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RESEARCH ARTICLE

Optimal commissions and subscriptions in mutual aid platforms

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Abstract

This paper investigates an operation mechanism for mutual aid platforms to develop more sustainably and profitably. A mutual aid platform is an online risk-sharing platform for risk-heterogeneous participants, and the platform extracts revenues by charging participants commission and subscription fees. A modeling framework is proposed to identify the optimal commissions and subscriptions for mutual aid platforms. Participants are divided into different types based on their loss probabilities and values derived from the platform. We present how these commissions and subscriptions should be set in a mutual aid plan to maximize the platform's revenues. Our analysis emphasized the importance of accounting for risk heterogeneity in mutual aid platforms. Specifically, different types of participants should be charged different commissions/subscriptions depending on their loss probabilities and values on the platform. Participants' shared costs should be determined based on their loss probabilities. Adverse selection occurs on the platform if participants with different risks pay the same shared costs. Our results also show that the platform's maximum revenue will be lower if the platform charges the same fee to all participants. The numerical results of a practical example illustrate that the optimal commission/subscription scheme and risk-sharing rule result in considerable improvements in platform revenue over the current scheme implemented by the platform.

1. Introduction

This paper investigates the operation mechanism for the sustainability and profitability of mutual aid platforms. Mutual aid platforms, as innovations of insurance and finance based on the internet and InsurTech, have attracted more than 300 million participants in China and have become the third most-popular choice for healthcare coverage after social health insurance and commercial health insurance. Platforms facilitate the process of risk exchange among participants by charging a commission (a percentage of the claim benefit), a subscription (a fixed fee that people pay to participate in the platform), or both. Most platforms implement simple payment schemes and charge the same fees to all participants. For example, Xianghubao charges an 8% commission to all participants, while participants in Waterdrop Mutual Aid pay a 9-yuan subscription fee to join the platform. Furthermore, participants in platforms share their risks by paying the same shared costs. However, most platforms do not consider the risk heterogeneity of participants when designing payment schemes and risk-sharing rules. For instance, people aged 59 years or younger are all eligible to be enrolled in Xianghubao. However, the loss probability of a person aged 59 years can be 47 times greater than that of a 19-year-old person. Charging the same fees to participants of different risk levels or setting the same shared costs can result in adverse selection and platform revenue loss. Because of the inappropriate payment schemes and risk-sharing rules, most mutual aid platforms are in deficit and face the risk of bankruptcy. Therefore, the objective of this paper is to find a better operation mechanism for the sustainable and profitable growth of mutual platforms and to offer guidelines to mutual aid platforms on establishing more sophisticated

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commission/subscription schemes and risk-sharing rules. Specifically, this paper aims to investigate the optimal design of commission rates and subscription fees, as well as a fair risk-sharing rule for mutual aid platforms to maximize their revenues by considering different types of participants in platforms.

In our opinion, the contribution of this paper is threefold. The first main contribution is that we propose a better operation mechanism for the sustainability and profitability of mutual aid platforms. The mechanism is based mainly on the establishment of commission/subscription schemes and risk-sharing rules. Mutual aid platforms can increase their revenues and be more sustainable by applying this mechanism. To achieve this goal, we propose a mathematical model to analyze mutual aid platforms by capturing the main features of these platforms and their participants. The second contribution is that we shed light on the design of commission/subscription schemes, based on our platform's revenue maximum problem. Previous papers studying mutual aid platforms have mainly focused on the risk exchange problem among participants (e.g., Denuit, 2019, 2020; Abdikerimova and Feng, 2022). In our work, we study the implications of commissions and subscriptions charged by mutual aid platforms and the effects of commissions/subscriptions on risk exchange process. To the best of our knowledge, this paper might be the first studying mutual aid platforms from the perspective of the payment scheme of the platform. The third contribution is that we emphasize the importance of accounting for risk heterogeneity when determining commissions and subscriptions. The importance of accounting for risk heterogeneity is neglected in the current mechanism of mutual aid platforms. We show that adverse selection occurs if all of the participants share the costs equally. Furthermore, charging the same fee to all participants can result in platform revenue loss.

Our work relates to models of risk-sharing mechanisms. Risk-sharing mechanisms, especially peer-to-peer (P2P) risk-sharing mechanisms, have been studied intensively in recent years since the emergence of P2P insurances and mutual aid platforms. Denuit (2019, 2020) and Denuit and Robert (2020) studied the risk-sharing rule and proposed a risk-sharing model in P2P insurance. The model has been extended to study the three business models in P2P insurance (Denuit and Robert, 2021b), pure premiums for a large number of heterogeneous losses (Denuit and Robert, 2021a) and the P2P insurance scheme where the higher layer is transferred to a reinsurer (Denuit and Robert, 2021c). Boyle *et al.* (2021) studied the application of blockchain technique on P2P insurance. Abdikerimova and Feng (2022) and Feng *et al.* (2020) proposed a risk-sharing rule called fair risk exchange, in which the variance in shared costs is minimal. Li *et al.* (2022) put forward an optimal design for the mutual aid platform. Our work is an extension of models in previous literature by incorporating payment method and platform's revenues.

Our work is also related to literature in platforms. In particular, mutual aid platforms are analogous to P2P service platforms, which include personal and professional services, such as product sharing, transportation, and food delivery. Such platforms have been studied intensively in the recent literature. Benjaafar *et al.* (2019) described a P2P product sharing model in which each individual decides on whether to own, rent from others who own or neither. They found that consumers always benefit from collaborative consumption. Jiang and Lin (2018) studied the strategic and economic impacts of product sharing among consumers and found that sharing of products is a win-win situation for both the firm and consumers if the firm's marginal costs are high. Other research topics on online service platforms have included the pricing, profits, and efficiency of platforms (see, e.g., Carlin, 2009; Bellos *et al.*, 2017; Bimpikis *et al.*, 2017), the performance of medical crowdfunding platform (Liu *et al.*, 2020), the impact of online inquiry services on gatekeeping systems (Li *et al.*, 2019) and the behavior equilibrium model with mode choice (Wang *et al.*, 2020). Different from other P2P service platforms, mutual aid platforms provide risk-sharing services to participants. We extend the horizon of online platform research topics by incorporating risk-sharing features into the platform.

The remainder of this paper is organized as follows. Section 2 presents an overview of the development, advantages and the mechanism of a mutual aid plan. Section 3 proposes a framework to model the main features of mutual aid plans and constructs the optimization problem to maximize platform revenues. The impact of adverse selection on mutual aid platforms is studied in Section 3 as well. Section 4

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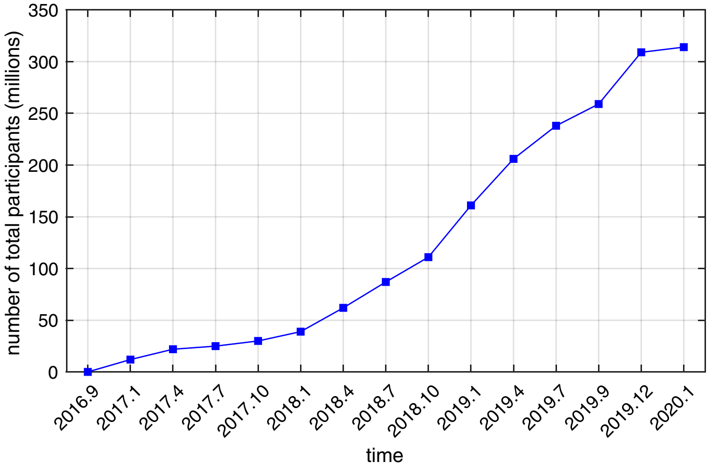

Figure 1. *Number of participants in mutual aid platforms in China (in millions).*

obtains optimal solutions to the optimization problem and considers a special case in which the platform charges the same fees to all participants. Section 5 presents a numerical example. Finally, noteworthy conclusions are summarized in Section 6.

2. Mutual aid platform

In this section, we provide a brief discussion of the development and the mechanism of the mutual aid platform.

2.1. Development of the mutual aid platform

With the development of internet technology and InsurTech, conventional insurance companies have shifted their focus to online insurance. Many insurance innovations specifically designed for online selling have been proposed. An online insurance platform has advantages over conventional insurance in a few aspects, such as convenience, efficiency, and ease of access. Along with the development of online insurance, a new type of online platform, the online mutual aid platform, has emerged in China. The online mutual aid platform is an innovation of both insurance and finance, and it is an internet-based platform adopting InsurTech during its entire operation process. In an online mutual aid platform, participants can share the risk of becoming critically ill and bear the medical expenses of patients collectively. Mutual aid platforms have quickly become popular because of their lower barriers to entry. An individual can pay a small fee to participate in a mutual aid plan provided by a platform and receive a considerable benefit if she is diagnosed with a critical illness covered by the plan. Mutual aid platforms typically provide participants with basic health plans covering more than 100 types of critical illnesses, including cancer, heart attack, critical brain injury, and acute myocardial infarction. Since the outbreak of COVID-19 (coronavirus disease 2019) worldwide, many mutual aid platforms have extended their coverage to include COVID-19.

Approximately 300 million members had enrolled in mutual aid platforms by early 2020 (See Figure 1). The annual growth rate of numbers of participants exceeded 100%. By 2025, the number of participants in China's online mutual aid platforms is expected to reach 450 million or nearly 32% of the country's population, according to Research Institute of Ant Group (2020).

According to Research Institute of Ant Group (2020), users of online mutual aid programs in China are primarily from low-income or middle-income households. Among more than 58,000 mutual aid

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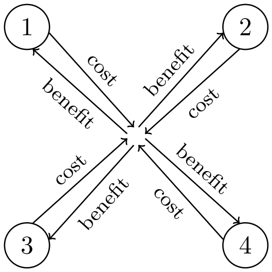

Figure 2. *A graphic illustration of a mutual aid platform.*

platform users who were surveyed, 80% earn less than 8333 yuan per month, while 72% come from third or lower-tier cities and rural areas. Therefore, mutual aid platforms are an alternative health coverage program for developing countries. The survey also indicated that online mutual aid platforms serve as a complement by further reducing out-of-pocket expenses accrued in the treatment of critical illnesses, bringing it from 40% to less than 20% for patients solely dependent on public healthcare coverage. While nearly 70% of online mutual aid participants surveyed by Research Institute of Ant Group (2020) said that they were not covered by commercial health insurance, more than 42% said they intend to purchase such insurance products in the future. This finding suggests mutual aid platform can increase awareness of the importance of insurance. The survey showed that the mutual aid platform is a complement of the insurance market and provides low-cost health protection to lower income households.

The costs of mutual aid plans are much lower than commercial insurance premiums. For instance, the cost of a mutual aid plan offering 500,000 yuan in coverage is usually less than 100 yuan per year, while the premiums for commercial critical illness insurance are approximately 4000 yuan. This lower price is possible and the reasons are as follows. First, mutual aid platforms are based on the internet, and costs such as operating and administrative expenses are much lower than those incurred in conventional insurance. Second, for conventional insurance, insurers have to rely on insurance agents and insurance brokers to sell insurance products. The commission fees of insurance agents and insurance brokers increase premiums of conventional insurance. Third, there are waiting periods in mutual aid plans. During the waiting periods, participants only pay the shared costs but cannot apply for benefits. As the participant populations of mutual aid platforms increase sharply in these years, many participants are still in the waiting period. There are more participants to share the costs of benefits and hence the shared costs are much lower.

2.2. Mechanism of the mutual aid platform

The organizational structure of a mutual aid platform is illustrated in Figure 2. Unlike conventional insurance companies that adopt a centralized model, mutual aid platforms apply noncentralized systems. Participants in the plan do not need to pay premiums in advance to pool their resources to pay for those who suffer critical illnesses. They collaboratively share the costs of benefits paid to those claimants at the end of each mutual aid plan period. The platform facilitates transfers between participants but is not allowed to take possession of participants' payments. According to the regulation issued by the China Insurance Regulatory Commission (CIRC), a mutual aid platform is not allowed to commit to risk insurance liability or use the word "guarantee" in any way. Furthermore, a mutual aid platform is not allowed to collect insurance premiums or establish pools of funds. As a result, a mutual aid platform does not guarantee the settlement of claims and does not incur the risk that a conventional insurance company takes. Therefore, the platform is not an insurer but acts as an intermediary and facilitator that provides participants with a risk-sharing platform and other related services.

The mutual aid plan operates as depicted in Figure 3. In general, there are two stages of a mutual aid plan: the beginning of the plan ($t = 0$) and the end of the plan ($t = 1$). At time $t = 0$, the mutual aid

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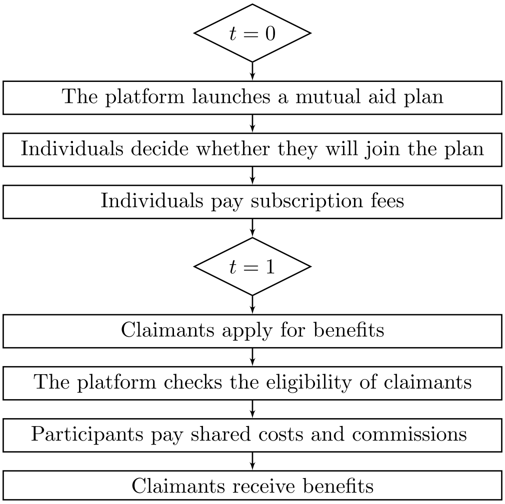

Figure 3. *Sequence of events.*

platform launches a mutual aid plan and releases the necessary information to the public. This information typically includes the rules of the mutual aid plan, benefit amount, how the costs of benefits are shared, and what critical illnesses are covered, among other factors. Individuals gather the information from the platform and decide whether to join the plan based on their own characteristics and valuations. People who decide to join the plan sign a contract and pay a subscription fee. These individuals form a risk-sharing community through the mutual aid plan. At time $t = 1$, some participants in the plan are diagnosed with a covered critical illness and claim benefits to cover their medical expenses. Their cases are subject to reviews by third-party investigation groups appointed by the platform. The platform checks the eligibility of every applicant to ensure that no applicant is cheating. The application information is also released to other participants, and they are part of the committee that makes the judgment. After all applications have been approved, the total benefits paid to claimants and the corresponding costs carried by the remaining participants in the plan are determined. Finally, all of the participants in the plan make the payments needed to share the costs of benefits paid to those who suffered losses, and the claimants receive benefits from the platform. The platform charges commissions when transfers are made between participants.

Having summarized the features of mutual aid plans, we conclude that there are two main factors that the platform must consider when designing a mutual aid plan, which are the risk-sharing rule and the payment scheme. The risk-sharing rule determines how participants share their risks. At the end of a period, participants share the costs of benefits used for claimants. In most platforms, participants pay the same shared costs, which is the total benefits divided by the number of participants. The payment scheme is comprised of commissions and subscription fees as follows.

- The commission, which is also called the management fee, is charged as a percentage (commission rate) of the shared costs paid by participants. Recall that participants' shared costs are for the benefits paid to those who suffer losses. Therefore, the commission is charged upon the occurrence of loss. More claims lead to more commissions. Commissions can be used to cover the costs and expenses incurred during the payment process. The excess portion of the commission can be viewed as part of the income of the platform. Without loss of generality, we assume that the transaction cost is 0 in this paper.
- The subscription fee, which is also called the member fee, is the fee that participants pay for entering the mutual aid plan. Some platforms waive the subscription fee to attract more

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Table 1. *Some examples of mutual aid platforms*

Platform	Subscription fee(yuan)	Commission rate
Xianghubao	0	8%
Waterdrop Mutual Aid	9	0
Qingsong Mutual Aid	10	0
e Mutual Aid	30	0
Kangai Gongshe	0	11%

individuals. Other platforms charge a small entrance fee. Unlike the commission, the subscription fee is independent of the shared costs paid by participants. Note that the subscription fee is different from an insurance premium because it is not used for claims reserves.

Some examples of mutual aid plans provided by different platforms in China are presented in Table 1. Xianghubao and Kangai Gongshe allow individuals to join the mutual aid plan for free and only charge commissions. In contrast, e Mutual Aid charges a high subscription fee, but there is no commission. The policies of mutual aid plans are adjusted constantly. For instance, the commission rate of Xianghubao was 10% in 2019 but decreased to 8% in 2020.

3. Modeling framework

In this section, a modeling framework is proposed to describe the main features of the mutual aid platform and participants. The impact of adverse selection on mutual aid platforms is also studied. An optimization problem is constructed for the platform to obtain its maximum revenue by setting the optimal commission rates and subscription fees.

3.1. Participants

We consider a monopolistic mutual aid platform that offers a general mutual aid plan for people to share their risks. People who have the potential to participate in the plan are all referred to as participants in this paper. We assume that participants are divided into finitely many types, denoted by $\mathcal{N} = \{1, \ldots, n\}$. We further suppose that participants of each type are infinitesimal, for example, there are a continuum of participants. We denote the total number of type-i potential participants by $m_i > 0$, $i \in \mathcal{N}$. Participants of type i have the probability p_i of being diagnosed with a critical illness and faced with a loss X_i. p_i is regarded as the loss probability of type-i participants. If participants are critically ill, they can receive the benefit I_i from the mutual aid platform. We assume that the loss is fully covered by the mutual aid plan, that is, $I_i = X_i$. If participants are not diagnosed with a critical illness, they must pay the shared cost S_i plus a commission to the platform. S_i is the cost of claim benefits carried by a type-i participant. The platform collects S_i from all healthy participants and pays the benefits to the claimants. S_i is endogenous and uncertain, and it depends on the total amount of benefits, total participant population in the plan and the risk level of each participant type. In our model assumptions, participants of each types are infinitesimal. This is a reasonable assumption when the number of participants is typically large enough and this assumption has been extensively used in economic models. As discussed by Lucas (1980) and Prescott and Townsend (1984a,b), with a continuum of participants, the loss probability p_i of type-i participants is also the fraction of type-i participants that will suffer a loss. Let s_i be the limit value of S_i for all i. As we consider the limiting case, we use s_i as the shared cost in the model. The calculation of s_i and the expression for s_i will be discussed in detail later. Since the costs of benefits paid to claimants are all carried by the rest of the participants in the plan, the total amount of fees collected from participants is equal to the amount of benefits paid to claimants. In our modeling setting, different types of participants have different risk levels (i.e., loss probability), benefits, and shared costs.

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Suppose that participants' valuation of participating in the mutual aid plan is denoted by v. This assumption is motivated by the work of Birge *et al.* (2021), in which sellers and buyers derive values from transacting with each other. Similar assumptions have been made by, for instance, Bellos *et al.* (2017) and Taylor (2018). v is used to measure participants' willingness to participate in a mutual aid plan. The participant value v may be explained by the following three terms. The first term is the value for which the participant can receive compensation for a potential loss. The second term is the value that comes from the satisfaction that the participant derives from being able to help others who are critically ill. The third term is the value derived from other related services offered by the platform. However, we do not specify how the participant value is expressed by these three terms. The participant value is the combination of all the personal factors those will affect the willingness to participate in the mutual aid plan. Thus, we only focus on the participant value. v is an exogenous variable in our model assumption and the value v is assumed to be a constant value for a specific participant. However, we assume that the values that participants of the same type derive from participating in the plan are heterogeneous. The distribution of v among type-i participants is described by the cumulative distribution function F_i:$[0, \bar{v}_i] \to [0, 1]$. Such assumption has been adopted in many literature on platforms, such as Taylor (2018), Bellos *et al.* (2017), Jiang and Lin (2018), and Birge *et al.* (2021). $\bar{v}_i \in \mathbb{R}^+ \cup \{\infty\}$ is the upper bound of the value derived from the plan by type-i participants. Suppose that $F_i(v)$, for all $i \in \mathcal{N}$, is continuously differentiable and strictly increasing in $v \in [0, \bar{v}_i]$ with respect to v. Under these assumptions, functions F_i are invertible. Let F_i^{-1}:$[0, 1] \to [0, \bar{v}_i]$ be the corresponding inverse function; we have $F_i^{-1}(F_i(v)) = v$ for $v \in [0, \bar{v}_i]$. For ease of convenience, we extend the domain of F_i to $\mathbb{R}$, that is, $F_i(v) = 0$ for $v \leq 0$ and $F_i(v) = 1$ for $v \geq \bar{v}_i$.

The platform can charge additional fees to participants to facilitate the processes of the mutual aid plan and provide services to participants. We assume that the platform chooses commission rates (a percentage of the total benefits) and subscription fees (lump-sum transfers to participate in the plan independent of total indemnity benefits). Suppose that the commission rates and subscription fees are the same for all participants of the same type but can be different for different types of participants. Let α_i represent the commission rate for type-i participants and β_i represent the subscription fee for type-i participants. We assume that the set of feasible commission rates is $\alpha_i \in [0, \infty)$, $\forall i \in \mathcal{N}$, and the set of feasible subscription rates is $\beta_i \in [0, \infty)$, $\forall i \in \mathcal{N}$. We denote by $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ with $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ the platform's commission and subscription vectors, respectively.

We next study behaviors of participants. Participants' decisions to join the mutual aid platform are based on their expected profits. Suppose that a participant of type $i \in \mathcal{N}$ with value $v \in [0, \bar{v}_i]$ decides to participate in the plan. Then, the participant pays the subscription fee β_i at time $t = 0$ and obtains the value v. At time $t = 1$, the participant will be faced with the loss X_i and receive the benefit I_i and if she is diagnosed with a critical illness (with probability p_i). In this case, the expected profit is $U_{i,c}(v) = v - \beta_i + I_i - X_i$. If the participant is healthy (with probability $1 - p_i$), she must pay the shared cost s_i plus the commission $s_i\alpha_i$ to the platform. Then her expected profit is $U_{i,h}(v) = v - \beta_i - s_i(1 + \alpha_i)$. Therefore, this participant's expected profit at time $t = 1$ if she joins the platform is

$$U_i(v) = pU_{i,c}(v) + (1 - p)U_{i,h}(v) = v - (1 - p_i)s_i(1 + \alpha_i) - \beta_i + p_iI_i - p_iX_i.$$

If the type-i participant decides not to participate in the mutual aid plan, the corresponding expected profit is $U_{i,n} = -p_iX_i$. If $U_i(v) \geq U_{i,n}$, which is $v \geq (1 - p_i)s_i(1 + \alpha_i) + \beta_i - p_iI_i$, the participant has a greater profit if she chooses to join the mutual aid plan. If $v < (1 - p_i)s_i(1 + \alpha) + \beta - p_iI_i$, that is, the value v is less than the expected outflow, the participant will choose not to participate in the plan because her expected profit is lower after she joins the plan. We assume that all participants' behaviors are based on their expected profits and that they are all profit maximizers. Thus, given commission rates and subscription rates, the action undertaken by each participant maximizes her profit. That is, the profit-maximizing participant participates in the mutual aid plan only if the profit is greater after she joins the mutual aid plan.

Let l_i be the total participant population of type-i participants who participate in the mutual aid plan, that is, the participant population of type-i participants. Given commission rates and subscription

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fees, all type-i participants who have greater expected profits ($U_i(v) \geq U_{i,n}$) will participate in the plan, meaning that the value of a type-i participant in the plan is at least $(1-p_i)s_i(1+\alpha_i)+\beta_i-p_iI_i$. Thus, l_i is given by

$$l_i = m_i\big[1 - F_i\big((1-p_i)s_i(1+\alpha_i)+\beta_i-p_iI_i\big)\big], \quad \forall i \in \mathcal{N}, \tag{3.1}$$

where m_i is the total potential population of type-i participants. It is obvious that $0 \leq l_i \leq m_i$ for $i \in \mathcal{N}$. Note that, in mutual aid plans, benefits paid to claimants all come from other participants. The platform only serves as an intermediary and charges a small amount of fees to participants. Therefore, we assume the total costs paid by all participants (excluding the fees paid to the platform) are equal to the total value of benefits received by claimants.

Suppose r_i is the the random percentage of population suffering the illness. The condition described above can be written as

$$\sum_i l_ir_iI_i = \sum_i l_i(1-r_i)S_i. \tag{3.2}$$

We assume that each type-i participant's shared cost S_i is determined through the weight factor w_i. w_i is exogenous in this paper and is determined before choosing optimal commission rates and subscription fees, while s_i is endogenous and is based on the participant populations, subscription fees, and commission rates. Participants of different types are assigned different weights to reflect the different risk levels of different participant types. The relationship between shared costs and weight factors of different participant types is given by

$$\frac{S_i}{S_j} = \frac{w_i}{w_j}, \quad \forall i, j \in \mathcal{N}. \tag{3.3}$$

Specifically, if we use S_1 as a basis, the shared cost of a type-i participant can be written as

$$S_i = \frac{w_i}{w_1}S_1, \quad \forall i \in \mathcal{N}.$$

The higher the value of w_i is, the higher the risk of type-i participants is; hence, a higher shared cost should be paid by type-i participants. An appropriate way to choose the value of w_i is discussed in the next subsection of this paper. From Equations (3.2) and(3.3), we have (for further details, see the Appendix)

$$S_i = \frac{w_i \sum_{j\in\mathcal{N}} l_jr_jI_j}{\sum_{j\in\mathcal{N}} l_j(1-r_j)w_j}, \quad \forall i \in \mathcal{N}. \tag{3.4}$$

As mentioned before, participants are assumed to be infinitesimal and therefore our model is under the limiting case. Such simplification is reasonable because the number of participants in mutual aid platforms are large enough. Taking the limit on both sides of (3.2) leads to (for further details, see the Appendix)

$$\sum_i l_ip_iI_i = \sum_i l_i(1-p_i)s_i. \tag{3.5}$$

Similarly, Equation (3.4) can be rewritten as the limiting case, and we have

$$s_i = \frac{w_i \sum_{j\in\mathcal{N}} l_jp_jI_j}{\sum_{j\in\mathcal{N}} l_j(1-p_j)w_j}, \quad \forall i \in \mathcal{N}. \tag{3.6}$$

Here, the limit s_i of S_i is in the sense of convergence almost surely because r_i converges to p_i almost surely as the number of participants tends to be infinity. In the rest part of this paper, we use Equations (3.5) and (3.6) in the optimization model under the limiting case. It is also worth noting that the shared cost is not predetermined in the mutual aid plan in practice. It is calculated by using Equation (3.6) using the values of w_i and I_i, which are predetermined by the platform.

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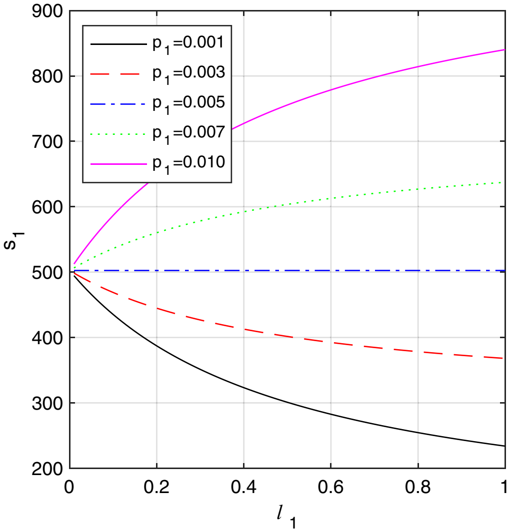

Figure 4. *s_1 against l_1 ($p_2 = 0.005$, $l_2 = 0.5$, $I_1 = I_2 = 100,000$ yuan, $w_1 = w_2 = 1$).*

We can see that $s = (s_1, \ldots, s_n)$ is endogenous and is determined by the populations of different types of participants in the mutual aid plan. Thus, s can be regarded as a function of l. We have the following proposition to illustrate the relationships between $s = (s_1, \ldots, s_n)$ and $l = (l_1, \ldots, l_n)$.

Proposition 1. *The relations between s_i and l_j, for $i, j \in \mathcal{N}$, are given by the following. (1) If $p_j I_j \geq (1 - p_j)s_j$, $\frac{\partial s_i}{\partial l_j} \geq 0$. (2) If $p_j I_j < (1 - p_j)s_j$, $\frac{\partial s_i}{\partial l_j} < 0$.*

Proposition 1 states that s_i is increasing in l_j if $p_j I_j \geq (1 - p_j)s_j$ and is decreasing in l_j if $p_j I_j < (1 - p_j)s_j$. Proposition 1 reveals the nature of participants' shared costs. Note that $p_j I_j$ can be regarded as the expected benefit of type-j participants, and $(1 - p_j)s_j$ is the expected outgo in the mutual aid plan. $p_j I_j > (1 - p_j)s_j$ indicates that a type-j participant's expected benefit is higher than her expected outgo. Thus, Proposition 1 implies that, if the expected benefit is higher than the expected outgo of type-j participants in the mutual aid plan, then increasing the population of type-j participants (l_j) will raise other participant types' s_i. Similarly, if the expected benefit is lower than the expected outgo of type-j participant in the mutual aid plan, increasing the population of type-j participants (l_j) will decrease other participant types' s_i. The main idea of this proposition is that the shared cost s_i of type-i participants is affected by populations of other participant types. Additionally, note that a higher $p_j I_j$ means that type-j participants' expected benefit is higher. Then, Proposition 1 can also be explained as follows: increasing the population of participants with high expected benefits will increase the costs of participants of all types, while increasing the population of participants with low expected benefits will decrease the costs of participants of all types. We consider a simple mutual aid plan that consists of only two participant types. The value of s_1 against l_1 is depicted in Figure 4. We can see that s_1 increases as l_1 increases when $p_1 > p_2$ and decreases when $p_1 < p_2$. This outcome is consistent with the fact described in Proposition 1.

According to Proposition 1, the platform can change the shared costs by adjusting the participant populations. Specifically, the platform can increase or decrease the shared costs by increasing or decreasing the populations of some participant types. However, the platform cannot directly change the participant population since participants decide whether they will participate in the plan by themselves. Participant populations should be adjusted by changing commissions and subscription fees, which are set by the

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platform. Therefore, it is essential to investigate how participant populations are affected by commission-subscription schemes. The relationship between participant populations and commissions, as well as subscriptions, is given by the following proposition.

Proposition 2. *The relations between l_j and α_i as well as β_i are given by*

1. *For $i = j$, $\frac{\partial l_i}{\partial \alpha_i} \leq 0$, $\frac{\partial l_i}{\partial \beta_i} \leq 0$;*
2. *For $i \neq j$,(1) if $p_j I_j \geq (1 - p_j)s_j$, $\frac{\partial l_j}{\partial \alpha_i} \leq 0$ and $\frac{\partial l_j}{\partial \beta_i} \leq 0$; (2) if $p_j I_j < (1 - p_j)s_j$, $\frac{\partial l_j}{\partial \alpha_i} > 0$ and $\frac{\partial l_j}{\partial \beta_i} > 0$.*

Proposition 2 states that l_i is decreasing in α_i and β_i. For $i \neq j$, l_j is increasing in α_i and β_i if $p_j I_j \geq (1 - p_j)s_j$ and is decreasing in α_i and β_i if $p_j I_j < (1 - p_j)s_j$. From Proposition 2, we know that populations of different types of participants in the plan can be adjusted by changing commission rates and subscription fees. The first part of Proposition 2 is obvious. Increasing commission rates or subscription fees will decrease the expected surplus of participants; thus, fewer participants will join the plan. The second part of Proposition 2 explains the relationships between the population of one participant type and the commission rates (or subscription fees) of other participant types. If the expected benefit is higher than the expected outgo of type-j participant in the mutual aid plan, that is, $p_j I_j \geq (1 - p_j)s_j$, increasing the commission rates (or subscription fees) for type-i participants will decrease the population of type-j participants. The reason is that increasing the commission rates (or subscription fees) for type-i participants will lower the participant population of type-i participants. Then, there are fewer participants sharing risks with type-j participants, whose expected benefit is greater than the expected outgo. Thus, fewer type-j participants will join the plan. In contrast, if $p_j I_j < (1 - p_j)s_j$, increasing the commission rates (or subscription fees) for type-i participants will increase the population of type-j participants.

From both Propositions 1 and 2, we know that commissions and subscriptions determine participant populations and hence shared costs. Keeping other parameters unchanged, if the population of riskier (i.e., with a high loss probability p_i) participants increases, the shared cost of each participant also increases. This outcome lowers the population of all participants of other types according to Equation (3.1). Similarly, increasing the population of participants of lower risk (i.e., with a low loss probability p_i) will lower the shared cost of each participant and hence raise the population of each type. Therefore, there exists an equilibrium for participant populations in the platform under which the participant populations are stable. If the platform is not in an equilibrium state, participant populations will move toward the equilibrium state. Given a feasible commission-subscription pair $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, weight factors $w = (w_1, \ldots, w_n)$ and benefits $I = (I_1, \ldots, I_n)$ chosen by the platform, the participant populations will reach equilibrium. This equilibrium must satisfy Equations (3.1) and (3.6). Thus, we have the following definition.

Definition 1. *Given a feasible commission-subscription pair $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, weight factors w and benefits I chosen by the platform, we say that a participant population vector $l = (l_1, \ldots, l_n)$ reaches equilibrium if Equations (3.1) and (3.6) for all $i \in \mathcal{N}$ are satisfied.*

Specifically, a commission-subscription pair $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ determines a participant population equilibrium. In the rest of this paper, when we refer to participant populations l corresponding to $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, we mean that l is under the equilibrium state. Furthermore, if we refer to optimal participant populations, we mean l is under the equilibrium state given optimal commissions and subscriptions $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$.

3.2. Risk-sharing rule and adverse selection

Before we construct the platform's revenue-maximization problem, we first discuss the adverse selection problem and introduce a fair risk-sharing rule of the mutual aid plan under our proposed modeling framework.

The pricing technique of traditional insurance is largely based on the law of large numbers. If policyholders' losses are independent and identically distributed, and the pool size of policyholders is

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sufficiently large, the average cost of insurance claims per policy is close to its theoretical expectation, leading to the net premium principle, which states that the expected premiums are equal to expected benefits. Most traditional insurance products are designed for covering homogeneous risks.

In contrast with conventional insurance, mutual aid plans allow participants' risks to be heterogeneous. For instance, in Xianghubao, participants aged from 1 year old to 60 years old all participate in the same mutual aid plan, which significantly increases the participant population size of risk-sharing groups. The results of our analysis in Appendix A also show that a mutual aid plan allowing participants of various types to join has a lower variance in shared costs than that in a homogeneous risk-sharing group. This outcome supports the idea that the mutual aid platform should enroll all participants of different risk levels in the same mutual aid plan to lower shared costs. However, risk heterogeneity leads to adverse selection problems, which is the major problem that concerns most mutual aid platforms. According to the data from China Insurance Regulatory Commission (2006), a 20-year-old individual's critical illness probability is 0.000340, while a 59-year-old individual's probability is 0.016086, which is 47 times greater than the 20-year-old individual's probability. If the platform set the same shared cost for all different participant types, participants with a higher risk level would benefit more than those with a lower risk level. For instance, if shared costs are the same for all participants, high-risk participants' expected benefits are higher than their expected costs. Thus, participants with a high loss probability are more willing to join the mutual aid plan. This finding raises the expected total benefits of the plan. The low-risk participants must pay more than their expected benefits, so they will decide to opt out of the plan, raising the expected total benefits again; ultimately, only high-risk participants remain in the plan. Thus, it is important to consider the differences between participant types and to offer them different services and charge them different fees.

Recall that w_i, $\forall i \in \mathcal{N}$, is the weight factor of the shared cost s_i of type-i participants. The platform can facilitate transfers between participants by assigning different weight factors to different types of participants. It is notable that the weight factors w_i and benefits I_i are exogenous variables in the revenue maximization problem and should be determined before finding optimal solutions. The values chosen for w_i and I_i determine how participants share their risks in the mutual aid platform. Thus, we investigate the risk exchange scheme by finding an appropriate w_i or I_i so that participants can share their risks fairly.

The actuarial fairness principle we consider in this paper is that the expected value of a participant's income should be equal to the expected value of her outgo. Such fairness principle is also described by Abdikerimova and Feng (2022), who study the fairness issues in mutual aid platforms. This is also in line with the definition in Donnelly (2015): *each member can expect to get back from the fund the same amount as what they have contributed or put at risk*. On one hand, the income of a type-i participant in a mutual aid platform is the benefit he receives when he is critically ill. Thus, the expected value of income is p_iI_i. On the other hand, the outgo of a participant is shared cost contributed to other participants. The expected value of outgo is $(1-p_i)s_i$. The fairness principle requires $p_iI_i = (1-p_i)s_i$. A mutual aid plan is actuarially fair if the equation $p_iI_i = (1-p_i)s_i$ is satisfied. Therefore, in a risk exchange scheme in which participants share their risks fairly, the expected value of benefits that a participant receives should equal the expected value of costs under the equivalence principle:

$$p_iI_i = (1-p_i)s_i = \frac{(1-p_i)w_i \sum_{j\in\mathcal{N}} l_j p_j I_j}{\sum_{j\in\mathcal{N}} l_j(1-p_j)w_j}, \quad \forall i \in \mathcal{N}. \tag{3.7}$$

We say the platform chooses the *fair risk exchange* scheme if Equation (3.7) is satisfied. This terminology was first introduced into the mutual aid platform by Abdikerimova and Feng (2022). In our work, we use a simpler version, in which the expected shared cost s_i only need to satisfy Equation (3.7), while Abdikerimova and Feng (2022) account for the minimum variance in the shared cost.

To have concise expressions for w_i, let the sum of type w_i be normalized to 1, that is, $\sum_{i\in\mathcal{N}} w_i = 1$, and from Equation (3.7), we have

$$w_i = \frac{p_iI_i/(1-p_i)}{\sum_{j\in\mathcal{N}} p_jI_j/(1-p_j)}, \quad \forall i \in \mathcal{N}. \tag{3.8}$$

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Equation (3.8) implies that, under the fair risk exchange scheme, the weight factor w_i is independent of participant population l_i, and it can be preset before the plan starts.

To understand the impact of adverse selection on the mutual aid platform, we elaborate on how participant populations change due to the effect of adverse selection. We consider two simple mutual aid plans in the rest of this subsection. (1) In the first mutual aid plan, participants' shared costs are based on the fair risk exchange scheme. (2) In the second mutual aid plan, participants' shared costs are the same. Suppose that there are only two participant types in these two mutual aid plans and $p_1 < p_2$, $m_1 = m_2 = 1$. Benefits, commission rates, and subscription fees are the same for these two participant types, that is, $I_1 = I_2 = I$, $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. The value distributions of the two participant types are identical and uniformly distributed, that is, $F(v) = \frac{v}{\bar{v}}$. Suppose that $(l_{1,1}, l_{1,2})$ is the equilibrium participant population vector under the first plan. From Equations (3.1) and (3.8), we have

$$l_{1,1} = 1 - \frac{p_1 I\alpha + \beta}{\bar{v}}, \quad l_{1,2} = 1 - \frac{p_2 I\alpha + \beta}{\bar{v}}. \tag{3.9}$$

Suppose that $(l_{2,1}, l_{2,2})$ is the equilibrium population vector under the second plan. From Equation (3.1), we have

$$\begin{cases} l_{2,1} = \frac{-b_1+\sqrt{b_1^2-4a_1c_1}}{2a_1}, \quad l_{2,2} = \frac{-b_2+\sqrt{b_2^2-4a_2c_2}}{2a_3} & , \quad (1-p_2)s(1+\alpha)+\beta-p_2 I \leq \bar{v}, \\ l_{2,1} = \frac{-b_3+\sqrt{b_3^2-4a_3c_3}}{2a_3}, \quad l_{2,2} = 1 & , \quad \text{otherwise}, \end{cases} \tag{3.10}$$

where the expressions of a_i, $b_{i,}$ and c_i for $i = 1, 2, 3$ are listed in the Appendix. To compare the total participant populations of these two plans, we consider the following numerical example.

Example 1. Suppose that $I = 100,000$ yuan, $\bar{v} = 500$, $\alpha = 0.05$, $\beta = 12$, and $p_1 = 0.001$ in both plans. We vary the value of p_2 from 0.001 to 0.005 to determine how adverse selection affects the population in the plan. Figure 5 displays the populations of two participant types for two plans. In the left panel of Figure 5, we see that the population of type-1 participants does not change as p_2 increases because the loss probability of type-2 participants does not affect the type-1 participant population. The decreasing trend of the type-2 population occurs because, as the loss probability increases, type-2 participants must pay more commissions and hence lower the marginal value. In the right panel of Figure 5, observe that the population of type-2 participants increases and reaches 1 as p_2 increases. The higher that the loss probability is, the more benefits that type-2 participants obtain from the platform and type-1 participants. However, since the expected value of costs is higher than the expected value of benefits, some type-1 participants choose to opt out of the plan. The higher that the loss probability is, the fewer type-1 participants that want to join the plan. Figure 6 depicts the total population of the two plans. We can see that the total population of plan 1 is higher than that of plan 2. As p_2 increases, the total population of plan 1 tends to be flat, while the total population of plan 2 sharply decreases. This finding implies that risk heterogeneity does not affect the plan if the shared cost's weight factor is set to satisfy fair risk exchange. In the plan in which participants share the costs equally, risk heterogeneity affects the total population of the plan and causes adverse selection problems.

From the discussion above, we know that the mutual aid plan can cause adverse selection problems because of risk heterogeneity. If the risk exchange is unfair, high-risk-level participants will benefit from the low-risk-level participants. This outcome will increase the population of high-risk-level participants and decrease the population of low-risk-level participants and hence increase the cost of each participant. To reduce the impact of adverse selection, fair risk exchange is introduced into the plan. Under the fair risk exchange scheme, each participant type has her own weight factor, and she needs only bear the costs that are related to her risk levels. Therefore, in the following sections, we assume that the platform adopts the fair risk exchange scheme when maximizing its revenue. The fees play an important role on adverse selection as well. If the fees are not fairly priced (e.g., charge the same fees to all participants),

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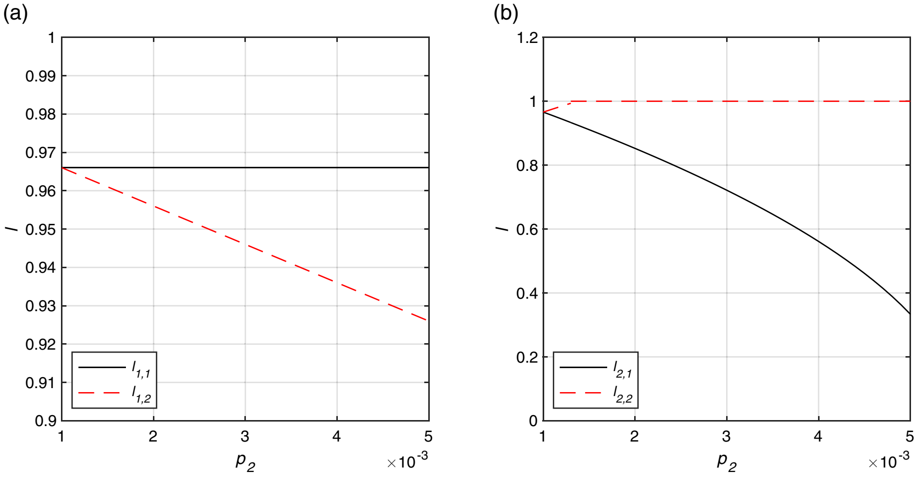

Plan 1 Plan 2

Figure 5. *Population of two participant types in plans.*

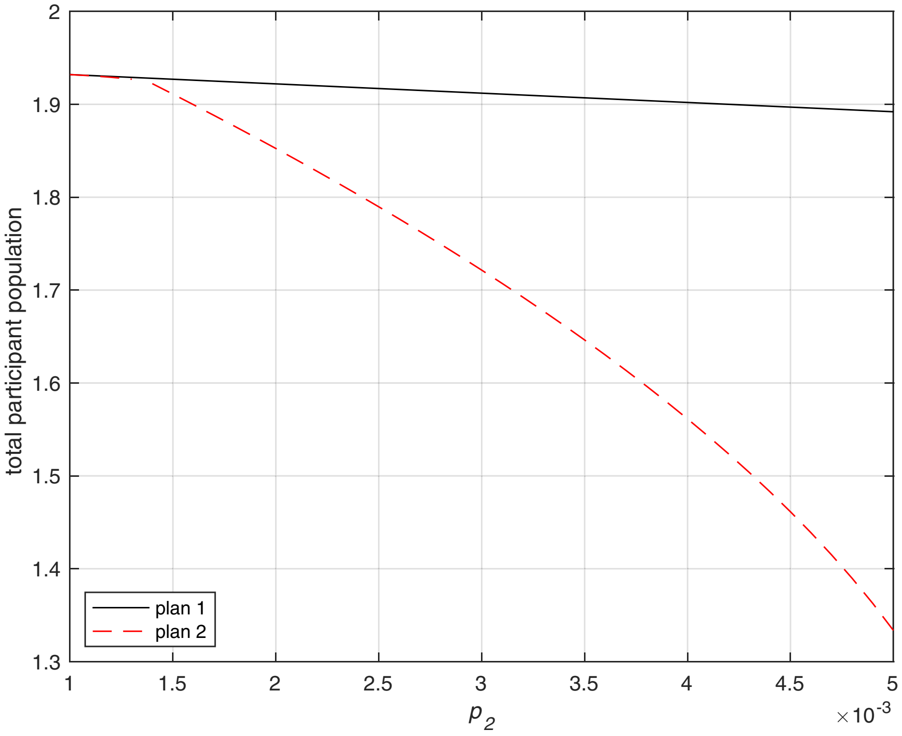

Figure 6. *Total participant population in plans.*

participants with lower values or lower loss probability will choose to opt out of the mutual aid plan while those with higher values of higher loss probability will join in the plan. We will also point out that charging different fees to different types of participants can increase platform's revenues in succeeding sections.

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3.3. Revenue maximization

In Subsection 3.2, we provided analytical results and examples to show that, when shared costs are the same for all participants, there is an adverse selection problem in the mutual aid platform, resulting in participant population loss and hence the platform's revenue loss. Therefore, we assume that the platform adopts the fair risk exchange scheme and discuss the commission/subscription scheme under the fair risk exchange scheme. That is, we select w_i to satisfy Equation (3.8) and allow the expected cost of a participant to equal her expected benefit. Then, the expected shared cost s_i is given by $s_i = p_i I_i/(1-p_i)$, and the expression for Equation l_i can be rewritten as

$$l_i = m_i\big[1 - F_i\big(\alpha_i p_i I_i + \beta_i\big)\big], \quad \forall i \in \mathcal{N}. \tag{3.11}$$

We assume that all participants and the platform have full information about the features of all of the participants and the platform. Therefore, no information asymmetry exists in our modeling framework. Thus, the participants' and the platform's behaviors are not affected by any information frictions. In practice, participants know all shared costs, which are released to public by the platform in each period. The shared cost reflects the information of each type of participants. As describe in Section 3.1, there are an equilibrium state of the mutual aid plan. Participant populations will eventually reach the equilibrium state where participant populations and share costs are stable. In our model, we consider the participant populations under the equilibrium state. Therefore, the information friction and the priori distribution are not incorporated in our model. Moreover, platforms such as Xianghubao have already adopted blockchain techniques in mutual aid plans. Claimants can submit their supporting documents on the blockchain, and investigation firms are able to obtain immediate access to this information. All of the participants in the platform can see the entire process, rendering the information in the plan more transparent and reducing information asymmetry.

The information disclosure mechanism in mutual aid platforms ensures that the platform cannot declare more claims. The detailed information of every approved claims is disclosed to public and other participants can check the eligibility of each claimant. For example, Xianghubao posts patients' information including their names, ages, illness information reports, medical bills, investigation reports and other related documents. Participants can challenge patients' eligibility if they find out some claimants do not meet the requirements. Therefore, the platform is supervised by all participants, and it is difficult for the platform to declare more claims. In conclusion, there is an incentive for the platform to declare more claims but the information disclosure mechanism makes the platform difficult to do that. It is a good point to consider that the loss probabilities are endogenous. However, as discussed above, it is difficult for the platform to declare more claims. Furthermore, since less claims lead to less commissions, there are no incentive for the platform to declare less claims. Thus, the loss probability will not either increase or decrease due to the platform's behaviors. Therefore, we assume that the mutual aid platform's behaviors do not affect participants' probabilities in the model.

The platform chooses commission rates and subscription fees to maximize its revenue. Recall that, given commission rates and subscription fees, a participant population equilibrium can be determined. Thus, when the platform chooses commission rates and subscription fees, a participant population equilibrium is also chosen. The platform's revenue maximization problem is given by

$$\text{(P1)} \quad V_{opt} = \max_{(\boldsymbol{\alpha},\boldsymbol{\beta})} \sum_{i\in\mathcal{N}} \alpha_i l_i p_i I_i + \sum_{i\in\mathcal{N}} \beta_i l_i, \tag{3.12}$$

$$\text{s.t.} \quad l_i = m_i\big[1 - F_i\big(\alpha_i p_i I_i + \beta_i\big)\big], \qquad \forall i \in \mathcal{N}, \tag{3.13}$$

$$\alpha_i \geq 0, \qquad \forall i \in \mathcal{N}, \tag{3.14}$$

$$\beta_i \geq 0, \qquad \forall i \in \mathcal{N}. \tag{3.15}$$

In problem (P1), $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are regarded as decision variables, and $l = (l_1, \ldots, l_n)$ can be derived when $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is determined. The first term in the objective function (3.12) is the revenue from commissions, and the second term is the revenue from subscription fees. Constraint (3.13) comes from Equation (3.1),

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which means that l is an equilibrium participant population vector corresponding to the optimal solution $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. Constraints (3.14) and (3.15) ensure that the solution $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is a feasible commission-subscription vector.

4. Optimal solution

In this section, we first find the optimal solutions to problem (P1). Then, we consider a special commission/subscription scheme, under which commission rates and subscription fees are the same for different participant types. We find the optimal solutions to a revised problem.

4.1. A general case

In this subsection, we find the optimal commission rates and subscription fees for problem (P1) and the corresponding population. The optimal solutions can be analytically derived by implementing the Karush–Kuhn–Tucker (KKT) condition. Therefore, we have the following theorem.

Theorem 1. *Suppose that* $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$ *is the optimal solution to problem (P1), and* l^* *is the corresponding equilibrium population given* $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$*. Under the fair risk exchange scheme, the optimal commission-subscription vector* $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$ *for problem (P1) is not unique, and the relation is given by*

$$\alpha_i^* p_i I_i + \beta_i^* = F_i^{-1}\left(1 - \frac{l_i^*}{m_i}\right), \quad \forall i \in \mathcal{N}, \tag{4.1}$$

where l^* *is determined by solving the following equation:*

$$F_i^{-1}\left(1 - \frac{l_i^*}{m_i}\right) + l_i^* \frac{dF_i^{-1}\left(1 - \frac{l_i^*}{m_i}\right)}{dl_i^*} = 0, \quad \forall i \in \mathcal{N}. \tag{4.2}$$

The platform's maximum revenue is

$$V_{opt} = \sum_{i \in \mathcal{N}} l_i^* F_i^{-1}\left(1 - \frac{l_i^*}{m_i}\right). \tag{4.3}$$

From Theorem 1, we know that the optimal commission rate and subscription fee for each participant type depend on their own characteristics, such as loss probabilities, benefits, and value distributions. Thus, to obtain the maximum revenue, the platform treats participants differently with respect to their types. Note further that, although l depends on $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, Equation (4.2) implies that the optimal equilibrium participant population vector l^* is independent of optimal solution $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$ and only depends on its own population size and value distribution. Platforms can first determine the participant populations of each participant type using Equation (4.2) and target the optimal participant population by choosing appropriate commission rates and subscription fees according to Equation (4.1). Furthermore, a platform's maximum revenue depends on the total potential participant population size of each participant type and their value distribution. This outcome implies that the maximum revenue is determined by participant type features and that the optimal participating population size is achieved by setting optimal commission rates and subscription fees.

We see that the optimal commission/subscription scheme is not unique. However, a platform might prefer a less complicated method to charge participants fees. Hence, the platform can charge only commissions or subscriptions to achieve the maximum revenue. It is straightforward to have the following results from Theorem 1.

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Proposition 3. *A simpler case of the optimal solution to problem (P1) can be obtained as follows:*

- *Setting* $\beta_i^* = 0$ *yields*

$$\alpha_i^* = \frac{F_i^{-1}\left(1 - \frac{l_i^*}{m_i}\right)}{p_i I_i}, \quad \forall i \in \mathcal{N}; \tag{4.4}$$

- *Setting* $\alpha_i^* = 0$ *yields*

$$\beta_i^* = F_i^{-1}\left(1 - \frac{l_i^*}{m_i}\right), \quad \forall i \in \mathcal{N}, \tag{4.5}$$

where l_i^* *is given by Equation (4.2).*

In Proposition 3, $\beta_i^* = 0$ means that the platform only charges participants via commission rates. Some platforms, such as Xianghubao and Kangai Gongshe, employ this method. The subscription fee is waived in this method; hence, participants do not pay any upfront fee to participate in the plan. $\alpha_i^* = 0$ corresponds to the situation in which the platform only charges participants via subscription fees. This method is implemented by platforms such as Waterdrop Mutual Aid and Qingsong Mutual Aid. In this method, the fees paid to the platform are fixed, and participants need not vary fees. Therefore, the platform does not need to consider a complex method to charge both commissions and subscription fees.

Next, we study the relationship between optimal solutions and exogenous variables p_i and I_i. We obtain the following results.

Corollary 1. *The relationships between optimal solutions* α_i^* *and* β_i^* *and* p_i *and* I_i *are given by the following.*

1. *If the platform only charges commissions,* α_i^* *is decreasing in* p_i *and* I_i*;*
2. *If the platform only charges subscriptions,* β_i^* *is independent of* p_i *and* I_i*; and*
3. *If the platform charge both commissions and subscriptions, both* α_i^* *and* β_i^* *are decreasing in* p_i *and* I_i*.*

Corollary 1 explains the differences between α_i^* and β_i^*. When charging only one type of fee, the optimal commission rate depends on the loss probability and benefit, while the optimal subscription rate is independent of them. This finding provides two different ways for the platform to charge fees to participants. These two methods each have advantages and disadvantages. Subscription fees are charged at time $t = 0$, and the total value is fixed, while commissions are charged at time $t = 1$, and they are based on the loss probability and benefit. Also note that the interest rate is not incorporated into our framework. Participants might prefer commissions if the interest rate is considered because they need not make any upfront payments.

We consider a special case of problem (P1) in which we are able to obtain explicit expressions for l and $\boldsymbol{\alpha}$, as well as $\boldsymbol{\beta}$. We assume there are only two types of participants in the plan, that is, $\mathcal{N} = \{1, 2\}$, and the value distributions follow the uniform distribution. The uniform distribution assumption for a continuum of individuals has been widely used in economic literature. For example, Belleflamme *et al.* (2014) consider a unit mass of individuals and assume their marginal utilities are uniformly distributed. These assumptions facilitate the calculations but still maintain the main features of mutual aid plans. Then, we have the following corollary.

Corollary 2. *Suppose there are only two types of participants in the plan, and their value distributions follow the uniform distribution, that is,* $F_i(v) = v/\bar{v}_i$ *for* $i = 1, 2$*, where* $\bar{v}_i$ *is the maximum value of type-i participants. Then, we have analytically optimal solutions to problem (P1). The optimal solution is given by the following.*

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- *If the platform only charges subscription fees, then*

$$\alpha_i^* = \frac{\bar{v}_i}{2p_iI_i}, \quad \beta_i^* = 0, \quad \textit{for } i = 1, 2.$$

- *If the platform only charges commissions, then*

$$\alpha_i^* = 0, \quad \beta_i^* = \frac{\bar{v}_i}{2}, \quad \textit{for } i = 1, 2.$$

The optimal participant population l_i^* *is given by*

$$l_i^* = \frac{m_i}{2}, \quad \textit{for } i = 1, 2.$$

The corresponding maximum revenue is

$$V_{opt} = \frac{m_1\bar{v}_1}{4} + \frac{m_2\bar{v}_2}{4}.$$

Under the uniform distribution, commission rates and subscription fees are considerably easier to compute. Optimal commission rates and subscription fees depend on the maximum values $\bar{v}_i$. The optimal participant population depends on the total potential participant populations m_i and maximum values $\bar{v}_i$. The platform's revenue is determined by m_i and $\bar{v}_i$.

In this subsection, we know that maximizing the platform's revenue by choosing subscription fees and commission rates is essentially achieved by choosing the optimal population of participants. The optimal solution is found by first finding the optimal population. Commission rates and subscription fees are determined through this optimal population. However, note further that the optimal revenue is achieved only if commission rates and subscription fees are allowed to differ for different participant types. If commission rates and subscription fees are the same for all participant types, as observed in many mutual aid platforms, the platform's optimal revenue in problem (P1) might not be achieved. This situation is discussed in the next subsection.

4.2. Same commissions and subscriptions

Note that the optimal commission/subscription scheme in problem (P1) typically requires treating participant types differently in terms of their commissions and subscription fees. However, in practice, many mutual aid platforms still charge the same commissions and subscriptions to all participants. That is, platforms charge the same commissions and subscriptions to all participants. Thus, we have $\alpha_1 = \alpha_2 \ldots = \alpha_n = \alpha$ and $\beta_1 = \beta_2 \ldots = \beta_n = \beta$. Then, the platform's revenue is revised to $\alpha \sum_{i\in\mathcal{N}} l_ip_iI_i + \beta \sum_{i\in\mathcal{N}} l_i$. Charging the same commissions and subscriptions makes it easier for the platform to manage the mutual aid plan and easier for participants to understand. Constraint (3.13) can be rewritten as

$$l_i = m_i\left[1 - F_i(\alpha p_iI_i + \beta)\right]. \tag{4.6}$$

Therefore, the optimization problem (P1) can be redefined as

$$\begin{aligned}
\text{(P2)} \quad V_{opt} = \max_{(\alpha,\beta)} \quad & \alpha \sum_{i\in\mathcal{N}} l_ip_iI_i + \beta \sum_{i\in\mathcal{N}} l_i, && (4.7)\\
\text{s.t.} \quad & l_i = m_i\left[1 - F_i(\alpha p_iI_i + \beta)\right], \qquad \forall i \in \mathcal{N}, && (4.8)\\
& \alpha \geq 0, && (4.9)\\
& \beta \geq 0. && (4.10)
\end{aligned}$$

The optimal solution to problem (P1) no longer applies in problem (P2). It is not possible to obtain an explicit solution to problem (P2) because of the restrictions $\alpha_1 = \alpha_2 \ldots = \alpha_n = \alpha$ and $\beta_1 = \beta_2 \ldots = \beta_n = \beta$. More specifically, in the constraint $l_i = m_i\left[1 - F_i(\alpha p_iI_i + \beta)\right]$ in problem (P2), F_i contains the

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same α and β for all i. Unlike problem (P1), the n constraints are dependent and the optimal solution α and β are related to all l_i, for $i \in \mathcal{N}$. We need to solve out a system of nonlinear equations. It is not possible to solve α and β as well as l_i analytically if the distribution F_i is not given. Thus, an explicit expression for the optimal solution to problem (P2) is not attainable. An approximately optimal commission and subscription can be obtained numerically by defining a grid over the two decision variables (α, β), computing participant populations using constraint (4.8) for each tuple of (α, β) and calculating corresponding platform revenues. Despite not being able to obtain an explicit solution, it is straightforward to have the following proposition from problem (P2).

Proposition 4. *Suppose that $V_{opt}^{(1)}$ is the maximum revenue in problem (P1) and $V_{opt}^{(2)}$ is the maximum revenue in problem (P1); then, $V_{opt}^{(1)} \geq V_{opt}^{(2)}$.*

The implication of Proposition 4 is intuitive. The feasible set of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in problem (P2) is a subset of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in problem (P1). Therefore, the maximum platform revenues in problem (P1) are higher than those in problem (P2). Thus, charging the same commissions/subscriptions is only suboptimal and leads to lower revenues than those under the payment scheme in which the platform charge different commissions/subscriptions.

To illustrate the differences between problems (P1) and (P2), we consider a special case for problem (P2). Suppose that there are two participant types, and their value distributions are uniform distributions. Let $\mathcal{N} = \{1, 2\}$, $F_1(v_1) = v_1/\bar{v}_1$ and $F_2(v_2) = v_2/\bar{v}_2$. Then, we have optimal solutions to problem (P2), the corresponding participant populations and platform revenues. as described in Theorem 2.

Theorem 2. *Suppose that a mutual aid plan consists of two participant types, that is, $\mathcal{N} = \{1, 2\}$. Further, we assume that the value distribution is a uniform distribution. Specifically, $F_1(v_1) = v_1/\bar{v}_1$ and $F_2(v_2) = v_2/\bar{v}_2$.*

1. *The optimal solution to optimization problem (P2) is given by*
   - *If $(\bar{v}_1 - \bar{v}_2)(p_1I_1 - p_2I_2) > 0$ and $(p_1I_1/\bar{v}_1 - p_2I_2/\bar{v}_2)(p_1I_1 - p_2I_2) > 0$,*

$$\alpha^* = \frac{1}{2}\frac{\bar{v}_1 - \bar{v}_2}{p_1I_1 - p_2I_2}, \quad \beta^* = \frac{1}{2}\frac{p_1I_1\bar{v}_2 - p_2I_2\bar{v}_1}{p_1I_1 - p_2I_2}; \tag{4.11}$$

   - *If $(\bar{v}_1 - \bar{v}_2)(p_1I_1 - p_2I_2) \leq 0$ or $(p_1I_1/\bar{v}_1 - p_2I_2/\bar{v}_2)(p_1I_1 - p_2I_2) \leq 0$ ($p_1I_1 \neq p_2I_2$),*

$$\alpha^* = 0, \quad \beta^* = \frac{\bar{v}_1\bar{v}_2}{2}\frac{m_1 + m_2}{m_1\bar{v}_2 + m_2\bar{v}_1}; \tag{4.12}$$

   - *If $p_1I_1 = p_2I_2$,*

$$\alpha^* p_1I_1 + \beta^* = \frac{\bar{v}_1\bar{v}_2}{2}\frac{m_1 + m_2}{m_1\bar{v}_2 + m_2\bar{v}_1}. \tag{4.13}$$

2. *Given the optimal solution (α^*, β^*), the corresponding participant population is given by*
   - *If $(\bar{v}_1 - \bar{v}_2)(p_1I_1 - p_2I_2) > 0$ and $(p_1I_1/\bar{v}_1 - p_2I_2/\bar{v}_2)(p_1I_1 - p_2I_2) > 0$,*

$$l_1^* = \frac{m_1}{2}, \quad l_2^* = \frac{m_2}{2}; \tag{4.14}$$

   - *Otherwise,*

$$l_1^* = \frac{m_1}{2}\left(1 + \frac{m_2(\bar{v}_1 - \bar{v}_2)}{m_1\bar{v}_2 + m_2\bar{v}_1}\right), \quad l_2^* = \frac{m_2}{2}\left(1 + \frac{m_1(\bar{v}_2 - \bar{v}_1)}{m_1\bar{v}_2 + m_2\bar{v}_1}\right). \tag{4.15}$$

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3. *Given the optimal solution* (α^*, β^*)*, the corresponding platform revenue is given by*
   - *If* $(\bar{v}_1 - \bar{v}_2)(p_1I_1 - p_2I_2) > 0$ *and* $(p_1I_1/\bar{v}_1 - p_2I_2/\bar{v}_2)(p_1I_1 - p_2I_2) > 0$,

     $$V_{opt,1} = \frac{m_1\bar{v}_1}{4} + \frac{m_2\bar{v}_2}{4};$$

   - *Otherwise,*

     $$V_{opt,2} = \frac{\bar{v}_1\bar{v}_2}{4}\frac{(m_1+m_2)^2}{m_1\bar{v}_2 + m_2\bar{v}_1}.$$

   We have $V_{opt,1} \geq V_{opt,2}$.

Recall that p_iI_i is the expected benefit of type-i participants. The condition $(\bar{v}_1 - \bar{v}_2)(p_1I_1 - p_2I_2) > 0$ in case 1 in the first part of Theorem 2 implies that (1) $\bar{v}_1 < \bar{v}_2$ and $p_1I_1 > p_2I_2$ or (2) $\bar{v}_1 < \bar{v}_2$ and $p_1I_1 < p_2I_2$. Thus, the optimal solution in case 1 is under the condition that participant types with higher values have higher expected benefits. If the participant type with a higher value has a lower expected benefit (e.g., case 2 of the first part of Theorem 2), the optimal α and β are negative, and the optimal solution in case 1 does not apply. Thus, optimal subscription fee and commission rate are different under different conditions. From Theorem 2, we observe that charging the same fees has less capacity to control the participant population because, unlike heterogeneous fees, the platform cannot target the population of a specific participant type. Furthermore, unlike heterogeneous fees, the optimal commission-subscription solution to problem (P2) is unique in some cases. Thus, there are fewer choices for the platform to design the mutual aid plan.

The second part of Theorem 2 implies that the optimal participant population varies under different cases. The optimal participant population is the same as that in problem (P1) in case 1, while it is different in case 2. The participant population of the participant type with higher maximum value in case 2 is larger than the participant population in case 1. Furthermore, note that the sum of total participant populations are the same in the two cases, that is, $l_1^* + l_2^* = \frac{m_1+m_2}{2}$. Although the total participant populations under different conditions are the same, the corresponding platform revenues are different. We can see that the maximum revenues are different under different cases from the third part of Theorem 2. It shows that charging the same fees does not always achieve the best maximum revenue. By comparing Corollary 2 and Theorem 2, we find that the maximum revenues in the second case of the third part of Theorem 2 are less than those in Corollary 2 because the feasible region of α_i and β_i in problem (P2) is smaller than that in problem (P1). In problem (P2), α_i and β_i are restricted to $\alpha_1 = \alpha_2 \ldots = \alpha_n = \alpha$ and $\beta_1 = \beta_2 \ldots = \beta_n = \beta$. Furthermore, the platform cannot target the optimal participant populations of all participant types when charging the same fees. Therefore, charging the same fees might only have suboptimal solutions and result in revenue losses. However, mutual aid platforms are still charging the same fees to all participants. A simple payment scheme in which the platform charges the same fees is easier for participants to understand and for platforms to implement. Platforms must balance complexity and profitability when designing mutual aid plans.

5. Numerical illustration

In this section, we use Xianghubao as an example to illustrate the implementation of our proposed model. Xianghubao is operated by Ant Financial, which is a financial services company of the internet giant Alibaba. The rise of Xianghubao has been remarkable in the mutual aid market. Xianghubao was introduced in October 2018 and had gained 10 million users by the ninth day after its introduction. As shown in Figure 7, more than 100 million people had participated in the plan by 2020, and it has become the largest mutual aid platform in the market. We implement the model proposed in our paper to analyze the optimal commission/subscription scheme for Xianghubao.

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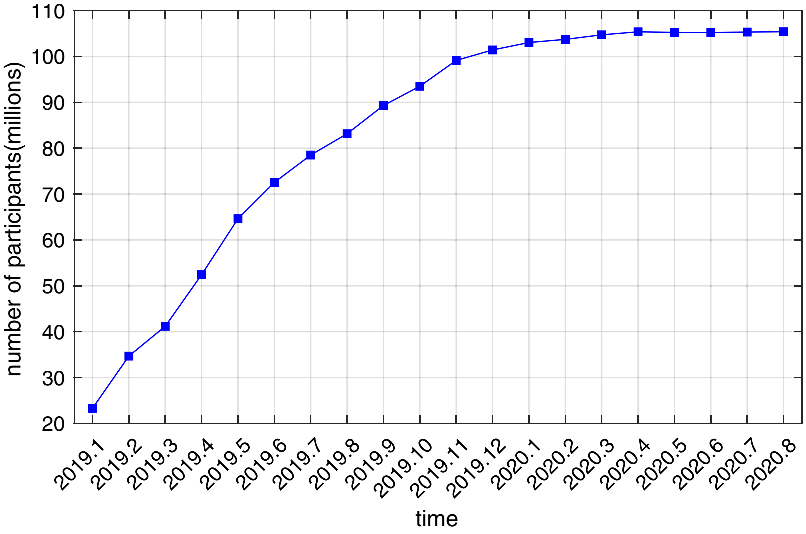

Figure 7. *Number of participants in Xianghubao (in millions).*

Xianghubao only charges commissions, which are also known as administrative fees, and it requires no upfront payment. Any Alipay user is qualified to join the plan for free if she meets basic health requirements, and her Zhima Credit Score, which is a credit system designed by Ant Financial, is 600 or greater. Alipay is a third-party payment platform established by Alibaba and has been the world's largest mobile payment platform since 2013. After claims are filed and confirmed at the end of the mutual plan, all of the participants evenly share the costs of claim benefits. Alipay automatically deducts the amount from participants' accounts. Participants in the mutual aid plan provided by Xianghubao are characterized as two types. The first type consists of participants aged younger than 39 years old, and the second type consists of participants aged between 40 and 59 years. Xianghubao charges the same commission rates to all participant types. Furthermore, the shared costs are the same for all participants in Xianghubao. The benefit amount is 300,000 yuan for the first type of participant and 100,000 yuan for the second type of participant. The current design of commission/subscription schemes does not consider the diversity of participants in the plan. Thus, we use the model proposed in this paper to revise the current mutual aid design of Xianghubao.

We assume that all Alipay users are potential users of Xianghubao. Alipay has a 54.5% share of the third-party payment market in mainland China. Therefore, it is reasonable to use the total population share of internet users in China as the total potential participant population share in the model, that is, m_i. The data on the internet user population can be found at China Internet Network Information Center (2019). Similarly, we use the data from China Insurance Regulatory Commission (2006) as the critical illness probability. People are characterized into 7 different types by age. We use the mean values of the critical illness probabilities of people of the same type to represent the critical illness probability of this type. Table 2 displays the critical illness probability and total population share of internet users for different age groups.

From the table, we find that the differences in critical illness probabilities between age groups are large. Therefore, it is better to divide participants into more than 2 types. Note that Xianghubao only allows participants younger than 60 to participate in the plan. Thus, we divide participants into 5 types accordingly. The total potential participant population is normalized to 1. For instance, the population of potential participants younger than 19 is $m_1 = \frac{23.2}{23.2+21.5+20.8+17.6+10.2} = 0.2487$. We keep the benefit amount the same as in the current plan provided by Xianghubao. The weight factor is calculated using Equation (3.8). Table 3 summarizes the parameter values used in the model.

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Table 2. *Critical illness probability and total internet user population share*

Age group	Critical illness probability	Total population share (%)
<19	0.000244	23.2
20–29	0.000514	21.5
30–39	0.001218	20.8
40–49	0.003997	17.6
50–59	0.011682	10.2
≥60	0.023845	6.7

Table 3. *Critical illness probability and total internet user population share*

Age group	Type(i)	p_i	m_i	I_i (yuan)	w_i
<19	1	0.000244	0.2487	300,000	0.0336
20–29	2	0.000514	0.2304	300,000	0.0709
30–39	3	0.001218	0.2229	300,000	0.1681
40–49	4	0.003997	0.1886	100,000	0.1844
50–59	5	0.011682	0.1093	100,000	0.5430

Table 4. *Current mutual aid plan design ($\alpha = 0.08$) if the value distribution is $F_i(x) = \frac{x}{\bar{v}_i}$*

Scenario	(1)			(2)			(3)		
Type(i)	l_i	s_i	Revenue	l_i	s_i	Revenue	l_i	s_i	Revenue
1	0.1030	73.2179	0.6034	0	0	0	0	0	0
2	0	0	0	0.1736	154.2793	2.1413	0	0	0
3	0	0	0	0	0	0	0.1071	388.5320	3.1307
4	0	0	0	0	0	0	0.1575	388.5320	5.0362
5	0	0	0	0	0	0	0	0	0
Total			0.6034			2.1413			8.1655

We use the current mutual aid plan offered by Xianghubao as a benchmark. As illustrated above, Xianghubao charges an 8% commission rate to all participants. We assume that all participant types' value distributions follow the uniform distribution $F_i(x) = \frac{x}{\bar{v}_i}$ but have various values of $\bar{v}_i$. The parameter value $\bar{v}_i$ of the distribution can be estimated using a questionnaire survey. The questionnaire method has been used widely for determining individual's risk preference. For example, a simple hypothetical gambling with 50% chance of gaining 1000 and 50% chance of loss X is adopted by Handel *et al.* (2015) and Breer and Novikov (2015). X can be used to determine the risk preference. In our manuscript, we can determine participant's value v in a similar way. More specifically, participants can choose a maximum payment X they are willing to afford in a mutual aid platform. Then X can be used to determine v. Three scenarios are considered here: (1) $\bar{v}_i = 10$ for $i = 1, 2, 3$ and $\bar{v}_i = 15$ for $i = 4, 5$; (2) $\bar{v}_i = 50$ for $i = 1, 2, 3$ and $\bar{v}_i = 75$ for $i = 4, 5$; and (3) $\bar{v}_i = 100$ for $i = 1, 2, 3$ and $\bar{v}_i = 150$ for $i = 4, 5$. The results are shown in Table 4. l is calculated by solving a system of nonlinear equations comprising of Equation (3.1) for different participant types. We can see that, when $\bar{v}_i$ is small, participants with a high loss probability do not join the plan because the commission rate is too high for them. When $\bar{v}_i$ increases, participants with a high loss probability are able to tolerate the commission rate and participate in the plan. As a result, participants with a low loss probability choose to opt out of the plan because of adverse selection problems. Note that the results in Table 4 are different from those in reality. There are several

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Table 5. *Optimal mutual aid plan design if charging the same commission/subscription ($F_i(x) = \frac{x}{\bar{v}_i}$)*

	Only commission ($\alpha^* = 0.0186$, $\beta^* = 0$)			Only subscription ($\alpha^* = 0$, $\beta^* = 5.6$)		
Type(i)	l_i^*	s_i(yuan)	Revenue(yuan)	l_i^*	s_i(yuan)	Revenue(yuan)
1	0.2148	73.21	0.2925	0.1094	73.21	0.6127
2	0.1643	154.27	0.4714	0.1014	154.27	0.5678
3	0.0714	365.84	0.4854	0.0981	365.84	0.5493
4	0.0951	401.30	0.7073	0.1182	401.30	0.6620
5	0.0000	1182.01	0.0000	0.0685	1182.01	0.3837
Total			1.9566			2.7755

reasons for this outcome. First, there are waiting periods in mutual aid plans. During the waiting periods, participants must bear the costs of claim benefits but cannot apply for benefits. Therefore, there are more participants to share the cost; hence, the actual shared cost is lower than expected. Second, some claimants' applications might be rejected because of strict regulations, lowering the loss probability and hence lower the shared cost. Third, the population share of younger people in platforms is much larger than that on the entire internet. Thus, the actual shared cost is lower than our results. However, as participants in platforms grow older and more older people join the platforms, the population share of older people will increase, and the shared cost will be higher than the current one.

We next consider the case in which the fair risk exchange scheme is adopted, while the platform still charges the same fees to all participants. We assume the platform only charges commissions or subscription fees. To compare platform revenues between same commissions/subscriptions schemes and different commissions/subscriptions schemes, we assume different types of participants have different value distributions. We choose the same parameter values as those in scenario (1) in the current mutual aid plan example. The value distribution for participants of types 1, 2, and 3 is $F_i(x) = \frac{x}{10}$, while the value distribution for participants of types 4 and 5 is $F_i(x) = \frac{x}{15}$. By solving optimization problem (P2) numerically by restricting $\beta = 0$, we have the optimal commission rate $\alpha^* = 0.0186$ when only charging commissions. Similarly, we can have $\beta^* = 5.6$ when only charging subscriptions. The results are displayed in Table 5. Note that both schemes have improvements in the total revenue compared to current mutual aid plan (0.6034). We further find that only charging commission results in a larger population of low-risk participants and smaller population of high-risk participants. Same commission rates under the fair risk exchange scheme lead to a higher commission for high-risk participants; thereby, fewer high-risk participants choose to participate in the plan. Therefore, the participant populations cannot reach the optimal ones. As a result, the total revenue of the only commission scheme is lower than that of the only subscription scheme.

Next, we propose a more sophisticated design of the commission/subscription scheme for mutual aid plans. The platform adopts the fair risk exchange scheme and charges different fees to different types of participants as illustrated in problem (P1). We use the parameter values in scenario (1) as described above. Additionally, to compare the results between different value distributions, we also adopt exponential distributions $F_i(x) = 1 - e^{-x/\kappa_i}$ for the value distribution here. κ_i is the parameter value of type-i participants' value distribution. The exponential distribution is used by Birge *et al.* (2021) to describe the value of hosts. Here, we assume that $\kappa_i = 5$ for $i = 1, 2, 3$ and $\kappa_i = 7.5$ for $i = 4, 5$. The optimal mutual aid plan under the uniform distribution is shown in Table 6. According to Theorem 1, the optimal solution is not unique. We consider two special cases: only commission and only subscription. The second column corresponds to the optimal solution, in which only a commission is applied, and the third column corresponds to the optimal solution, in which only a subscription fee is applied. The corresponding optimal participant populations, shared costs, and revenues are listed in the fourth, fifth, and sixth column, respectively. The optimal mutual aid plan under the exponential distribution is shown in Table 7.

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Table 6. *Optimal mutual aid plan design ($F_i(x) = \frac{x}{\bar{v}_i}$)*

	Only commission	Only subscription			
Type(i)	α_i^*	β_i^*(yuan)	l_i^*	s_i (yuan)	Revenue (yuan)
1	0.0683	5.0	0.1243	73.21	0.6271
2	0.0324	5.0	0.1152	154.27	0.5761
3	0.0137	5.0	0.1115	365.84	0.5573
4	0.0188	7.5	0.0943	401.30	0.7074
5	0.0064	7.5	0.0547	1182.01	0.4100
Total					2.8725

Table 7. *Optimal mutual aid plan design ($F_i(x) = 1 - e^{x/\kappa_i}$)*

	Only commission	Only subscription			
Type(i)	α_i^*	β_i^* (yuan)	l_i^*	s_i (yuan)	Revenue (yuan)
1	0.0683	5.0	0.0915	73.21	0.4574
2	0.0324	5.0	0.0848	154.27	0.4239
3	0.0137	5.0	0.0820	365.84	0.4101
4	0.0125	7.5	0.0694	401.30	0.5205
5	0.0043	7.5	0.0402	1182.01	0.3016
Total					2.1134

By comparing the results in Tables 4 and 6, we see that the platform's total revenue under the optimal commission/subscription scheme (2.8725) is 475% greater than that under the current mutual aid design (0.6043). Thus, charging different commissions/subscriptions can significantly improve the platform's revenues. Additionally, by comparing Tables 5 and 6, we observe that the total revenue of charging different fees is greater than that of charging the same commissions (1.9566) or subscriptions (2.7755). This finding means that using both the optimal commission/subscription scheme and the fair risk exchange scheme is better than only using the fair risk exchange scheme in mitigating the impact of risk heterogeneity. Thus, only using the fair risk exchange scheme, which has been adopted by some platforms, is not sufficient to reach the platform's maximum revenue. Along with the fair risk exchange scheme, charging different commissions and subscriptions to different types of participants can target the optimal participant population and maximize the platform's revenue.

From the results in Tables 6 and 7, we find that the participant populations under the exponential distribution are lower than those under the uniform distribution. The reason is that the exponential distribution is right skewed; hence, there are fewer participants with lower values. Note further that, subscription fees and commission rates as well as the shared costs under the exponential distribution are the same as those under the uniform distribution. Although maximum revenues are different, the optimal strategy for commissions and subscriptions may not be affected by the choice of F_i. We can also note that the commissions and subscriptions are based on the expectation of distributions in our cases. This also implies that optimal solutions are less affected by the choice of distributions.

In order to further investigate the effect to F_i on the maximum revenue, we calculate the maximum revenue by varying values of parameters in F_i. More specifically, we change values of $\bar{v}_1$ and $\bar{v}_4$ when $F_i(x) = \frac{x}{\bar{v}_i}$ and values of κ_1 and κ_4 $F_i(x) = 1 - e^{x/\kappa_i}$ in the example we discuss above. Numerical results are depicted in Figure 8. Note that parameters $\bar{v}$ and κ can be regarded as the maximum value and the average value of participants, respectively. From the plot, we can see that the maximum revenue is

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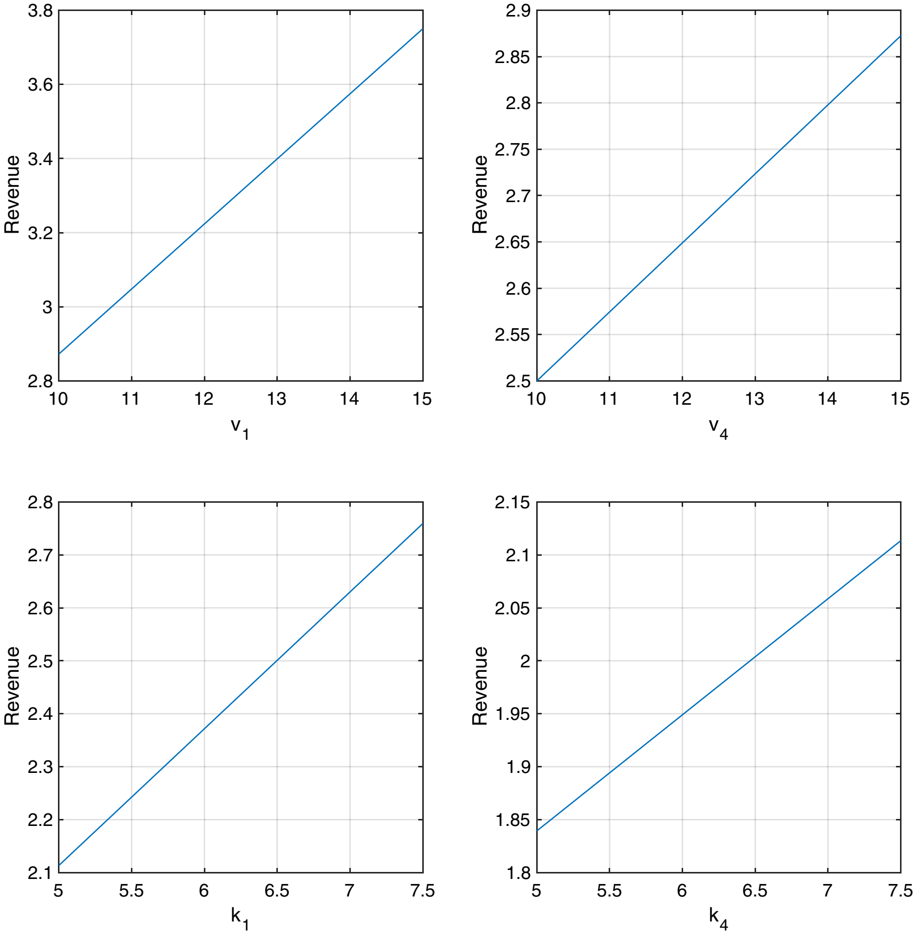

Figure 8. *Maximum revenue against parameter values in F_i.*

increasing in the maximum value or the average value. This suggests that the platform can get higher profits if participants have higher value on the platform. Furthermore, the changing percentages of the maximum revenue with respect to the four parameter values are 61%, 30%, 61%, and 30%, respectively. This means parameter values of F_i have a strong influence on the maximum revenue. Thus, the parameter estimation of F_i is important for determining the maximum revenue. Therefore, optimal solutions and the corresponding maximum revenues depend on distribution parameters.

In this section, we provide new commission/subscription schemes for Xianghubao according to the results of our proposed model. Compared to the current commission/subscription scheme, the new schemes divide participants into more than two types because we consider the importance of risk heterogeneity in the mutual aid plan. The expected shared cost s_i varies across different types and relates to the risk level of the participant type, which is different from the current scheme, in which participants make equal payments. Furthermore, participants pay different fees based on their types under the new scheme, also reducing the effect of risk heterogeneity and increasing platform revenue. Therefore, although the new scheme is much more complicated than the current scheme, the new scheme has many advantages over the current scheme.

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6. Conclusion

In this paper, we propose a modeling framework to describe the commission/subscription scheme of mutual aid platforms. Participants are divided into different types with respect to their loss probability and value distribution. Participants of the same type are heterogeneous in terms of their valuations of the plan. Participants' decisions about whether to participate in the plan depend on their profits. The platform facilitates risk exchanges between participants and charges commissions and subscriptions to participants. The shared costs used to pay benefits to claimants are borne by all of the participants and are determined endogenously. We construct an optimization problem on how to charge commissions and subscriptions to maximize the platform's revenues. There are three key takeaways that we conclude from our analysis results.

- We show that, in general, the platform obtains its maximum revenue by choosing different commission rates/subscription fees for different participant types depending on their total populations and value distributions. Furthermore, charging the same commissions can result in revenue loss. These results emphasize the importance of accounting for risk heterogeneity when designing the commission/subscription scheme in a mutual aid platform.
- Our results show that increasing the population of higher risk participants will raise the shared costs of all participants, and increasing the population of lower risk participants will decrease the shared costs. Changing the proportions of participant types in the plan leads to different shared costs. Moreover, the participant population is affected by commission rates and subscription fees; hence, the platform can control the participant population by setting appropriate commission rates and subscription fees. The optimal commission rates and subscription fees are affected by participants' loss probabilities and value distributions, while the corresponding optimal participant populations depend on participants' total potential participant populations and value distributions.
- Adverse selection has a strong influence on mutual aid plans. Risky participants benefit from less-risky participants if participants pay the same shared costs. A fair risk-sharing rule, which is called the fair risk exchange scheme, was developed to avoid adverse selection problems. Our example results show that fair risk exchange can effectively decrease the influence of adverse selection.

We shed light on how commissions and subscriptions should be set in mutual aid plans. Our proposed modeling framework and numerical results provide mutual aid platforms, insurance companies, and other online platforms with guidelines on how to establish a commission/subscription scheme, as well as a fair risk-sharing rule for a risk-sharing platform. Our results also underscore the importance of accounting for the risk differences between participant types, while many mutual aid platforms do not place notable emphasis on such considerations.

Since we are interested in the optimal design of mutual aid plans in terms of how to charge fees, we neglect other important issues that affect mutual aid plans, such as transaction costs, information friction, default risk, and other externalities. Externalities of individual activities that are not internalized in our proposed model can lead to different results. Furthermore, we focus on a monopolistic mutual aid platform. The optimal design in our model is restricted to the maximum revenue of a monopolistic mutual aid platform. We could extend our model in several ways in the future.

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