Proceedings of the Edinburgh Mathematical Society (1990) 33, 333–336 ©

BOOK REVIEWS

REES, D. *Lectures on the asymptotic theory of ideals* (London Mathematical Society Lecture Note Series 113, Cambridge University Press, Cambridge, 1989), 224 pp. 0 521 31127 6, £15.

For a thumbnail sketch of the basis of the classical theory underlying this book, consider the following. Given a commutative noetherian domain A with identity element, and an ideal I of A, the Rees ring $R=A[It,t^{-1}]$ is formed as a subring of the Laurent polynomial ring $A[t,t^{-1}]$. Thus R encodes information about each I^n, $n\geqq 1$. R can be viewed as an algebraic description of a blow-up, in the sense of algebraic geometry, in that we have moved from I, corresponding to a subvariety, to the principal ideal $t^{-1}R$, corresponding to a hypersurface. If, further, one normalises by forming the integral closure $\bar{R}$ of R, a Krull domain is obtained; so $\bar{R}$ comes equipped with its set V of essential valuations, which encode all important information about $\bar{R}$. The intersection of $\bar{R}$ with $A[t,t^{-1}]$ can be viewed as a type of generalized Rees ring, one which encodes information about the integral closure of each I^n, $n\geqq 1$. This process can also be considered in terms of the valuations comprising V. More recently, other types of generalised Rees rings have been investigated, often using the general language of filtrations, the filtration in the classical case being the set $\{I^n|n\geqq 1\}$.

The present set of lecture notes by D. Rees consists of a highly refined and polished account of filtrations, generalized Rees rings and related valuations, specializing after a while to the classical situation (essentially). Applications are made to the theory of analytically unramified rings and of quasi-unmixed rings, and to multiplicity theory. Great stress is given to associativity formulae for multiplicity and degree functions. Other highlights include an exposition of the classic theorems of Matijevic and Mori-Nagata, following ideas of Kiyek and Querré. The book closes with a discussion of the very intriguing theory of general extensions of local rings and of general elements, with applications to Teissier's mixed multiplicities.

There are many beautiful arguments and ideas in this book. As already mentioned, the exposition is highly polished (though, now and then, a little on the terse side). The material is an interesting blend of the classical and the modern. Undoubtedly, the book richly repays close study. My one cavil (leaving aside the occurrence of a few misprints) is that the presentation is almost too refined—a beginner or interested amateur would have to work hard before finding a foothold or perspective from which to view the subject matter of the book in the large. The contrast between this account and the one in the first part of the book *Equimultiplicity and blowing up*, by M. Herrmann, S. Ikeda and U. Orbanz (Springer-Verlag, 1988) is very interesting in an almost (one might say) meta-mathematical way.

L. O'CARROLL

DUDLEY, R. M. *Real analysis and probability* (Wadsworth and Brooks/Cole Mathematics Series, Pacific Grove, California, 1989), xii+436 pp. 0 534 10050 3, $52.95.

This book is an introduction to probability theory including a substantial amount of real analysis and measure theory for the necessary background knowledge. It is a text book for students at beginning graduate level based on lectures given by the author, but a few parts also contain material for researchers in the field.

Chapters 1 to 5 provide a one-semester course in real analysis, in particular basic measure theory. Topics covered are elementary set theory, general topology, measure and integration, basic functional analysis, and L^p-spaces. Chapters 6 and 7 can be considered as supplements containing further material from functional analysis and measure theory on topological spaces, especially metric spaces.

Chapters 8 to 10 and 12 are designed for a subsequent one-semester course of probability theory. The first half consists of the standard topics up to the central limit theorem, to be followed by fundamental martingale theory, the sub/superadditive ergodic theorem and finally stochastic processes in Chapter 12, mainly concentrating on a detailed account of Brownian motion. Chapter 11 and the final Chapter 13 contain additional material on convergence of laws on separable metric spaces, on standard Borel spaces and analytic sets, respectively.

The appendices include among other things a section on axiomatic set theory and in particular a very interesting discussion of measure-theoretic pathologies of compact non-metric spaces.

This book can be used as an excellent base for a one-year course in probability theory (including prerequisites), well preparing a student to proceed to research in many different possible directions. The author (an expert who has worked in the area for many years) has taken great care to give a complete and pedagogically perfect presentation of both the necessary preparatory material of real analysis and the proofs throughout the text. Some of the topics and proofs are rarely found in other text books, for example Strassen's "nearby variables/nearby laws" theorem or Kindler's proof of the Daniell–Stone theorem. The well-organized references at the end of each chapter provide a very good guide to the related literature. Furthermore, extensive notes on the history of important results and theorems (obviously thoroughly researched by the author) contain interesting and sometimes surprising additional information.

I think that this book is a marvellous work which can be highly recommended and will soon become a standard text in the field both for teaching and reference.

M. G. RÖCKNER

ATIYAH, MICHAEL *Collected works*, Vol. 1: Early papers, general papers (Clarendon Press, Oxford, 1988), 386 pp., 0 19 853275 X, £30.

This is the first of the five volumes of Michael Atiyah's works. The second volume is on K-theory, two are on index theory and the fifth is on gauge theories. They include everything published by him up to 1985 other than the text book written with Ian Macdonald; a set of lecture notes on representation theory appears for the first time. Each main section starts with a commentary by Atiyah giving some of the historical background to the papers. The decision to arrange the papers chronologically within each subject brings out the mathematical development clearly: classical algebraic geometry gradually influenced by modern methods; algebraic topology leading to K-theory; the index theorem in its many forms and finally the mathematical aspects of gauge theories. There are numerous papers applying these topics to solve a variety of problems. The author is as imaginative and quick to see connections between different parts of mathematics as he ever was and is still very prolific. Several more volumes will be needed to cover the period from 1985.

The volume under review has two sections. The "Early Papers" are on algebraic geometry, published between 1952 and 1958. This was a very exciting period in algebraic geometry. Jean Leray had introduced both sheaves and spectral sequences and these techniques were ripe for exploitation. Homology had been clarified and put into its present day form. Kunihiko Kodaira had proved his basic vanishing theorem yielding the finite dimensionality of certain cohomology groups. Atiyah was well aware of these developments and of their potential; he met the leading exponents during his year in Princeton in 1955–56 and several of them became his lifelong friends and collaborators.

His first paper, written whilst he was an undergraduate, studies the tangents to the twisted cubic via the Klein representation. This fascinating technique, which captured Atiyah's imagination as well as Roger Penrose's at that time, was to prove extremely important to both of them more than twenty years later through twistors and gauge theory. As a graduate student Atiyah became interested in bundles and this led to three important papers: the first applies the method to the study of ruled surfaces, the second studies connections on complex analytic bundles, and the third classifies analytic vector bundles over an elliptic curve. The straightforward parts of topological bundle theory do not extend to algebraic geometry and these papers explore the new