

BAYES SEQUENTIAL DESIGN PROCEDURES:  
THEORY AND APPLICATIONS

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This thesis considers the problem of the sequential design of experiments from a Bayesian standpoint. Viewed as a sequential decision problem with a choice of experiments at each stage, the Bayes (that is, optimal) sequential design procedure is constructively determined for the case when the number of experimentation stages is bounded. Under reasonably general conditions, it is shown that the Bayes sequential design procedure exists in the unbounded case and may be taken as the appropriate limit of optimal procedures in the bounded case. Also, an essentially complete class of sequential design procedures is characterised using generalised definitions of statistical sufficiency. These results parallel the work of Haggstrom [2], but they are derived and considered in terms of the sequential decision problem (see Ferguson [1]) rather than an optimal stopping problem for a stochastic process.

The constructive theory of Bayes sequential design procedures is then applied to two multiple decision problems, an identification problem and a ranking problem. Whilst the former illustrates the power of using the optimal procedure, the latter reveals the formidable computational difficulties involved in carrying out the backward induction for a non-trivial problem. Consequently, an alternative sequential design procedure is proposed, which, although based on the optimal procedure, is nevertheless easy to compute in most situations of interest. By modifying the goal of the experimenter in an appropriate fashion, it is shown that the alternative procedure possesses an optimality property for an

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identification problem concerning the parameters of two binomial populations. Finally, the results of computer simulation of the alternative procedure's performance in this problem and a similar problem concerning the parameters of two normal populations are presented, which suggest that the procedure is superior to those procedures currently proposed for the problem. The computer programs used are reproduced in an appendix.

### References

- [1] Thomas S. Ferguson, *Mathematical statistics; a decision theoretic approach* (Probability and Mathematical Statistics, 1. Academic Press, New York and London, 1967).
- [2] Gus W. Haggstrom, "Optimal stopping and experimental design", *Ann. Math. Statist.* 37 (1966), 7-29.