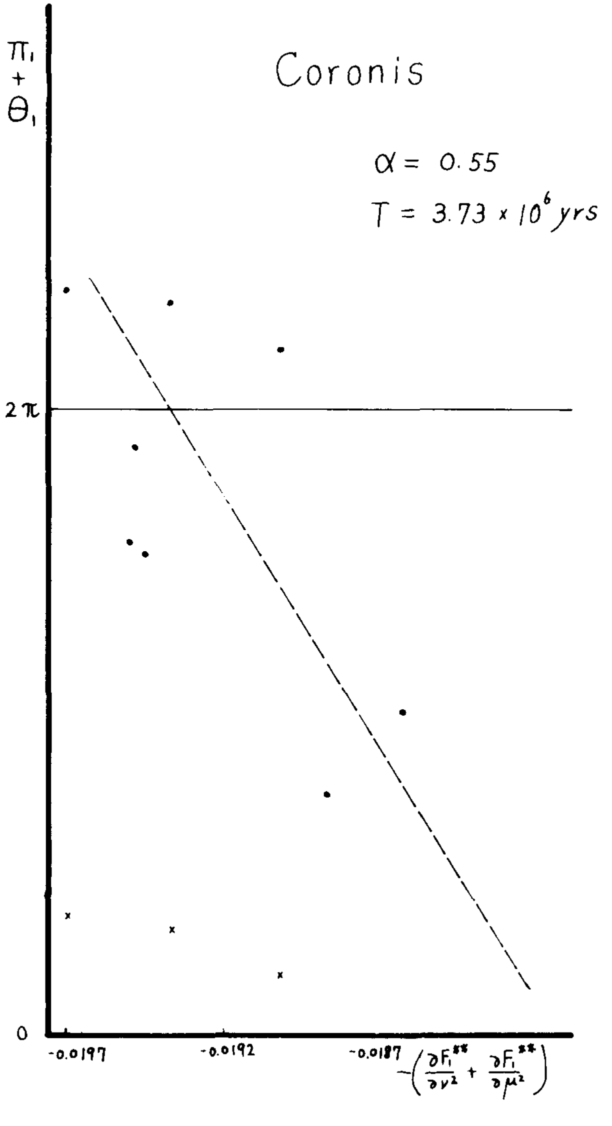

Figure 1. $\Pi_1+\theta_1$ against the coefficient of time for Coronis family. Points x are shifted vertically by 2π upward.

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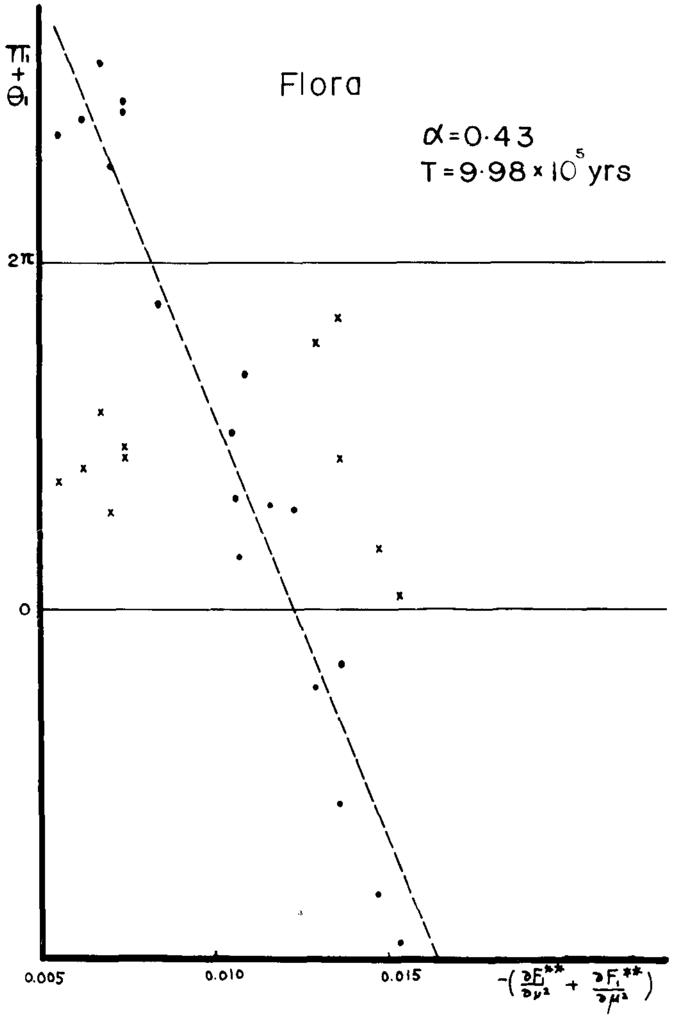

Figure 2. $\Pi_1+\theta_1$ against the coefficient of time for Flora family. Points x are shifted vertiaclly by 2π upwards in the left-side and downwards in the right-side.

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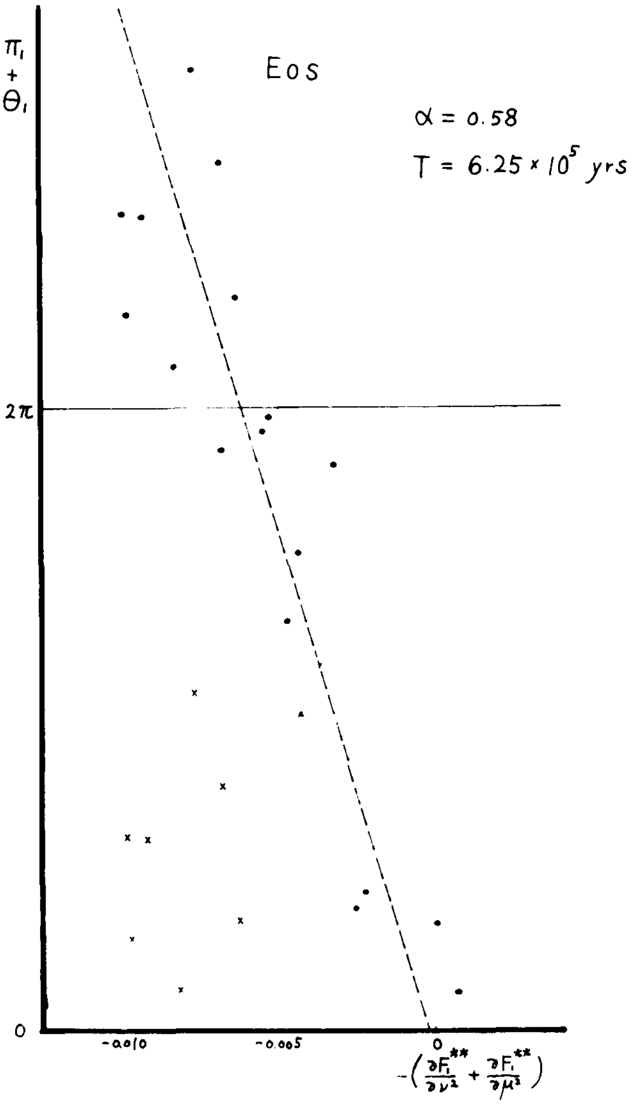

Figure 3. $\Pi_1+\theta_1$ against the coefficient of time for Eos family. Points x are shifted vertically by 2π upwards.

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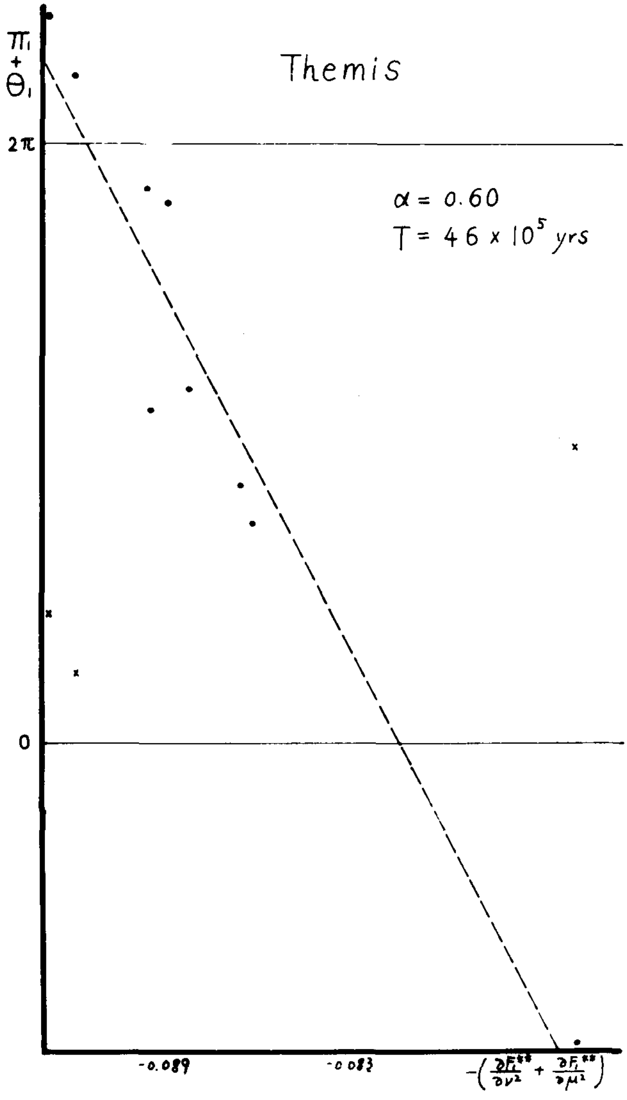

Figure 4. $\pi_1+\theta_1$ against the coefficient of time for Themis family. Points x are shifted vertically by 2π upwards in the left-side and downwards in the right-side.

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precise theory, namely a more higher order secular perturbation theory. In addition, the data used in Figures 1-4 are not sufficient in number, and now we are preparing the recalculation by adopting more data. When the recalculation is finished whether obtained values are the intrinsic ages or the minimum ages of the asteroid families may become more clear.

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