PROBLEMS FOR SOLUTION

P. 149. Find all solutions, other than the trivial solution $(a, b, c) = (1, 1, c)$ of the simultaneous congruences:

$ab \equiv 1 \mod c$, $bc \equiv 1 \mod a$, $ca \equiv 1 \mod b$ where a, b, c are positive integers with $a \leq b \leq c$.

G.K. White, University of British Columbia

P. 150. Let S be a set of commuting permutations acting transitively on set Ω. Prove that S is a sharply transitive abelian group.

A. Bruen, University of Toronto

P. 151. Given 8 points in the Euclidean plane forming two squares ABCD and A'B'C'D', neither congruent nor homothetic, use a ruler not more than ten times to locate their centre of similarity (that is, O such that $\triangle OAB \sim \triangle OA'B'$, etc.)

A. L. Steger, University of Toronto

P. 152. The classical Jordan-Dirichlet theorem states that if $f: [-\pi, \pi] \to R$ is continuous and of bounded variation, then the Fourier series of f converges to f uniformly. Find an example of a continuous f which is not of bounded variation, but whose Fourier series converges pointwise. Can you find one whose Fourier series converges uniformly ?

J. Marsden, University of California, Berkeley

SOLUTIONS

P. 141. Let $\nu_i = (\alpha_{i1}, \ldots, \alpha_{in})$, $i = 1, \ldots, m$ be vectors where α_{ij} are integers such that the greatest common divisor of all the α_{ij} is 1. Prove that there exist integers k_i such that the greatest common divisor of the components of $\nu = k_1\nu_1 + \ldots + k_m\nu_m$ is 1.

A.M. Rhemtulla, University of Alberta

Solution by D.Ž. Djoković, University of Waterloo

If A is the matrix (α_{ij}) then the assertion of the problem is that there exists a row vector K and a column vector R such that $KAR = 1$. This follows from well-known theorems about the canonical form

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of matrices over a principal ideal domain, here the integers Z. Indeed, if B and C are invertible matrices in Z_m and Z_n respectively such that BAC has canonical form, we can take K to be the first row of B and R to be the first column of C. The necessary theorems can be found, for instance, in N. Jacobson's "Lectures in Abstract Algebra", Vol. II (Theorems 5, 11, 12 on pages 79, 91 and 92).

Also solved by the proposer.

P. 142. Let A be a commutative noetherian ring, S a multiplicatively closed subset of A, M an A-module, $s \in S$ and $m \in M$, so $m/s \in M_S$. If ann denotes annihilator in A, prove that $\mathrm{ann}(m/s) = \mathrm{ann}(s'm)$ for some $s' \in S$. (This is used implicitly in Lang, Algebra, p. 151, Proposition 10).

K. Taylor, McGill University

Solution by D. Ž. Djoković, University of Waterloo

Let $P = \mathrm{ann}(m/s)$. P is finitely generated as an A-module, and let $a_1, \ldots, a_n$ be a set of generators. There exists $s_1, \ldots, s_n \in S$ such that $a_i s_i m = 0$ $(i = 1, \ldots, n)$. Then clearly $P = \mathrm{ann}(s'm)$ where $s' = s_1 s_2 \ldots s_n \in S$.

Also solved by the proposer.

P. 143. Find all metric spaces which have no infinite compact sets.

J. Marsden, Princeton University.

Solution by R. Tate, University of California, Berkeley

We shall do better. Let X be T_1 and first countable. Then X is discrete if and only if every compact subset is finite.

If X is discrete it is clear that every compact set is finite. Conversely, suppose X is not discrete. Then we can choose $x \in X$, a base of neighbourhoods U_n of x and $x_n \in U_n$ such that $\{x_n\} \cup \{x\}$ is compact and infinite.

One cannot remove the T_1 assumption, as is trivial to see, nor the first countable assumption as we see from the following example of E. H. Connell, Proc. Amer. Math. Soc., 10 (1959) p. 974: Let X be $[0, 1]$ with open sets of the form $A \setminus B$ where A is open in the usual topology and B is finite or countable. Then **X** is Hausdorff, non-discrete but every compact set is finite.

Also solved by J. Baker, H. Niederreiter, S. Reich, J. Wilker and the proposer.