IV. JOINT DISCUSSION ON TURBULENCE IN STELLAR ATMOSPHERES

(5 September 1955)

INTRODUCTION

By Prof. M. G. J. Minnaert, *Chairman*

During the meeting of the I.A.U. at Rome in 1952, three astrophysicists conceived the project of a joint discussion on turbulence in stellar atmospheres. These three, Messrs de Jager, Pecker and Thomas, submitted their plan to Dr Chalonge, Dr Greenstein and myself; each of the initiators suggested a tentative programme. The plan was approved by the Executive Committee, and at the proposition of Dr Greenstein, an organizing committee was appointed, consisting of Dr Schwarzschild (*chairman*), Dr Cowling, Dr Unsöld and myself. From then on, Dr Schwarzschild took the lead, keeping in touch with all of the others but taking care to come to practical decisions. There were quite a lot of difficulties before this little programme was settled; among others, it was really bad luck when Dr Cowling became ill and when we understood that his further co-operation in the active sense would be impossible. But finally the speakers were found, things were arranged; several commissions expressed their interest in the subject and gave their members the opportunity to attend this discussion. Schwarzschild had managed to have me appointed as the chairman of this meeting and he himself took care of assembling the reports of the speakers and the discussion. Dr Pecker acted as a secretary.

One of the highlights of the Dublin meeting has been the presentation of some sections of the solar spectrum, photographed at the McMath Observatory by the joint work of Doctors McMath, Mohler and Pierce. Two of these records are reproduced here, as a striking introduction and illustration to this Joint Discussion on stellar turbulence.

1. MOTIONS IN THE SOLAR ATMOSPHERE

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In this review we confine ourselves to the description of motions in the 'normal' undisturbed parts of the solar atmosphere, excluding the corona and the spots, faculae, flares, surges and prominences.

1. Granulation

The most conspicuous feature in photospheric aerodynamics is the granulation. According to the best observations it seems to consist of bright, more or less roundish blotches, surrounded by dark matter. It seems that the description of the granulae as a complex of bright and dark mottles, about equivalent in sizes and areas, cannot be maintained against the best observations, according to which the bright elements seem to be the fundamental ones. Thiessen (1955), observing visually with a 60 cm. refractor and using the full aperture, notes even their polygonal forms, which observation, together with the measured upward and downward motions, may support the idea that the granules are convective elements in the solar photosphere. One of the crucial points in the modern granulation observations is the problem of the true sizes and brightness fluctuations of the granules. This problem will first be discussed.

Sizes of granules

Janssen's classical observations (1896) indicate mean granule diameters between 0″.5 and 1″.5. On his photographs there are also bigger elements, but it seems that these are generally complexes of smaller ones. Most previous and modern observations confirm these results: Hansky (1905) notes a diameter of 1″; Chevalier (1912) and Keenan (1938, 1939) measure mean diameters of 1″.5. More recently Macris (1953), reducing

Lyot's observations, found a mean diameter of $1''.5$; these results are confirmed by Miller (unpublished) who, from a discussion of about 30,000 granulation observations made from 1952 to 1954, concluded that 'our measures are not discordant with the 1–2″ size range'. Also Rösch (1955*a*, *b*), observing at the Pic du Midi with a refined observing technique, agrees with this conclusion. His observations show granules with diameters down to $0''.5$, which is the theoretical resolving power of the objective. Naturally, smaller granulae can hardly be observed. Nevertheless, according to Krat (1952, 1954) they should exist and Krat advances the hypothesis that the true sizes of the granules are of the order of $0''.2$ or even less. The proof of this statement is given by observations on four plates, obtained by Krat on 24 August 1952, where the total half-widths of the granules were enclosed between $0''.24$ and $0''.35$. Since the half-width of the diffraction image of his objective is $0''.35$, this should mean that the real sizes of the granules are considerably smaller than $0''.35$, perhaps smaller than $0''.2$ (=140 km.). It seems, further, that the mutual distances of the granules are about $0''.35$.

The above results have all been obtained with objective diameters smaller than 30 cm. Really new and reliable results can only be acquired by observing with a much greater objective diameter (although we realize, of course, that such observations are tremendously difficult). The first attempts in this direction have been made by Thiessen (1955) and Rösch (unpublished). Thiessen observed visually granules with diameters down to a fraction of a second of arc, but he is still of opinion that the frequency curve of the granule sizes has a maximum between 1 and 2″. New photographic observations by Rösch with a 54 cm. objective seem to confirm these results.

So there remains a remarkable difference between the majority of the observers finding mean diameters of 1″ to $1''.5$, that is of 750–1000 km., and Krat, who finds diameters of near 100 km., a value also demanded by theoretical considerations (cf. Dr Biermann's lecture).

Another observation that seems worth mentioning, but that must necessarily be confirmed by more homogeneous observations, is that of a possible variability of the granulation in the course of the solar cycle. Miller (unpublished) remarks that his observations show conclusively that the granule diameter changes with the solar cycle, at least it did so during the recent years of low sunspot activity. This latter remark should be compared with one of Macris and Elias (1955) based, however, on an inhomogeneous material, stating that the number and the brightness of the granules varies with the Wolf number. Also, Krat notes the variation of the number of the granules with the sunspot cycle.

A third very important problem is that of the upper limit of the granulation layer. It has sometimes been announced (Waldmeier, 1945) that the granules are invisible when closer than 0·05 R to the Sun's limb. But Rösch's observations show the granulae still as close as 5″ from the limb. This means that the objects which are observed as granules do in any case still exist up to an optical depth $\tau \approx 0{\cdot}1$ in the photosphere. This upper limit is nearly equal to the one computed by E. Vitense (1953) on the basis of a semi-theoretical convection theory of the granulation elements.

Brightness and velocity fluctuations

After Waldmeier (1940) and Thiessen (1955), measurements yield $\Delta I/I \approx 35\%$ ($\lambda = 5650$ Å) (granules with respect to the surroundings); occasionally other authors published smaller values, down to some percents, but these observations are certainly influenced by stray light and by lack of resolving power and may refer to conglomerates of granules. The above brightness difference as directly observed corresponds to a difference in radiation temperature of 350°.

Radial velocity measurements show that the bright granulae have an upward motion with respect to the surrounding dark matter (Richardson and Schwarzschild 1950; Goldberg, 1955). A transverse (horizontal) motion has never been detected, which is not surprising considering the sizes (10^3 km.), life times (3 min.) and velocities (1–2 km./sec.) of the granulae. A r.m.s. vertical velocity component of 0·37 km./sec. was determined by Richardson and Schwarzschild from measured radial velocities of middle-strong photospheric lines; De Jager (1956) found 0·50 km./sec., whereas Reichel (1953), from a simple consideration of the visibility of granules in d'Azambuja's spectro-heliograms of strong metal-lines (1930), found a similar value: 0·28 km./sec. (However, these latter observations do probably not refer to the same elements as the former two.) These earlier observations have been overtaken by the recent results obtained by the Michigan astronomers (communication by Dr Goldberg at meeting of Commission 12 of the I.A.U. in Dublin) who, thanks to a better observational technique, were able to photograph spectra of complexes of granules, in some cases not bigger than a few seconds of arc, and measured Doppler velocities up to about 1·5 km./sec. The correct value may be still greater than this latter one, since the true velocity fluctuations in these tiny structures are generally greatly reduced by lack of resolving power (apparatus; scintillation) and by stray light. The corrections which should be applied to the velocity measurements depend on the brightness differences and the sizes of the granules. With the data collected above, it is now possible to correct Richardson and Schwarzschild's velocity measurements. This has been done by Thiessen (1955); the correct value should be $\sqrt{\overline{v^2}} = 1{\cdot}85$ km./sec., which is of the same order of magnitude as the micro-turbulent velocities of the upper photosphere, deduced from curves of growth and the macro-turbulent velocities deduced from Doppler cores of Fraunhofer lines. Thiessen applied the observational correction for the influence of stray-light, but in interpreting these corrected results one might still ask whether they correctly represent the true velocities and temperature variations inside the photosphere. A stable absorbing layer lying above the inhomogeneous elements would act as a veil and this veiling effect would reduce the intensities both of the velocity- and of the light-fluctuations (De Jager and Pecker, 1951). In view of the above results on the upper limit of the granulation zone, we think that this latter correction is unimportant.

In interpreting the observational results we should finally like to point out that the information collected here from several sources does not always refer to the same layers of the photosphere: the velocity measurements (Fraunhofer lines) refer to the highest layers ($\tau_\lambda \approx 0{\cdot}1 \ldots 0{\cdot}3$), while the continuous radiation (brightness measurements) is produced by layers about 100 km. deeper ($\tau_\lambda \approx 1{\cdot}0$). This should be borne in mind when discussing the influence of the granulation on the strengths of Fraunhofer lines.

2. Temperature Inhomogeneities in the Photosphere

The granulae, with their great temperature differences, should manifest themselves in certain Fraunhofer lines as appreciable small-scale fluctuations of their profiles. The investigation of Fraunhofer lines with the aim to detect such temperature fluctuations would inform us about other layers of the photosphere than does the continuous radiation. Direct observations of the line-profiles or equivalent widths inside and outside the granulae have not yet been made, but the concept of temperature fluctuations, first suggested by A. Unsöld (1953), has already proved to be successful in explaining the profiles of certain Fraunhofer lines and of the solar U.V. continuous spectrum. Böhm (1954*b*, Na, D and Fe lines); de Jager (1955*a*, Balmer lines)*, (1957, U.V. continuous spectrum).

The method followed in the latter case in computing the amount of temperature fluctuations is rather simple and probably so rough that it only gives the value of the T fluctuations to about a factor 2. First, with a given model of the photosphere the

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* A previous announcement (de Jager, 1954) that the concept of temperature fluctuations was also necessary in order to explain the profiles of the Paschen and Brackett lines has proved erroneous, since this conclusion was based on incorrectly computed widening parameters. (I thank Mrs Böhm-Vitense for having drawn my attention to this error.)

It has now been found that the profile of Brackett α is entirely explained by the classical theory for a homogeneous atmosphere if due account is taken of the twenty-nine individual Stark components of the line. The influence of temperature inhomogeneities on this line should still be examined in detail. It seems not impossible that temperature inhomogeneities would have much less effect on the Brackett lines than on those of the Balmer series. New computations for Paschen β confirm the old value, $\Delta\theta \approx 0{\cdot}1$.

profiles or the equivalent widths of certain Fraunhofer lines are computed. It then appears that there are systematic differences between the observed and the computed line-profiles, and as a second step two new models of the photosphere are introduced, each with the same relation between τ_λ and P and P_e as the old model but differing in that the original T (τ) relation is replaced by two others:

$$T_{1,2}(\tau)=T(\tau)\pm\Delta T\ \left(\text{or }\theta_{1,2}(\tau)=\theta(\tau)\pm\Delta\theta;\ \theta=\frac{5040}{T}\right).$$

For simplicity it is assumed that ΔT (or $\Delta\theta$) is constant over the whole atmosphere or at least over the region that is important for the formation of the lines which are studied. Let the solar photosphere consist of two kinds of adjacent cylinders, each characterized by one of the two T functions. Their areas are supposed to be equal. The spectrum is computed for each of these cylinders and the weighted mean is then taken (weighted according to the intensity of the computed continuous spectrum). By choosing various ΔT or $\Delta\theta$ values the 'best' value is found from a comparison with the observations. In this way we obtain from the Balmer lines: $\Delta\theta\approx 0{\cdot}1$ or $\Delta T\approx 750°$ referring to $\tau_{5000}\approx 1{\cdot}5$. We note that the value for the Balmer lines is perhaps an upper limit, since shock-tube laboratory experiments seem to indicate that the Balmer lines are already wider than is predicted by the Holtzmark theory. This should reduce the 'observed' ΔT to a still unknown value lower than 750°.

Böhm's three-streams model is more refined in two respects. In the three-streams model 50% of the photospheric surface consists of columns stratified according to Böhm's previous model of the photosphere (1954*a*; this model is in perfect radiative equilibrium). In the two other columns, corresponding each to 25% of the area, Böhm adopted the same pressure stratification as a function of geometrical depth. This assumption is necessary in order to avoid strong horizontal currents (Unsöld, 1948). It is further assumed that at the same geometrical depth h, the black-body radiation in one of the 25% columns is equal to 1·6 times the black-body radiation in the main column; in the other 25% column it is 0·4 times this value.

Böhm's model was refined by H. H. Voigt, who could make acceptable that the temperature differences are small or zero for optical depths smaller than about 0·1. An argument for this assertion is also the fact that the intensity of the solar radiation measured with the rocket technique near 2000 Å is about equal to the intensity computed theoretically (De Jager, 1957).

An advantage of the simple two-streams model is that this enables us to estimate quickly the ΔT, best adapted to the observations, while a complete calculation of a three-streams model would require a considerable amount of work. On the other hand, it is clearly an important advantage that Böhm has been able to justify his choice of ΔB by referring to E. Böhm-Vitense's theoretical discussion of the granulation zone. Böhm's three-streams model permits a reasonably correct explanation of the centre-limb variations of an Fe I multiplet (as a further refinement, the deviations from local thermodynamic equilibrium in the upper photosphere are taken into account). Also for the wings of the NaD lines a better agreement is obtained than with the homogeneous model.

The table below presents a comparison between Böhm's ΔT values and the ΔT values determined observationally with the two-streams model or from granulation observations; the temperature differences refer to the optical depths given in the first and fifth columns.

	Böhm's model		From two-streams model		
τ_{5000}	ΔT	ΔT	Object	τ_{5000}	ΔT
0·001	−800° and	+400°	Sun's limb	0·004	small
0·01	−700° and	+350°			
0·1	−750° and	+600°	Thiessen, granulae	1·0	200°
1·0	−500° and	+300°	Balmer lines	1·5	$\leqslant$750°

Böhm's values are in reasonable agreement with the two-streams values determined in a more empirical way. We want to stress that the (still rather uncertain) *values* of the T

fluctuations are for the time being not so important as the fact that Unsöld's concept of photospheric temperature fluctuations will prove very stimulating and incite to investigate from that point of view the profiles, equivalent widths and centre-limb variation of Fraunhofer lines, as well as the detailed temperature distribution in the granules, especially in the uppermost parts thereof.

However, we are only at the beginning, and a correct treatment of this problem should necessarily be based not on a two- or three-streams model, but on a continuous statistical distribution of temperatures and densities at each geometrical depth. Attempts should be made to find these functions observationally as well as theoretically and their variation with depth. Theoretical calculations should be based on a detailed theory of convection and turbulence in the photosphere, which is in turn based on a theory of compressible as well as of incompressible turbulence in a gravitational field (formulation by R. N. Thomas).

3. TURBULENCE IN THE CHROMOSPHERE

If the turbulence field in the upper photosphere and in the low chromosphere may be described (*a*) as a field of sound waves, (*b*) propagating without dissipation of energy, the value of $\rho v^2 c_v$ must remain constant (Unsöld, 1952); this means that v must increase with height. For this reason it is important to determine observationally the variation of v_{turb} with height in the chromosphere. It appears that the law is fulfilled qualitatively only, showing that at least one of the above conditions is not met.

In Fig. 1 some recent v determinations are collected (cf. de Jager, 1957).

Apparently there are no clear differences between the radial velocity components (filled symbols) and the sight-line velocity components (open symbols), so that we may

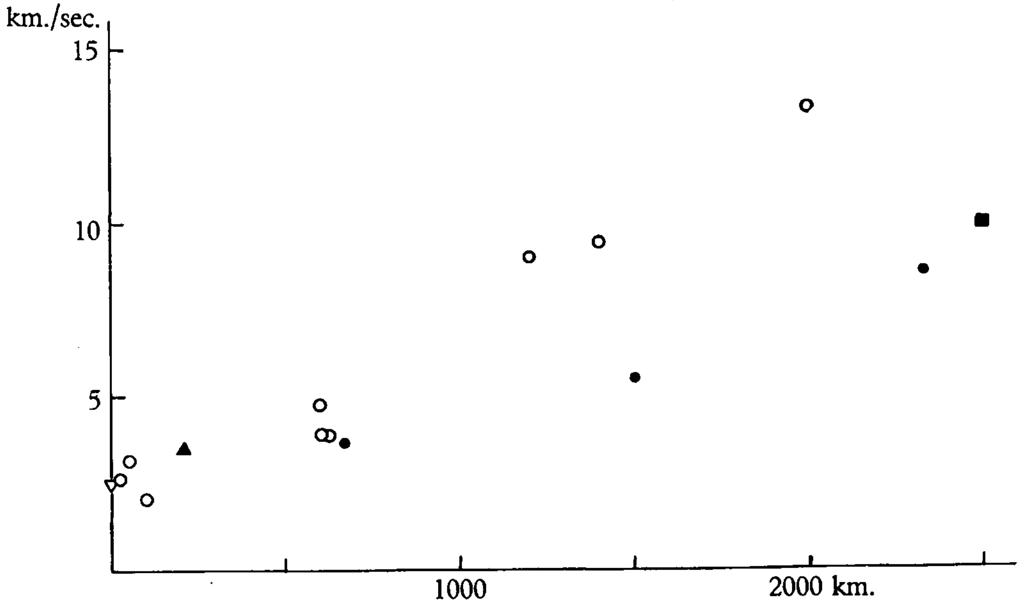

Fig. 1. Variation of turbulent velocity with height in the low chromosphere.

(*a*) At $h=0$ we assume Reichel's value (1953) deduced from cores of Fraunhofer lines: $\sqrt{\overline{v^2}}=2\cdot5$ km./sec. (open triangle) (micro- and macro-turbulence; component radial to the solar surface).

(*b*) At $h=300$ km. we assume $\sqrt{\overline{v^2}}=4$ km./sec. (full triangle), determined from the intensity gradient at the Sun's limb, taking the T gradient into account (component radial to the surface).

(*c*) The full circles give Redman and Suemoto's values (1955), deduced from line-profiles in a chromospheric eclipse spectrum (line of sight component). The He line data for $h<2000$ km. are not used, since the He lines seem to be formed preponderantly between $h=2000$ and 3000 km.

(*d*) The open circles give Mrs Böhm-Vitense's (1955) density gradients in the chromosphere, determined from Menzel and Cillié's 1935 flash spectra (radial component).

(*e*) The full rectangle gives the radial component deduced for the infra-red helium line, observed in the Fraunhofer spectrum (de Jager, 1957). (*Note, added in proof*: new observations (Mohler) lift this value to 15 km./sec.)

conclude that the turbulent field in the low chromosphere ($h < 3000$ km.) is, as far as we can see it now, isotropic.

Although we have the impression that in each individual spicule the motions are not isotropic, the mean field of motion in the higher chromosphere still seems to be isotropic. At $h \approx 6000$ km., where the spicules can be seen separately, the mean radial velocities are about 20 km./sec. and they increase linearly with height up to maximum values of about 80 km./sec. near the 30,000 km. level (Rush and Roberts, 1954). According to Michard (1954 and personal communication), the velocities tangential to the surface have random values and are also of the order of 20 km./sec. at a height of 6000 km.

4. TEMPERATURE AND DENSITY FLUCTUATIONS IN THE CHROMOSPHERE

That density and/or temperature fluctuations occur in the high chromosphere is evident from the very existence of the spicules. In the parts of the chromosphere above 5000 km. these objects occupy about 2% of the solar surface (Woltjer, 1954; Rush and Roberts, 1954) and they can easily be seen on a Hα photograph of the Sun's limb. In the lower parts of the chromosphere the spicules cannot be distinguished at the limb, but there are certainly inhomogeneous columns of matter that are comparable to the photospheric granulae and which occur in great numbers, as can be inferred from $H_{\alpha 3}$ spectroheliograms: from these observations it is estimated that at $h \approx 3500$ km. a considerable part of the Sun's surface is occupied by bright and dark chromospheric granules. The precise ratio between both areas is difficult to estimate from spectro-heliograms.

The 'high spicules' have a mean lifetime of 3·5 min. and have diameters of 2–4″ (Rush and Roberts, 1954), whereas the fine mottling on $H_{\alpha 3}$ spectro-heliograms has sizes of 2000–4000 km. and a mean lifetime of 5 min. (de Jager, 1957). Hence, it seems not improbable that the fine mottling of $H_{\alpha 3}$ spectro-heliograms corresponds with the lowest parts of the spicules. Since this mottling is mainly visible for $\Delta\lambda < 0{\cdot}4$ or 0·5 Å, this means that their radial velocities are smaller than 20–25 km./sec., which is again in agreement with a statement by Rush and Roberts who, from their investigation of spicules, conclude that the low chromosphere must consist of an overwhelming number of small spicules with $|\bar{v}| < 20$ km./sec. It is these same structures, perhaps, which are also visible on the beautiful Michigan photographs which Dr Goldberg showed us at this congress. The photographs of Hα he projected showed a number of tiny broadenings, often more or less symmetrical to the line itself and extending up to $\Delta\lambda = 0{\cdot}5$ Å or more. Apparently these same structures are seen in emission as small bright knots in K_2 spectra. These two data might give us information about the temperature- and density-differences between these elements and their surroundings.

An interesting point raised by Miller (1953) is that the low spicules or similar structures may sometimes also produce some continuous absorption. His white-light photographs show local regions of partial obscuration as if dark clouds were floating above the photosphere. Their life time is 4 min. and they occupy one-half of the solar surface.

After this qualitative description we shall try to summarize current views on the *temperature distribution* in the chromosphere. The extreme complexity of this problem is due to the fact that we are dealing with at least two different elements (whether these might be vertical columns or smaller elements) and that it is impossible to speak of 'the' temperature of the chromosphere. The ratio between the electron temperature at a certain level and the radiation temperature in the centre of a line emitted by that layer may easily amount to a factor 2 to 5 or even higher. The same applies to the differences between the electron temperature and the excitation and ionization temperatures.

Since the spicules emit Hα radiation and hence cannot have too high a temperature, and since some of them are visible up to 20,000 km. or higher above the Sun's limb, where the coronal temperature is of the order of 10^{6}° K, this means that the *upper parts of the spicules* must certainly be considered as *cold masses of gas* penetrating into a hot corona. At the same time we observe that the chromosphere emits the light of the Ca^{+} lines, visible up to 14,000 km., for which Miyamoto and Kawaguchi (1950) compute an electron temperature of 6000°. It seems likely that this radiation is also emitted by the spicules, although this has still to be investigated. We estimate that these high spicules have electron temperatures perhaps above 2×10^4, but this has still to be derived from spectral or photometric observations of the high spicules. Nothing is known about their densities.

The temperature of the spicules in the *intermediate region* is somewhat better known. Here Michard (1955) has measured the width of the Hα line emitted by some spicules. He found this to correspond to a T_{el} value of approximately 10,000° if micro-turbulence is neglected; hence this value is an upper limit. These measurements refer to heights somewhat near 5000 km.

From a photographic investigation of the spicules the variation with height of the number n_2 of hydrogen atoms in the second quantum level can be determined (Woltjer, 1954). The value of n_2 is here about fifty times greater than in the mean chromosphere. Since the values of N_{e} in the spicules are supposed to be known from the heights of the Balmer lines, the electron temperature T_{e} can be found with the aid of tables computed by Giovanelli (1948). In the chromosphere between 4000 and 8000 km. Woltjer finds in this way $T_{\mathrm{e}}=21{,}000°$. This result is practically in agreement with that of Athay and Menzel (1956) who find from the chromospheric helium spectrum that at 6000 km. $T_{\mathrm{e}}\approx 2\times 10^4$ °K. (Note that the *radiation temperature* of Hα in the upper chromosphere is much lower: $T_{\mathrm{rad}}\approx 4000°$.) Combining these three T_{el}-determinations we assume $T_{\mathrm{el}}\approx 10{,}000°$ to 15,000° at 5000–6000 km.

The most difficult part of the chromosphere is *the region below* 4000 *km*. Here the greater amount of information still leads to contradicting results. It is not yet certain whether the spicules are the hot or the cold elements in the low chromosphere. Arguments that in parts of the low chromosphere T_{el} is about 5000° are: (1) the low temperatures deduced from the strengths of neutral and singly ionized metal lines (visible up to 4000 km.); (2) the non-occurrence of forbidden lines (Wurm, 1948); and (3) the widths of metallic lines in the low chromosphere (Redman, 1942; Unsöld, 1952). Arguments for the low temperature based on radio data are not strong for this part of the chromosphere: a properly chosen high-temperature model of the medium chromosphere explains equally well the radio-radiation of the Sun as does a low temperature model; but the other arguments, especially those based on the profiles of the metal lines, seem stronger.

On the other hand, the far U.V. solar emission and the hydrogen emission indicate the existence of hotter elements, in which T_{el} rises according to various observers from 4500° to about 10,000° at 2000 km. (Fig. 2; cf. de Jager, 1957). Also the greatly temperature-dependent infra-red helium line at 10,832 Å, visible as an absorption line in the Fraunhofer spectrum, may be produced by these elements near $h=2500$ km.

Finally, Athay and Menzel (1956) find that the helium lines observed on eclipse spectra must be emitted by regions having up to 3000 km. a temperature of about 20,000°, which then increases upward. This high temperature is mainly required by the He II line at 4686 Å. It should be examined whether this high T_{el}-value may (partially) be explained by absorption of coronal U.V. radiation.* We should remember further that a part or perhaps the whole of the differences between Woltjer's and Athay's models arises from the fact that Woltjer finds that the hot elements are at the same time the most dense, while the Harvard workers base their discussion on the pre-supposition of pressure equality at the same geometrical height, thus making the hottest elements the less dense.

In fact there are five unknowns, all being functions of height: T_1, T_2, ρ_1, ρ_2 and a_1, the latter being the relative portion of the chromosphere occupied by the elements (T_1, ρ_1). All five unknowns should be found from the observations by a combined discussion of all available chromospheric observational data. Only then will it be possible to decide with how many kinds of elements we are dealing in the low chromosphere and what part of the chromosphere is occupied by them. The present author's belief is that perhaps a model with elements of 5000° and 10,000° at 2000 km. is the most probable (cf. Fig. 2).

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* *Note, added in proof:* a rediscussion of the chromospheric helium spectrum (De Groot and De Jager) shows that the observations can be explained quantitatively with Woltjer's model, with $T_{\mathrm{el}}=9000°$ at 2000 km.

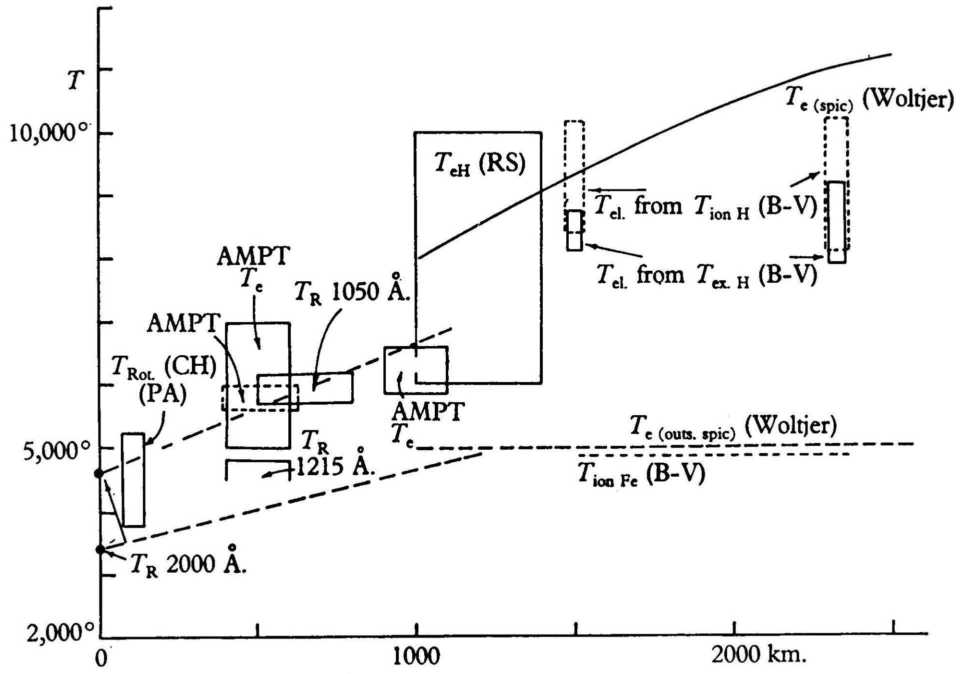

Fig. 2. Variation of temperature with height in the low chromosphere, indicating the existence of hot and cold elements.

T_R 2000 Å, etc.: radiation temperature derived from intensities in the far ultra-violet continuous spectrum at 2000 Å, 1215 Å, etc. (*Note, added in proof*: new computations make $\Delta T=0$ at $h=0$);
$T_{Rot.}$ (CH) PA: rotation temperature of CH bands found by Pecker and Athay;
T_e (AMPT): electron temperatures found by Athay, Menzel, Pecker and Thomas;
T_{eH} (RS): electron temperatures found from profiles of H lines by Redman and Suemoto;
T_{el} from $T_{ion\ H}$ and from $T_{ex.\ H}$ (B-V): electron temperatures computed from the ionization temperatures, and from the excitation temperatures found for hydrogen by Mrs Böhm-Vitense;
$T_{ion\ Fe}$ (B-V): ionization temperature for iron found by Mrs Böhm-Vitense.

Large-scale structures

In the foregoing discussion we have been dealing throughout with the small-scale elements (diameters of the order of some seconds of arc). There are, however, indications of bigger elements occurring mainly in the chromosphere and in the uppermost part of the photosphere, since they are visible in a great part of the Hα line-profile in spectro-heliograms. The $H\alpha_{12}$ granulation or coarse mottling becomes visible in the very line-centre, where it is emitted at a height of 4000–5000 km.; it extends up to $\Delta\lambda=1{\cdot}1$ Å, where it is mainly emitted by layers with $\tau_{5000}=0{\cdot}25$; $h\approx 250$ km. below the surface of the Sun. The same elements with correlation distances of 15,000–20,000 km. (Rogerson, 1955) are clearly visible in K_3 spectro-heliograms, where they have been known already since the early days of spectro-heliography (Deslandres, Hale); and they are also, but faintly, discernible in the normal photospheric spectrum, as has been demonstrated by statistical investigations. This latter observation agrees with their above-mentioned visibility in $H\alpha_1$ spectro-heliograms up to $\Delta\lambda=1{\cdot}1$ Å. In this line of thought it is obvious why Frenkiel and Schwarzschild (1950) could find evidence for large-scale motions, from a secondary maximum, near 15,000 km. in the auto-correlation curve for the ***radial velocities***. The r.m.s. velocity component is about 0·15 km./sec. It was not possible to find an indication for analogous large-scale structures in the ***brightness distribution*** in the continuous spectrum between the lines on this same plate (Stuart and Rush, 1954). This might seem evident from the foregoing, for the continuous spectrum is emitted principally by the deeper regions of the photosphere where, as we believe, the big-scale elements do not occur. This argument has, however, been made invalid by the same authors by a reduction of Miller's plates (1953), which does show a secondary maximum near 18,000 km. in the auto-correlation function for the brightness. The r.m.s. intensity fluctuations are about 1% of the mean solar surface brightness. If this secondary maximum is not due to spurious effects, like Janssen's photospheric network, the question could be raised whether the appearance of the large-scale pattern on Miller's plates could be due to the many Fraunhofer lines in the blue part of the spectrum, which influence must somehow exist, since Miller used orthochromatic plates and observed in 'white' light.

We interpret these bigger structures as huge elements, rising and descending with a r.m.s. velocity of ± 1 km./sec., as it is found from measurements of the line displacements in Hα. This line shows characteristic large-scale widenings and narrowings, correlated with a small displacement of the line. Our measurements refer to parts of the line formed in the chromosphere somewhere between 2000 and 4000 km., and the widening and narrowing, being of the order of ± 15%, can be interpreted as an increase or decrease in the ionization and excitation temperatures by ± 100°, the ascending elements being the coldest (de Jager, 1957). The lifetime of these structures is of the order of half-a-day to a day. Above, we noted that *in the photosphere* the big elements have a r.m.s. velocity component of 0·15 km./sec. Between 2000 and 4000 km. this value has increased to 1·05 km./sec. It may now be asked what is the velocity in the intermediate levels. Could the answer perhaps be read in Reichel's investigation (1953) on d'Azambuja's metal spectro-heliograms (1930), where the visibility curves of the 'granules' could be explained by a r.m.s. velocity component of 0·28 km./sec.?

Summarizing the foregoing it seems that we must interpret the big structures as typical chromospheric objects having their basis, however, in the upper photosphere, at $\tau_{5000} \approx 0{\cdot}5$, somewhere near the top of the granulation zone or still lower. They can still be detected at the 5000 km. level. As yet they have not been discovered in higher regions, but this need not necessarily mean that they do not exist in the high chromosphere.

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Discussion

In reply to a question from Dr Schwarzschild, Dr Goldberg replies:

I am unable to guess at the root mean square velocity that will be obtained from the Lake Angelus measurements, in as much as the measurements have not yet been made. I will say, however, that the maximum displacements of the metallic lines appear to correspond to about 1·5 km./sec.

Dr Schatzman asks about the total amount of energy radiated away by the solar chromosphere and corona.

Dr de Jager, in reply to Dr Schatzman:

This should still be computed.

Dr R. N. Thomas:

I have three points I would like to raise, two in the form of questions, and a third filling in specific values in the discussion several nights ago by some of us, trying to reconcile our varying ideas on the chromosphere.

First, in connexion with the use of Dr Michard's measures as evidence of sight-line velocities, I wonder whether Michard might care to comment on how much of the effect he believes might be associated with a projection of radial spicule motions along the line-of-sight?

Secondly, Dr de Jager remarked on the possibilities of investigating the large-scale atmospheric structure and its physical interpretation by a correlation between the line-shift and line-widening observed in the hydrogen lines. I agree strongly that this procedure is most important, particularly from the standpoint of any self-consistent aerodynamic interpretation of the phenomenon. It would seem to me that the same sort of thing might also be done by correlating the results from the hydrogen line with those from the infra-red helium line. I wonder if Dr Goldberg would care to comment, from the standpoint of the results by visual inspection of these very first spectra from the McMath-Hulbert vacuum spectrograph?

Thirdly, I would like to fill in just a few of the details of the results of that informal get-together the other night of those interested in the attempts at non-uniform models of the chromosphere. Let me emphasize that the following simply represents an attempt to delineate the areas of agreement, and to suggest possible lines of investigation which might clarify the areas of disagreement. Simply for convenience, this summary is presented in terms of two models, denoted by Athay-Thomas and Woltjer, because they were available as a basis for discussion. The actual summary, however, represents the end-product of an evening's discussion of a group as a whole (Athay, de Jager, Michard, Newkirk, Pagel, Pecker (Ch. and J.-C.), Schatzman, Thomas, van de Hulst, Warwick, Woltjer) and I hope only that I am an accurate reporter.

Region 6000–8000 km.: coronal line emission present at 8000 km.

Woltjer model	$T_{S1} < T_{C1}$	T_{C1} ?	$T_{S1} \sim 2 \times 10^4$ °K.
Athay-Thomas model	$T_{S1} < T_{C1}$	$T_{C1} = ? \sim 10^5 \cdot 10^6$ °K.	$T_{S1} \sim 2 \times 10^4$ °K.

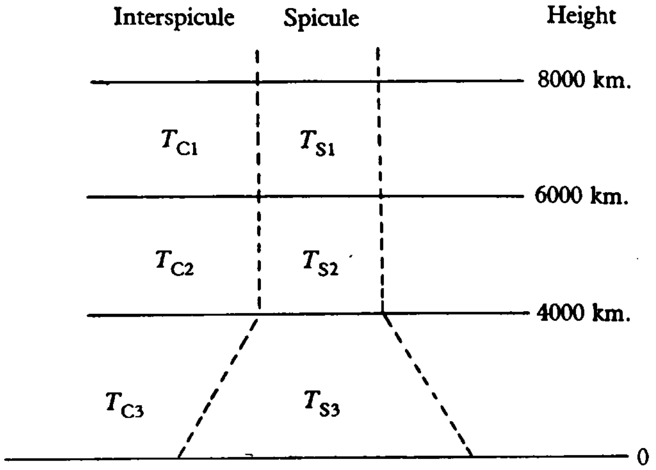
$6000 \leqslant h < 4000$ km.

Woltjer model	$T_{S2} > T_{C2}$	$T_{C2} < 5000$ °K.	$T_{S2} \sim 2 \times 10^4$ °K.
Athay-Thomas model	$T_{S2} < T_{C2}$	$T_{C2} \sim 10^5$ °K.	$T_{S2} \sim 2 \times 10^4$ °K.

$h < 4000$ km. (number of spicules increases towards lower heights)

Woltjer model	$T_{S3} > T_{C3}$	$T_{C3} \sim 5000$ °K.	$T_{S3} \sim 7 \times 10^3 – 2 \times 10^4$
Athay-Thomas model	$T_{S3} < T_{C3}$	$T_{C3} \sim 2 \times 10^4$ °K.	$T_{S3} \sim 6500–9000°$

Schematic features



(Contour indicates area occupied by spicules increases downward)

Fig. 4

The above table and figure represent an attempt to represent two 'mean' types of regions. In the actual case, temperature may vary within the region and the detailed properties of a spicule may vary from one spicule to another.

Approaches to resolution of the above conflict:

(i) Data on the metallic lines for $h < 4000$ km. (in progress at Harvard). Data on the radio results for $h \lesssim 4000–5000$ km. (preliminary results by Hagen presented at Manchester agree with Athay-Thomas, but more detailed reduction is necessary).

(ii) Spicule spectra for $4000 < h < 6000$ km. would be particularly valuable. Detailed data on spicule luminosity (in progress at Sac. Peak) (filter observations).

(iii) Details on line-widths of coronal lines at the lowest observable height.

(iv) Interpretation of non-eclipse, high resolution spectrophotometry on both limb and disk.

(v) Theoretical work: (*a*) Need of inelastic collision cross-sections for metals in order to evaluate the contribution of non-equilibrium terms in the Boltzmann equation, for thermal electrons $T_e \gtrsim 6000°$. (*b*) Attempts to represent observed data on velocities and brightnesses of spicules by a self-consistent theory.

Dr Michard, in reply to Dr Thomas:

My measurements are sight-line velocities; they refer to Doppler shifts in the Hα light when the spectrograph slit cuts the spicules. It seems to me that the results of sight-line velocities, velocities in the plane of the sky, and angles of spicules with the vertical on the Sun, are consistent with the assumption of essentially vertical motions: the interpretation of motions in the high chromosphere ($\sim$5000 km.) as an isotropic turbulence can only be a very rough approximation. The observed sight-line velocities are a component of these radial motions.

Dr de Jager, in reply to Drs Thomas and Michard:

According to a short computation which I have made, an optical depth unity is reached in Hα, observed at about 5000 km. beyond the Sun's limb, in a point on the line of sight, which lies about 50,000 km. nearer the observer than the geometrical point closest to the Sun. This means that the angle between the normal to the solar surface and the plane perpendicular to the line of sight is about 4°. In that case the effect to which Dr Thomas referred is not important and the conclusion remains that either the field of motions in the median chromosphere is isotropic or that the motions in the spicules are anisotropic while at the same time the spicules have considerable inclinations to the normal.

Dr Goldberg, in reply to Dr Thomas:

The Lake Angelus observations of both Hα and He I 10,830 in absorption on the solar disk demonstrate that both lines show large fluctuations in line-width, roughly by 30% in Hα and between zero and about 1 Å for 10,830. It would obviously be of great importance to examine the correlations between the widths of the two lines if the observational difficulties of making simultaneous photographs of the two lines can be overcome.

Dr Schwarzschild:

It was emphasized that the size of the true, average granules is of basic interest not only to the dynamics of the photosphere, but also to the internal structure of the Sun. It is felt that the presently available observations are inconclusive because of the lack of objective means of determining the observational resolution reached. It appears entirely possible that the most energetic granules have diameters of about 300 km. and have temperature contrasts twice as large as even the largest contrasts as yet observed ($\pm 400°$ instead of $\pm 200°$).

Dr Minnaert, in reply to Dr Schwarzschild:

When speaking about the size of the granulation, it seems important not to state simply one value, which might be the smallest observed size, but to give the complete distribution curve of the sizes or the Fourier analysis of the brightness distribution.

Dr Unsöld:

The profiles and displacements of Fraunhofer lines at the limb can give detailed information on velocities and temperature differences in the granulation. A theoretical discussion of existing observational material is going on at Potsdam and Kiel. More and better observations are highly desirable.

Dr Rösch:

Si les granules apparaissent ronds et de 1″5 au centre parce qu'ils sont petits et agrandis par diffusion, etc., ils doivent *aussi* apparaître ronds et de 1″5 au bord. Or, ils sont allongés (environ $3 \times 0''5$) ce qui ne peut s'obtenir que pour les granules de 1″5 aplatis par la perspective. Il semble donc qu'il y a au moins une majorité de granules de diamètres voisins de 1″5 et une décroissance du spectre vers les petits diamètres.

Dr ten Bruggencate:

We started in Göttingen a programme to study the stratification of the solar photosphere. By measuring the profiles of the infra-red O lines at the centre of the disk, we found a distinct asymmetry of the line-profile; the violet wing being stronger than the red wing. This can be understood by using the three streams model of Böhm. The excitation potential of the O lines is so high that the hot rising elements of granulation contribute much more to the line-profile than the cooler elements going downwards. The asymmetry disappears if the lines are observed near the solar limb.

Dr Athay:

Dr de Jager presented a diagram showing a compilation of the measures of isotropic turbulence in the chromosphere that showed turbulent velocities increasing with height. Some of these results, particularly those of Suemoto and Redman (*M.N.* 1955), were based on the assumption of a uniform, spherically symmetric chromosphere. If one includes temperature fluctuations as large as those we have been discussing, the increase of turbulence with height will not be as rapid as indicated by de Jager's diagram. Would Dr de Jager comment on this point and also comment on the extent to which the other results may be influenced by such non-uniformities?

Dr de Jager, in reply to the questions from Dr Athay:

The data derived from the Redman-Suemoto observations have been recomputed by using the temperature distribution given in Fig. 2 of my lecture and the non-uniformity there assumed. An exception has been made for the helium lines: from an unpublished computation made at Utrecht, I have the impression that the helium lines are mainly formed in a layer between 2000 and 3000 km. high. The same follows from the v_{turb} values, derived from the He lines by Redman and Suemoto: at $h<2000$ km. these are equal to the values at $h=2000$ km.; and afterwards they increase with h. I have excluded the helium points for $h<2000$ km. from Fig. 2.

2. EVIDENCE FOR TURBULENT MOTIONS IN STELLAR ATMOSPHERES

By K. O. Wright, *Dominion Astrophysical Observatory*

The application of the principles of hydrodynamics has become an important feature of modern astrophysical investigations, but the first mention of turbulence in stellar atmospheres was made by Rosseland(1) as long ago as 1928 in his paper on 'Viscosity in the Stars'. He showed that if there were any differential motions taking place in these atmospheres, the scale must be so great that turbulence of a 'bewildering complexity' would result. By 1934 the word 'turbulence' appeared frequently in the literature and, for some fifteen years, was often used somewhat indiscriminately to explain any stellar phenomenon relating to stellar spectra not otherwise accounted for. However, about 1949 students of hydrodynamics and astrophysics began to pool their ideas and it is believed that, although many of the problems of astrophysics are exceedingly complex, the implications of turbulent phenomena and their applications to astrophysics are becoming better known.

This paper presents in some detail the spectroscopic evidence for turbulence in the atmospheres of stars other than the Sun as obtained from curves of growth and from line-profiles, and also discusses some observations made during the atmospheric eclipses of the ζ Aurigae-type of stars. Other data related to turbulent phenomena will be discussed only briefly.

Following Rosseland's paper, Unsöld(2) and McCrea(3) discussed turbulence in the solar atmosphere and Menzel(4) noted that the systematic differences in the velocity-curves of different elements observed in the spectra of Cepheid variables might be evidence of this phenomenon. However, Struve and his students at the Yerkes Observatory were the first to discover a new phenomenon in stellar spectra that could be ascribed to 'turbulence'. The 'gradient effect' was first noted by Struve(5) in the spectrum