(7) The triangle formulae for $\cos A$ and $\tan \frac{B-C}{2}$ are derivable in the same way. Also, by using the formulae corresponding to $\tan \frac{A}{2} = \sqrt{\frac{rr_1}{r_2 r_3}}$, $\tan \frac{B-C}{2}$ can be shown equal to $\frac{r_2 - r_3}{r_1 + r} \tan \frac{A}{2}$.

As a final example,

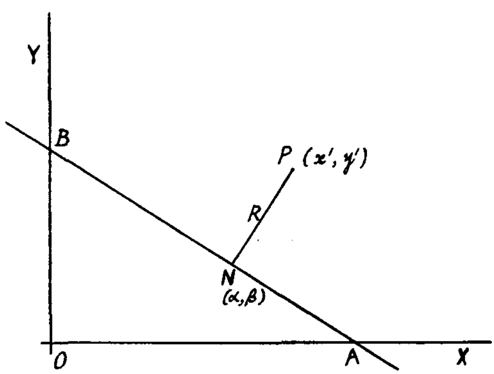
$$\Sigma\left(\tan \frac{B}{2} \tan \frac{C}{2}\right) = \Sigma \sqrt{\frac{(s-a)(s-c)}{s(s-b)} \cdot \frac{(s-a)(s-b)}{s(s-c)}} = \Sigma\left(\frac{s-a}{s}\right) = 1$$

or $$\Sigma\left(\tan \frac{B}{2} \tan \frac{C}{2}\right) = \Sigma \sqrt{\frac{r r_2}{r_1 r_3} \cdot \frac{r r_3}{r_1 r_2}} = \Sigma\left(\frac{r}{r_1}\right)$$

$$\therefore \quad \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}.$$

A. G. BURGESS.

The Distance of a given Point from a given Line.—If $P(x', y')$ is the given point and $ax + by + c = 0$ the equation of the given line, the expression for the distance of P from the line and the coordinates of the foot of the perpendicular from P to the line can be obtained by projections as follows:—



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Let $NP = R$ and have projections X and Y on OX and OY respectively. Then if NP makes an angle θ with OX

$$X = R\cos\theta \qquad Y = R\sin\theta$$

$$\therefore \quad R = X\cos\theta + Y\sin\theta.$$

But $$\tan\theta = \text{gradient of } NP = \frac{b}{a}$$

$$\therefore \quad \cos\theta = \frac{a}{d} \quad \text{and} \quad \sin\theta = \frac{b}{d} \text{ where } d = \pm\sqrt{a^2+b^2}.$$

Again, if N is (α, β), $X = x' - \alpha$ and $Y = y' - \beta$,

$$\therefore \quad R = \frac{a(x'-\alpha) + b(y'-\beta)}{d}$$

$$= \frac{ax' + by' + c}{d}$$

since $$a\alpha + b\beta + c = 0.$$

Also $$x' - \alpha = R\cos\theta = \frac{a(ax' + by' + c)}{d^2}$$

Hence $$\alpha = x' - \frac{a(ax' + by' + c)}{a^2 + b^2}$$

and $$\beta = y' - \frac{b(ax' + by' + c)}{a^2 + b^2}$$

R. J. T. Bell.

Note on the Determination of Centres of Curvature. —For determining the Cartesian co-ordinates of the centre of curvature of a plane curve two methods are principally used in the text-books. One of these (see for example, Edwards, "Differential Calculus," p 266, § 339) having previously established the formula for the radius of curvature, derives the coordinates of the centre of curvature by using the circular functions of the angle "ψ" which the tangent to the curve at the point considered makes with OX. Since, however, for the same tangent, and therefore the same "ψ," the curve may be either convex or concave towards OX (and accordingly

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