In this case (B), the introduction of imaginary numbers into the determination of the real foci and directrices may be avoided by writing the original equation

$$(a+\mu)x^2+2h+y+(b+\mu)y^2=\mu(x^2+y^2)-2gx-2fy-c$$

and proceeding as above.

Example of Numerical Case.

C. Smith, p. 210.

$$x^2-6xy+y^2-2x-2y+5=0 \qquad (\Delta \textit{ negative})$$

can be written

$$(\lambda-1)x^2+6xy+(\lambda-1)y^2=\lambda(x^2+y^2)-2x-2y+5.$$

Choose λ to satisfy

$$(\lambda-1)^2=9, \text{ so that } \lambda_1=4,\ \lambda_2=-2.$$

then λ_1 gives $\qquad 3(x+y)^2=4(x^2+y^2)-2x-2y+5$

$$\text{i.e.,}\quad 3(x+y+\nu)^2=4\left\{\left(x+\frac{3\nu-1}{4}\right)^2+\left(y+\frac{3\nu-1}{4}\right)^2\right\}$$

if ν be so chosen that $\qquad (3\nu-1)^2=2(3\nu^2+5)$

$$\text{i.e.,}\quad 3\nu^2-6\nu-9=0 \quad \text{or} \quad \nu^2-2\nu-3=0$$

$$\therefore\quad \nu_1=3,\ \nu_1'=-1.$$

The directrices are $x+y+3=0,\ x+y=1$;

the corresponding foci are $(-2,\ -2)$ and $(1,\ 1)$

and the eccentricity is $\sqrt{\frac{3}{2}}$.

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By Professor A. Brown.

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