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ARTICLE

Costly enforcement in credit economies*

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Abstract

In a monetary model based on Lagos and Wright (2005) where unsecured credit and money are used as means-of-payments, we analyze how the cost and quality of the record-keeping technology affect welfare. Specifically, monitoring agents' debt repayment is costly but is essential to the use of unsecured credit because of limited commitment. To finance this cost, fees on credit transactions are imposed, and the maximum credit limit that is incentive compatible depends on such fees and monitoring level. Alternatively, the use of money avoids such costs. A higher credit limit does not necessarily improve welfare, especially when the limit is high: the benefit from increased trade surpluses from a higher credit limit is offset by the increased cost of monitoring to achieve the improvement. Moreover, under the optimal arrangement, optimal credit limit decreases with the marginal cost of monitoring. When the cost is sufficiently low, a pure credit equilibrium is optimal. When the marginal cost is high, it is optimal to have a pure-currency economy. But when the cost is at an intermediate level, we show that credit is sustainable but not socially optimal. In this range, the implementable credit limit leads to a higher trade surplus than in a pure monetary economy, but owing to the cost of operating the record-keeping system, social welfare in credit equilibrium is lower than the welfare in a pure monetary equilibrium. In addition, we show that there can be a non-monotonic relationship between the optimal record-keeping level and the optimal credit limit.

Keywords: Money; Credit; Limited commitment; Costly record-keeping; Enforcement

JEL classifications: E44; E51

1. Introduction

Nowadays, technological progress has made it faster, cheaper, and more reliable to keep track of records in the financial system. The records include detailed information such as the amount, time, and parties involved in each transaction, laying the foundation for credit arrangements in the financial system (Kocherlakota and Wallace, 1998). However, the record-keeping technology does not eliminate the basic friction that hinders credit transactions, namely, limited commitment of economic agents. Moreover, the cost of establishing a modern credit system which allows for accurate updates of transaction data is still nontrivial. In contrast, an economy can avoid this costly credit arrangement by using money as the only payment instrument. Indeed, in many economies today, most transactions are accomplished with cash alone.[1] These observations naturally lead to two research questions when we consider endogenous record-keeping and enforcement: first,

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when is it feasible to adopt the technology and sustain a credit economy? Second, when the use of credit is sustainable, would it be socially optimal to use it?

In this paper, we propose a monetary model based on Lagos and Wright (2005) with endogenous liquidity needs and introduce a costly and incomplete record-keeping technology to answer these questions. Specifically, we endogenize the level of record-keeping and study the socially optimal use of payment instruments when agents cannot commit to repay their future obligations. Previous literature, as Gu et al. (2016) and Bethune et al. (2018) have shown that in an environment with complete record-keeping and limited commitment, a pure credit economy is better than a pure currency economy in terms of welfare, a result consistent with Kocherlakota (1998) in more general settings.[2] The gain in welfare is that as agents bear the opportunity cost of holding fiat money and thereby bring an inefficient level of liquidity to trade, credit arrangement relaxes this liquidity constraint and hence improves gains from trade. Thus, when the cost of record-keeping is zero, it is optimal to use credit. But once we introduce the cost of record-keeping, where better enforcement requires more real resources from society, this comparison will then depend on the marginal cost of providing a better quality record-keeping service.

Indeed, by endogenizing the record-keeping technology, we capture a nontrivial trade-off that is missing in the literature: while record-keeping of higher quality directly increases liquidity provided by the credit system, its cost requires real resources which will affect liquidity provision indirectly. Unsurprisingly, credit equilibrium is sustainable only when the marginal cost of improving record-keeping is not too high. However, for a range of intermediate marginal costs, we find that there can be a non-monotonic relationship between credit limits and welfare. Moreover, for a range of such marginal costs, a pure monetary economy generates higher welfare than a credit economy, even though a higher level of liquidity can be sustained in a credit economy.

Following Hu et al. (2009) and Andolfatto (2013), we assume that the government has limited coercion power, and the cost of record-keeping has to be borne by private agents in a way that this burden also affects liquidity provision due to limited commitment. Our model shares the standard environment according to Lagos-Wright, where agents meet bilaterally in a decentralized market and use money and/or credit to trade. This decentralized market is followed by a centralized market, where agents repay obligations and rebalance money holdings. Following Kocherlakota and Wallace (1998) and Bethune et al. (2015), we introduce a record-keeping system which is subject to stochastic record-updating; that is, it may fail to record a default when it happens and it is costly to lower the rate of such failures.

Coupled with the limited commitment friction, the accuracy of the record-keeping technology implies a level of self-enforcement that is linked to the incentive-compatible credit limit. To finance a more accurate record-keeping system and hence a higher level of enforcement, agents who access the technology in order to issue credit have to pay a fee used to finance the cost of operating the record-keeping technology. Thus, in principle, better record-keeping does not necessarily lead to higher incentive-compatible credit limit, a result that is missing in the literature when the quality and cost of record-keeping are not endogenized. More precisely, while a higher level of record-keeping has encourages the buyers to repay, the cost of better record-keeping technology also affects incentive constraints through higher fees to finance it. For low marginal costs of record-keeping, better record-keeping increases the incentive compatible credit limit, however, this may not be true for higher marginal costs.

In the normative analysis, our first result characterizes the social welfare that is achievable in a credit equilibrium with different incentive compatible credit limits. For a given marginal cost of record-keeping, we find that the welfare can be non-monotonic in the credit limit. In particular, when the credit limit is low, and below the level of liquidity achievable with money alone, the buyer optimally use money to compensate for insufficient credit. Therefore, higher credit limit would not enhance the overall liquidity but only raise the social cost for better enforcement, thereby reducing welfare.[3] However, while welfare would increase with credit limit after that, it might decrease again when the credit limit is high. The intuition is that the marginal benefit from an increase in credit limit becomes smaller as the limit increases, but the marginal cost that is required remains constant or becomes higher. As a result, the optimal credit limit may not be the highest implementable one.

Our second main result then characterizes the regions under which credit is better than money and vice versa. We characterize the optimal level of record-keeping and credit limit for a given marginal cost of enforcement. By comparing the welfare derived from the optimal credit arrangement with that in a pure currency economy, we explore the socially optimal decision between using money, credit, or both, and the liquidity provided by the optimal means-of-payment. When the marginal cost of enhancing enforcement is low, it is optimal to use credit and optimal credit limit decreases as such marginal cost increases. When such cost is very high, money dominates; in particular, agents would not even be willing to access the credit given the choice as it requires a high credit fee. By contrast, the optimal arrangement in the intermediate range of such marginal costs is more subtle. In such range, the credit limit is high but the cost of record-keeping is also high, leading to the fact that the welfare from the pure credit equilibrium is lower than the welfare from a pure monetary equilibrium. However, agents would be willing to access the credit, because though the credit fee is high, it saves the opportunity cost of holding money.

1.1. Related literature

This paper is related to a growing literature that studies endogenous credit limits by taking limited commitment and enforcement seriously. Conceptually, our formulation of endogenous credit limits follows that in Kehoe and Levine (1993) and Kocherlakota and Wallace (1998), and is similar to those taking the same inheritance to the Lagos-Wright (2005) framework for tractability, including Sanches and Williamson (2010), Chiu et al. (2018), and Bethune et al. (2018), among others.

The paper is also related to the literature that discusses the optimal use of means-of-payments that follows Kocherlakota (1998). We have a similar model setup to Gu et al. (2016), but in their paper the record-keeping technology is complete and costless. Their findings suggest that credit is neutral in terms of both allocation and welfare when money is valued, which is a special case of our results by letting the marginal cost of enforcement be zero. Other relevant papers include Bethune et al. (2018); whose focus is more on the non-stationary credit equilibrium that is welfare-improving, and Araujo and Hu (2018); in which the access to money and credit is exogenous.

Other papers emphasize the imperfection of record-keeping technology. In Kocherlakota and Wallace (1998) and Bethune et al. (2015), the record-keeping technology is incomplete and it is shown that better quality of record-keeping always improves credit condition because it is costless. For the same reason, more credit always has a welfare improving effect. Still others have introduced an exogenous cost to access credit but they assume perfect commitment therefore credit is unconstrained, as in Wang et al. (2020) and Dong and Huangfu (2021). In contrast to these papers, we characterize both the incompleteness and the cost of record-keeping technology and endogenize it by taking the limited-commitment friction seriously.

Another related paper is Lotz and Zhang (2016): in their work, the cost of accessing credit is paid by sellers. This leads to complementarities between buyers and sellers and hence multiple equilibria issue; moreover, it generates inefficiency from bargaining-related holdup problems. Instead, we let buyers pay the credit cost and have all the bargaining power, by means of which we have more tractable equilibrium analysis and can focus more on endogenizing the record-keeping technology and welfare analysis.

Earlier papers indicating that credit is not always welfare-improving include Chiu et al. (2018). The authors capture a general equilibrium effect of better access to credit as it increases credit user's consumption, but also drives up the equilibrium price, which harms the money users. In our work here, agents endogenously choose the means-of-payment and the negative effect of credit comes from the cost of enforcement and limited commitment friction.

2. Environment

The baseline model is based on Lagos and Wright (2005). Time is discrete and has an infinite horizon. The economy is populated by two continuum sets of agents, labeled *buyers* and *sellers*. Every period is divided into two stages. In the first stage there is a decentralized market (DM), where buyers and sellers meet in pairwise meetings, and only buyers desire to consume and only sellers can produce the DM good. The probability that a buyer meets a seller (and vice versa) is $\sigma \in (0, 1]$ and the buyer makes a take-it-or-leave-it-offer (TIOLI) to the seller. In the second stage there is a centralized market where all agents meet. All agents can consume and produce a single good, called the CM good, with linear preferences and a linear technology to produce. Both DM goods and CM goods are perfectly divisible and nonstorable.

The instantaneous utility functions of buyers and sellers are given by

$$U^b(y, x) = u(y) + x \text{ and } U^s(y, x) = -y + x,$$

respectively, where y is the amount of DM consumption (production) and x is the amount of CM consumption (negative number is interpreted as production). The utility function is twice continuously differentiable with $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'(0) = +\infty$, $u'(\infty) = 0$ and $u(0) = 0$. Let y^* denote the consumption level that maximizes the match surplus $u(y) - y$, which is determined by $u'(y^*) = 1$. The cost function for the seller is assumed to be linear for expositional simplicity, and it has no bearing on our results. The discount factor is β, and denote the discount rate as $r = 1/\beta - 1$.

The government operates a financial system with a costly and imperfect monitoring technology. The buyer can access the financial system in the DM by incurring a credit fee χ (in terms of the CM goods) that is due in the CM.[4] The monitoring technology records the buyer's credit transaction in the DM, payment of the credit fee and repayment of debt up to a limit D in the coming CM. The technology updates the buyer's status to be either of good standing (g) or bad standing (b), depending on the buyer's payments. The status of a buyer is observable to all sellers, but the status is updated imperfectly. Any repayment beyond the credit limit D cannot be recognised and recorded. Moreover, if a buyer of status g who incurs debt d in the DM repays at least $\min\{D, d\}$ to the seller and the credit fee χ to the government in the CM, he remains in status g. If he fails to repay $\min\{D, d\}$ to the seller or the credit fee χ in the CM, his status will be updated to b with probability $\epsilon \in [0, 1]$.[5]

The probability of successful record-updating ϵ measures the enforcement level, or the quality of record-keeping.[6] It is endogenously determined and is costly to implement. In particular, the government needs to spend $\xi v(\epsilon)$ units of CM good (in per buyer terms) to sustain ϵ, and $v(\cdot)$ is a convex cost function. We start with a linear cost function $\xi v(\epsilon) = \xi\epsilon$ and generalize the main results under any convex cost function in the last section. With linear cost function, the parameter ξ is the marginal cost of monitoring and enforcement. Such a cost will depend on the technology for reviewing transactions and updating records, but it will also depend on the difficulty to overcome rent-seeking opportunities that would allow a defaulter to avoid a bad record. The government uses the credit fee, imposed on buyers who use credit, to finance this cost.[7] As mentioned above, the credit fee is denoted by χ, and it occurs when the buyer issues credit in a meeting, but is payable only in the following CM.[8]

There is only one asset, fiat money, which is intrinsically useless but is perfectly durable and recognizable. Let M_t denote the aggregate money supply at the beginning of period t, and in the first period, M_0 is given to the buyers evenly. In the baseline model we assume money supply is constant over time, and we discuss the case where money grows at a constant rate in the last section. There is a competitive market for money and CM goods in the CM and we express real balances as z (in terms of the CM goods). We consider stationary equilibrium where the aggregate real balance is constant over time.

In a DM meeting a buyer can always use cash to finance his consumption. Or he can choose to issue credit but subject to the credit fee and repayment constraint. If a buyer defaults on his obligation and is detected with probability ϵ, he is labeled as *bad*. The punishment of a bad record is that the buyer will lose his bargaining power in all the future DM markets, and the seller makes TIOLI offer.[9]

3. Equilibrium analysis

Here we characterize implementable equilibrium allocations. In section 3.1, we begin with a partial equilibrium analysis for an exogenously given credit limit D and credit fee χ. We fully characterize the range of (D, χ) such that different equilibrium exists, according to the use of different means-of-payment. In section 3.2, we move on to endogenize the credit limit. We characterize the *implementable* credit limit D for given enforcement rate ϵ, subject to the incentive constraint and government budget constraint. In section 3.3, we further discuss the social welfare when the planner takes ξ as given and chooses ϵ and D, subject to implementability. We find that money and credit are never co-essential, and an increase in the debt limit may strictly reduce welfare.

3.1. Equilibrium under a given credit limit

In this subsection, we fully characterize different stationary equilibrium that exists under exogenously given credit limit, $D \leqslant y^*$ and credit fee $\chi \geqslant 0$. In particular, we show that a monetary equilibrium ($z > 0$) exists only when the credit limit is not too high, and there is a threshold for the credit limit above which money is dominated by credit and only a pure credit equilibrium exists. When credit limit is below that threshold, depending on the value of credit limit and credit fee, either a coexistence equilibrium exists where buyers use both money and credit in the decentralized trade, or a pure monetary equilibrium exists where only money is used even though credit is provided.

We begin with assuming that buyers always repay their debts and credit fees in the CM if they choose to access the financial system in the DM trade. In the next subsection we shall consider the incentive problem of repayment and endogenize this limit D.

For a given D, and assuming that buyers always repay their debts up to D, a stationary equilibrium consists of equilibrium real balance, z, that each buyer holds when leaving the CM and equilibrium DM trade, (y, p, d), where $p \leqslant z$ is the equilibrium money transfer (denominated in CM-goods terms) from the buyer to the seller, $d \leqslant D$ is the equilibrium credit (denominated in CM-goods terms) that each buyer issues to seller and y is the equilibrium output. Further, equilibrium requires that z be the optimal money holding for each buyer, and, whenever $d > 0$, buyers are willing to access the financial system in the DM and repay the debt d and pay the credit fee χ. Finally, sellers are willing to produce the amount y in exchange for the payment $p + d$.

We first consider the buyer's CM problem. To do so, denote the value function for a buyer who did not access the financial system in the previous DM and enters CM with z real balances by $W^m(z)$, and denote a buyer who accessed the financial system in DM and enters CM with z real balance and d debt by $W^c(z, d)$. We assume that sellers do not carry money across periods and sell all their money holding accumulated from the DM in the coming CM, an assumption that is with no loss of generality, and hence we only consider buyers' problems. The buyers CM values follows:

$$W^m(z) = z + W^m(0) \text{ and } W^c(z, d) = z - d - \chi + W^m(0). \tag{1}$$

Note that $W^c(0, 0) = W^m(0) - \chi$, which reflects the fact that the buyer is obligated to pay the credit fee χ, independent of his debt issuance. Note also that $W^m(0)$ solves the CM problem for a buyer who enters the CM with no money and did not access the financial system, and, by linearity,

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this represents the typical CM problem for the buyer, regardless of whether he has accessed the financial system in the previous DM.

Now we move to the DM problem. Denote the DM value function for a buyer with real balance z by $V(z)$, and it satisfies

$$V(z) = \sigma \max\{V^m(z), V^c(z)\} + (1-\sigma)W^m(z), \tag{2}$$

where $V^m(z)$ is the continuation value without accessing the financial system and solves

$$V^m(z) = \max_{y^m \leqslant p, p \leqslant z} \left\{u(y^m) + W^m(z-p)\right\}, \tag{3}$$

in which (y^m, p) is the offer made to the seller, subject to the liquidity constraint $p \leqslant z$, and the seller participation constraint $y^m \leqslant p$; and where $V^c(z)$ is the continuation value by accessing the financial system and solves

$$V^c(z) = \max_{y^c \leqslant p+d, p \leqslant z, d \leqslant D} \left\{u(y^c) + W^c(z-p, d)\right\}, \tag{4}$$

in which (y^c, p, d) is the offer made to the seller, subject to the liquidity constraint $p \leqslant z$, the credit limit $d \leqslant D$ and the seller participation constraint $y^c \leqslant p + d$. If a buyer is not matched with a seller, the buyer takes real balance z to the CM without debt or tax obligation.

By (1) and (3), the consumption of a buyer who only uses money in the DM is determined by $y^m = y^m(z) = y^*$ if $z \geqslant y^*$, and $y^m = y^m(z) = z$ otherwise. Similarly, by (1) and (4), the consumption of a buyer who accesses the financial system in DM is determined by $y^c = y^c(z, D) = y^*$ if $z + D \geqslant y^*$, and $y^c = y^c(z, D) = z + D$ otherwise. In both cases buyers will use all available payment capacity in DM to consume, until the first-best level of output is achieved.

Now we consider the CM problem, which is given by

$$W^m(0) = \max_z \left\{-z + \beta V(z)\right\}. \tag{5}$$

When choosing his money holding, the buyer anticipates his DM decision to access the financial system or not (which is embedded in the Bellman equation for V in (2)), and plans accordingly. To characterize the optimal decision, define

$$\Psi^m \equiv \max_z \left\{-rz + \sigma[u(y^m(z)) - y^m(z)]\right\}, \tag{6}$$

the solution to which is denoted by $\bar{z}$, and it solves $u'(\bar{z}) = \frac{\sigma + r}{\sigma}$, and define

$$\Psi^c(D) \equiv \max_z \left\{-rz + \sigma[u(y^c(z+D)) - y^c(z+D) - \chi]\right\}, \tag{7}$$

with solution $z = 0$ if $D > \bar{z}$; $z = \bar{z} - D$ otherwise. The following lemma shows that the CM problem can be solved by comparing Ψ^m and $\Psi^c(D)$, and it summarizes equilibrium allocations and means-of-payments as well.

Lemma 3.1. *Let* $\bar{\chi} \equiv \frac{\sigma[u(y^*) - y^*] - \Psi^m}{\sigma}$, *we characterize the monetary equilibrium with the highest* z *below, in which* $y = z + d$.

1. *If* $\chi > \bar{\chi}$, *then in equilibrium real balance* $z = \bar{z}$ *and buyers do not use credit in the DM.*
2. *Otherwise, there exists* $\tilde{D} = \tilde{D}(\chi)$, *determined by* $\Psi^m = \Psi^c(\tilde{D})$ *such that*

   (2.1) *if* $D < \tilde{D}$, *in equilibrium* $z = \bar{z}$ *and buyers do not use credit in the DM;*
   (2.2) *if* $D \in [\tilde{D}, \max\{\bar{z}, \tilde{D}\})$, *in equilibrium* $z = \bar{z} - D$ *and* $d = D$ *in the DM;*
   (2.3) *if* $D \geqslant \max\{\bar{z}, \tilde{D}\}$, *there is no monetary equilibrium.*

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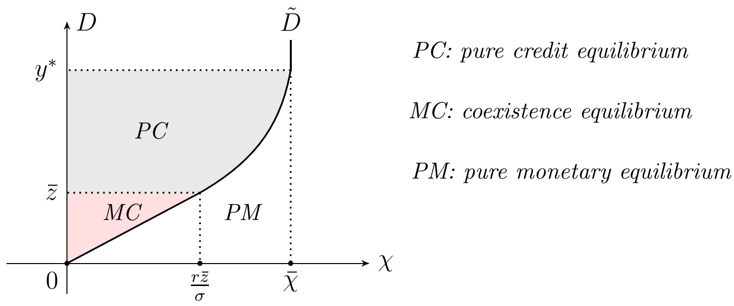

Figure 1. The equilibrium for given credit limit D and credit fee χ.

As shown in Lemma 3.1, the buyer's decision to access the financial system depends on the potential gain from trade achievable, which depends on the credit limit, D, and the cost of accessing it, namely the credit fee χ.

In the extreme case where the credit fee χ is zero, the buyer's decision only depends on the credit limit D. Then the set up is the same as Gu et al. (2016), resulting in similar outcomes: money is not used in equilibrium if the debt limit is high enough, while money and credit are both used otherwise.[10]

However when χ increases, the equilibrium deviates from those observed in Gu et al. (2016), as the buyer now considers both the potential trade surplus from accessing credit, dependent on D, and the associated cost, χ. When χ is above zero but not too high, the buyer's choice between using money or credit hinges on a critical value determined by the threshold $\tilde{D}(\chi)$, derived from $\Psi^c(D) = \Psi^m$. The buyer then maximizes between $\Psi^c(D)$ and Ψ^m to decide whether to access credit. According to Lemma 3.1 (2.1), if the credit limit falls below $\tilde{D}$, the buyer only uses money and holds a real balance of $\bar{z}$. Then in Lemma 3.1 (2.2), when the credit limit lies between $\tilde{D}$ and $\bar{z}$, the buyer uses both money and credit. But as the credit limit is low, the buyer supplements credit with cash one-to-one, and the total liquidity held by buyers are constant at $z + d = \bar{z}$. Finally, when the credit limit exceeds a certain threshold, specifically $D \geqslant \max\{\tilde{D}, \bar{z}\}$, the buyer solely use credit, as outlined in Lemma 3.1(2.3).

When χ is high enough, only money is used in equilibrium. This outcome comes from the fact that the benefit derived from credit trades is bounded above by the first-best gain from trade. As a result, a very high fee for credit trades will deter the buyer from using credit. Lemma 3.1 (3) precisely defines the upper bound $\bar{\chi}$, beyond which buyers never use credit, irrespective of the credit limit. In this scenario, the equilibrium real balance is $z = \bar{z}$.

In Figure 1, we present a summary of the findings outlined in Lemma 3.1. It illustrates the equilibrium outcomes for given credit fee and credit limit. Specifically, when the credit fee is relatively high for a given credit limit, the buyer chooses to only use money, resulting in the existence of a pure monetary equilibrium (PM). Otherwise, when the credit fee is low, the buyer's choice between using money or credit depends on the credit limit. When the credit limit is low, the buyer use both means of payment, leading to a coexistence equilibrium (MC). However, when the credit limit is high, only credit is used, giving rise to a pure credit equilibrium (PC).

3.2. Endogenous credit limit and incentive compatibility

Here we focus on credit equilibrium and endogenize the credit limit D for given monitoring level ϵ, by considering the buyer's incentive to repay and government budget constraints. In particular, for a candidate credit limit D and enforcement level ϵ, the pair is said to be *implementable* if buyers are willing to access the financial system and repay up to D plus the credit fee, and if the credit fees collected are sufficient to finance the cost associated with ϵ, given the equilibrium allocation characterized by Lemma 3.1.

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In the previous section, we have characterized the optimal behavior for a buyer with good standing. Note that since for a buyer with bad standing, it is the seller who makes the TIOLI offer, and since the cost of holding money across periods is positive, the continuation value for such a buyer is zero.

For incentive compatibility, consider a buyer with good standing in the CM, with a loan equal to the credit limit $D \leqslant y^*$ from the previous DM and the credit fee χ. If the buyer repays, his continuation value is $W^c(z, D) = z - D - \chi + W^m(0)$. Alternatively, the buyer could default. Since the record updating technology does not depend on the size of the default up to the credit limit plus the credit fee, it is optimal to default on the total obligation if the buyer chooses to default on any amount. By doing so, with probability ϵ the buyer's standing will be changed to bad, and, as we have seen, this implies a zero continuation value. Thus, to ensure that such a buyer repays his obligation, we need

$$W^c(z, D) \geqslant z + (1-\epsilon) W^m(0), \tag{8}$$

where the left-side is the continuation value if the buyer repays, and the right-side is the continuation value otherwise: z is from the buyer selling his money holding and with probability $1-\epsilon$ he is not caught and can continue as a buyer with good standing. As mentioned, we focus on the case where $D \geqslant \tilde{D}(\chi)$, hence $\Psi^c(D) \geqslant \Psi^m$, and we can rewrite the condition as

$$r(D+\chi) \leqslant \epsilon \Psi^c(D). \tag{9}$$

The government uses the credit fees to finance the cost of monitoring, and only buyers who access the financial system pay those fees. Assume then that all buyers with access use the system, and the government faces the following budget constraint:

$$\sigma\chi = \xi\epsilon. \tag{10}$$

In principle, budget constraint would only require the left-side of (10) to be no less than the right-side; we only consider equality because it is welfare-maximizing and simplifies the notation.

To summarize, the pair (D, ϵ) is implementable if, for χ satisfying (10), the incentive constraint (9) holds, and if the credit equilibrium exists, as in Lemma 3.1. As we focus on the set $D \in [\tilde{D}(\chi), y^*]$, by (10) we can express $\tilde{D}$ as a function of (ϵ, ξ) and hence we use the notation $\tilde{D}(\epsilon; \xi)$ from this point. The following lemma characterizes implementable (D, ϵ) that $D \in [\tilde{D}, y^*]$.

Lemma 3.2. *For a given marginal cost of enforcement, ξ, the pair (D, ϵ) is implementable with $D \geqslant \tilde{D}(\frac{\xi\epsilon}{\sigma}) \equiv \tilde{D}(\epsilon; \xi)$ if and only if*

$$\begin{cases} \Psi^m + rD - \xi\epsilon \geqslant \frac{r}{\epsilon}D + \frac{r\xi}{\sigma} & \text{if } D \in [\tilde{D}, \max\{\bar{z}, \tilde{D}\}); \\ \sigma[u(D) - D] - \xi\epsilon \geqslant \frac{r}{\epsilon}D + \frac{r\xi}{\sigma} & \text{if } D \in [\max\{\bar{z}, \tilde{D}\}, y^*); \\ \sigma[u(y^*) - y^*] - \xi\epsilon \geqslant \frac{r}{\epsilon}D + \frac{r\xi}{\sigma} & \text{if } D = y^*. \end{cases} \tag{11}$$

Moreover, for each pair (ϵ, ξ) there is a threshold $\bar{D} = \bar{D}(\epsilon; \xi)$ such that D satisfies (11) if and only if $D \in [\tilde{D}, \bar{D}]$ (which might be empty in case $\tilde{D} > \bar{D}$).

Given Lemma 3.2, we can then fully characterize implementable (D, ϵ) when $D \in [\tilde{D}, y^*]$.

Proposition 3.1.

1. *For all $\xi > \frac{\sigma}{\sigma+r}\Psi^m$, there is no implementable (D, ϵ) such that $D \geqslant \tilde{D}(\epsilon;\xi)$.*
2. *For all $\xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$, there exists a threshold $\bar{\epsilon}(\xi) > 0$ such that for any $\epsilon \in [0, \bar{\epsilon}(\xi)]$, $\tilde{D}(\epsilon; \xi) \leqslant \bar{D}(\epsilon; \xi)$. Moreover, in this range,*

   (2.a) $\frac{\mathrm{d}\bar{D}(\epsilon;\xi)}{\mathrm{d}\epsilon} \geqslant 0$ and $\frac{\mathrm{d}\bar{D}(\epsilon;\xi)}{\mathrm{d}\xi} \leqslant 0$, with strict inequality if $\bar{D} < y^*$;

   (2.b) $\bar{D}(0; \xi) = 0$, $\bar{D}(\bar{\epsilon}(\xi); \xi) \geqslant \bar{z}$ and the inequality is strict for all $\xi < \frac{\sigma}{\sigma+r}\Psi^m$.

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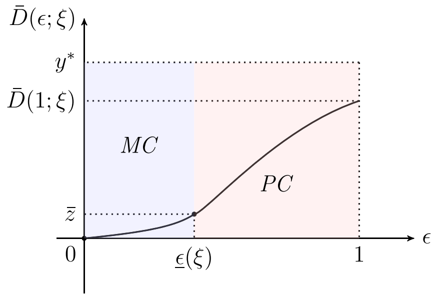

Figure 2. The highest implementable credit limit $\bar{D}$ for a given enforcement rate ϵ.

Proposition 3.1 fully characterizes implementable (D, ϵ) when $D \in [\tilde{D}, y^*]$. Part (1) above shows that when the marginal cost of enforcement is too high ($\xi > \frac{\sigma}{\sigma+r}\Psi^m$), there is no implementable (D, ϵ) such that $D \geqslant \tilde{D}$. Otherwise, for any enforcement rate ϵ below the threshold $\bar{\epsilon}$, Lemma 3.2 implies that any $D \in [\tilde{D}, \bar{D}]$ is implementable under ϵ and Proposition 3.1 (2) implies that the range is not empty. As typical of the credit economies with endogenous credit limits, this implies that there is a continuum of credit limits that are incentive compatible when ξ is low.[11] Here we show more: there is also a range of enforcement rates that are both incentive feasible and fiscally feasible as enforcement is costly.

Our main focus, however, is on optimal arrangements that maximize social welfare. For any given $\xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$ and $\epsilon \leqslant \bar{\epsilon}(\xi)$, as we will later see, the highest implementable credit limit is $\bar{D} = \bar{D}(\epsilon; \xi)$. Proposition 3.1 (2.a) then shows that $\bar{D}$ is an increasing function of ϵ for any $\epsilon \in [0, \bar{\epsilon}(\xi)]$. Interestingly, better record-keeping can influence the highest implementable credit limit $\bar{D}$ through two opposing effects. The first effect arises from an increased probability of being detected and losing all trade surplus, thereby encouraging buyers to fulfill their obligations, leading to a higher credit limit. However, the second effect comes from an increase in the credit fee, χ, which raises the cost of accessing the financial system and exacerbates the incentive to default. Furthermore, a higher credit fee decreases the buyer's continuation value of accessing credit, $\Psi^c(D)$, further increasing the incentive to default and reducing the incentive-compatible credit limit. In the baseline model with linear cost of record-keeping, it turns out that the first effect is stronger. The reason for this is that, to make the arrangement incentive compatible, the marginal cost ξ cannot be too high to begin with and this limits the magnitude of the second effect.[12] In contrast, an increase in ξ (but holding ϵ at constant) only has the second effect, and it decreases $\bar{D}$.

Proposition 3.1 (2.b) describes two boundary conditions of $\bar{D}(\epsilon; \xi)$. Clearly, when $\epsilon = 0$, buyers face no penalty by defaulting, and, anticipating this, sellers would never issue any credit, $\bar{D}(0; \xi) = 0$. When the enforcement is the highest, $\epsilon = \bar{\epsilon}(\xi)$, the credit limit exceeds $\bar{z}$ and hence the buyer does not carry cash. Since $\bar{D}$ is continuous in ϵ, this also implies that there is a cutoff point, $\underline{\epsilon}(\xi)$, below which the credit limit $\bar{D}$ is low, and hence the buyer carries cash to compliment the difference between $\bar{z}$ and $\bar{D}$, but above which the buyer only uses credit and does not carry cash. See Figure 2 for a depiction of the relationship between $\bar{D}$ and ϵ, and the ranges for the optimal portfolio of the buyer.

3.3. Welfare analysis

Here we study the welfare of credit economies. We consider a social planner who takes ξ as given and chooses ϵ and D to maximize welfare, subject to implementability and budget balancing. We do this in two steps. First, for a given ξ, we trace the welfare change as D varies and show that because credit is costly, it is not necessarily optimal to implement the highest possible credit limit. This is also related to the non-monotonicity of debt limit and welfare, noted in the introduction.

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Second, we show that the optimal credit limit decreases with ξ, and it collapses to zero for higher ξ's even though a pure credit equilibrium is implementable.

We first define welfare. A stationary allocation in our economy consists of only the enforcement rate, ϵ, and the level of DM trade, y. Given the allocation (ϵ, y), the corresponding welfare is

$$\mathcal{W}(\epsilon, y) = \sigma[u(y) - y] - \xi\epsilon. \tag{12}$$

In the following analysis we also maintain budget balance according to (10).

If the social planner decides not to use credit, it is then optimal to choose $\epsilon = 0$. In that case, welfare is given by

$$\mathcal{W}^m = \sigma[u(\bar{z}) - \bar{z}], \tag{13}$$

where $\bar{z}$ solves (6), the equilibrium real balance holding in a pure monetary equilibrium where credit is not used. Since the pure monetary equilibrium is implementable for any ξ by setting $\epsilon = 0$, $\mathcal{W}^m$ serves as a lower bound to the optimal welfare. Moreover, it will not be optimal to use credit unless it can achieve a higher welfare than $\mathcal{W}^m$. Finally, since a credit equilibrium is implementable only if $\xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$, we only need to consider that range.

To maximize the welfare from a credit equilibrium, the social planner chooses a pair (ϵ, D) that is implementable. Note that for given ϵ, the social welfare increases with D as long as it is implementable, $\tilde{D}(\epsilon;\xi) \leqslant D \leqslant \bar{D}(\epsilon;\xi)$. Therefore, we can simplify the social planner's decision to the choice of an level of record-keeping ϵ. However, Proposition 3.1 (2.a) implies that there is a one-to-one relationship between the enforcement level ϵ and the highest implementable credit limit $\bar{D}$, (as long as $\bar{D} < y^*$). Hence the social planner's problem can be reduced to choose a level of credit limit D, and then choose the lowest enforcement level such that the credit limit is implementable, denoted as $\epsilon(D;\xi)$, as enforcement is costly. So the social planner's problem then further reduced to

$$\max_{D \in [0, \bar{\bar{D}}(\xi)]} \mathcal{W}^c[D, \epsilon(D;\xi)], \tag{14}$$

where $\bar{\bar{D}}(\xi)$ is the highest D implementable under the marginal cost of enforcement ξ. In this maximization problem, any choice of D is implicitly already the optimal one for the enforcement level $\epsilon(D;\xi)$. Since $\mathcal{W}^c(D,\epsilon)$ is increasing in D and decreasing in ϵ, equation (14) highlights the essential trade-off involved in the optimal credit limit when the level of enforcement is endogenous: a higher D increases the trade surplus in the first term in (12), but it also requires a higher ϵ that increases the cost in the second term. The next proposition shows that this is a nontrivial trade-off.

Proposition 3.2. *Let $\xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$ be given.*

1. *For any $D < \bar{z}$, $\mathcal{W}^c[D, \epsilon(D;\xi)]$ strictly decreases with D.*
2. *Suppose that $u(y) = \frac{y^\alpha}{\alpha}$ with $\alpha \in (0,1)$. If $r < \sigma\frac{1-\alpha}{\alpha}$, then $\mathcal{W}^c[D, \epsilon(D;\xi)]$ first increases with D but then decreases with D for $D \in [\bar{z}, \bar{\bar{D}}]$.*

Proposition 3.2 shows a non-monotonic relationship between credit limits and social welfare. According to Proposition 3.2 (1), it is never optimal to choose a $D < \bar{z}$. Indeed, for implementable D in that range, in equilibrium the buyer uses both money and credit, but equilibrium DM trade is not affected by the credit limit and stays at $\bar{z}$ as D increases. However, higher D still requires higher ϵ and hence welfare strictly decreases with D. For higher value of D, Proposition 3.2 (2) gives a sufficient condition for the welfare to be non-monotonic in D. Indeed, a straightforward differentiation yields

$$\frac{\mathrm{d}\mathcal{W}^c}{\mathrm{d}D} = \sigma[u'(D) - 1] - \xi\frac{\mathrm{d}\epsilon(D;\xi)}{\mathrm{d}D}. \tag{15}$$

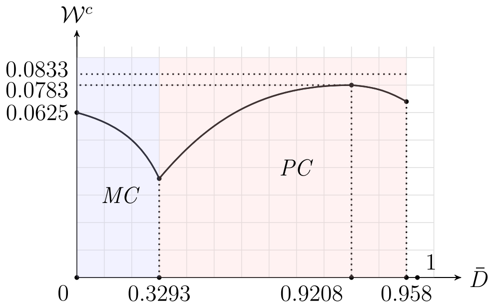

Figure 3. The social welfare for given highest implementable credit limit.

Since the first term decreases with D because of the concavity of u, when D is relatively high, the second term dominates the first. When the highest implementable credit limit approach y^*, a higher credit limit always decreases welfare, because the marginal trade surplus approaches zero but the marginal cost of enforcement remains positive. But even when first-best y^* is not implementable, this non-monotonic relationship still exists, as illustrated in Figure 3.

In the numerical example, $r = 0.08$, $\sigma = 0.25$, $\alpha = 0.75$ and $\xi = 0.005$. Thus the first-best trade outcome is $y^* = 1$ with the first-best social welfare $\mathcal{W}^* = 0.0833$, and the DM trade in a pure monetary equilibrium is $\bar{z} = 0.3293$ with welfare $\mathcal{W}^m = 0.0625$. In a pure credit equilibrium the highest implementable pair is $(D, \epsilon) = (0.958, 1)$. The curve represents the social welfare that obtains when credit is used. In the coexistence equilibrium, social welfare strictly decreases with D, as in Proposition (3.2)(1). In the pure credit equilibrium, social welfare first increases and then decreases with D, and reaches the highest value when $(D, \epsilon) = (0.9208, 0.9596)$. The highest social welfare is 0.0783, accounting for 94.03% of the first-best social welfare, and improves 25.28% from a pure monetary social welfare.

Now we turn to the optimal arrangement, and examine how changes in the marginal cost of enforcement, ξ, affect the optimal means-of-payments. First we give a proposition to characterize the optimal means-of-payment to be used.

Proposition 3.3. *Let $\xi < \frac{\sigma}{\sigma+r}\Psi^m$ be given. Denote the optimal welfare in a credit equilibrium by $\mathcal{W}^c(\xi)$, then there exists $\hat{\xi} < \frac{\sigma}{\sigma+r}\Psi^m$ such that*

$$\mathcal{W}^c(\xi) \geqslant \mathcal{W}^m \text{ if and only if } \xi \in [0, \hat{\xi}]. \tag{16}$$

Hence, it is optimal to implement a pure credit equilibrium for $\xi \in [0, \hat{\xi}]$ and to implement a pure monetary equilibrium for $\xi > \hat{\xi}$. Moreover, $\mathcal{W}^c(\xi)$ strictly decreases with ξ.

Proposition 3.3 shows that the solution to the optimal welfare under credit arrangement always exists. Moreover, it shows that there is a threshold for the marginal cost of enforcement, $\hat{\xi}$, below which a pure credit equilibrium is optimal and above which a pure monetary equilibrium is optimal. Note that above the threshold $\hat{\xi}$ a pure credit equilibrium is implementable but dominated by a pure monetary equilibrium. The intuition for this result is simple. When ξ is close to the threshold $\frac{\sigma}{\sigma+r}\Psi^m$, although a pure credit equilibrium is implementable, the maximal credit limit D is close to $\bar{z}$ and hence the trading surplus in credit trade is not significantly than the monetary trade in a pure monetary equilibrium. However, the credit economy requires a cost of enforcement that is not needed in a monetary equilibrium, and hence welfare in the credit economy must be lower. Our next result states that for ξ of relatively small value, the optimal credit limit decreases with the marginal cost of enforcement.

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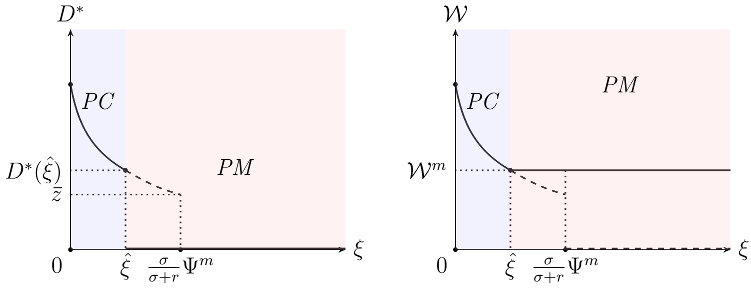

Figure 4. Optimal credit limit and social welfare.

Proposition 3.4. *There exists a threshold $\tilde{\xi} \in (0, \hat{\xi}]$ such that for all $\xi < \tilde{\xi}$, the optimal credit limit, denoted by $D^* = D^*(\xi)$, decreases with ξ. If $u(y) = \frac{y^\alpha}{\alpha}$ with $\alpha \in (0, 1)$, then $\tilde{\xi} = \hat{\xi}$ and D^* strictly decreases with ξ for all $\xi \in [0, \hat{\xi}]$.*

Proposition 3.4 shows that the optimal credit limit decreases with ξ, at least for a range of small ξ's; and, if u is CRRA with $u(0) = 0$, then this holds for all ξ's under which it is optimal to use credit. In Figure 4, we depict the optimal credit limit, D^*, and the optimal social welfare, $\mathcal{W}$, both as functions of ξ. As can be seen from the right panel, around $\xi = \hat{\xi}$ there is a kink to the optimal welfare, which results from the discontinuity in the amount of liquidity under socially optimal means-of-payment in the economy. Indeed, the left panel shows that for any $\xi < \hat{\xi}$, the optimal credit limit is bounded away from $\bar{z}$, but that for $\xi > \hat{\xi}$ the real balance in the pure currency economy is constant at $\bar{z}$. As a result, the output level in the DM will have a discontinuous increase when the marginal cost of enforcement decreases below $\hat{\xi}$. Note that this last result does not depend on the functional form of the utility function u, as $D^*(\xi) > \bar{z}$ for all $\xi < \hat{\xi}$.

4. Extension

4.1. Convex cost of enforcement

We remark here that while we have assumed a linear enforcement cost, our results can be extended to a convex cost case, $\xi v(\epsilon)$ with $v'(\cdot) > 0$, $v''(\cdot) > 0$, $v(0) = 0$, $v'(0) = 0$ and $v(1) = 1$. The analysis then follows the same path as the linear cost case, and we highlight only a few key results. We will show that the monotonic relationship between the highest implementable credit limit and enforcement rate still hold but only when the cost parameter ξ is sufficiently small, and, in this range, the welfare in pure credit equilibrium decreases with ξ. Interestingly, with convex cost, there can be a non-monotonic relationship between the cost parameter and the optimal enforcement level.

With government budget $\sigma\chi = \xi v(\epsilon)$, the incentive constraint, as equation (9), is given by

$$r(D + \frac{\xi v(\epsilon)}{\sigma}) \leqslant \epsilon \Psi^c(D), \tag{17}$$

and the next proposition gives a sufficient condition for implementability:

Proposition 4.1. *When $\xi \leqslant \min\{\frac{1}{v'(1)} \frac{\sigma}{\sigma+r}\Psi^m, r\bar{z}\}$, for any $\epsilon \in [0, 1]$, $D \in [\tilde{D}(\epsilon;\xi), \bar{D}(\epsilon;\xi)]$ is implementable and is not empty, with $\frac{\mathrm{d}\bar{D}(\epsilon;\xi)}{\mathrm{d}\epsilon} \geqslant 0$.*

It is important to note that the impact of enforcement rate on the highest implementable credit limit depends on the balance between two opposing effects. Unlike the linear cost scenarios, here, there may exist a non-monotonic relationship between $\bar{D}$ and ϵ. Specifically, when the enforcement rate is sufficiently high, an increase in enforcement rate can actually reduce the highest

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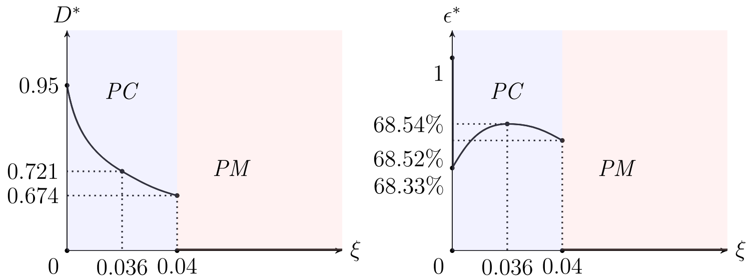

Figure 5. Optimal credit limit and enforcement rate.

implementable credit limit. This phenomenon occurs because although a higher probability of monitor generally enhances the incentive to repay, the associated cost of financing a high enforcement rate through credit fees may become too expensive. Consequently, the incentive constraint for buyers to pay credit fee, on top of the repayment for consumption, may be too stringent. In such cases, the negative effects of high enforcement rate outweigh the positive effects, particularly when the cost function exhibits convexity, leading to rapidly increasing marginal credit fees at higher enforcement rates. In Proposition 4.1, we show that only when ξ is sufficiently small or $\nu'(1)$ is low, does better enforcement lead to a higher implementable credit limit.

Given ξ is sufficiently small, the optimal welfare and credit limit satisfies:

Proposition 4.2. *Let $\xi \leqslant \min\{\frac{1}{\nu'(1)}\frac{\sigma}{\sigma+r}\Psi^m, r\bar{z}\}$, the optimal welfare in the pure credit equilibrium strictly decreases with ξ: $\frac{d\mathcal{W}^c(\xi)}{d\xi} < 0$ with $\mathcal{W}^c(0) > \mathcal{W}^m$. Moreover, optimal credit limit $D^*(\xi)$ decreases with ξ for sufficiently small value of ξ.*

The proof of Proposition 4.2 is same as for Proposition 3.2 and 3.4. Interestingly, with convex cost, the optimal credit limit still decreases with the marginal cost when it is small, but there can be a non-monotonic relationship between the cost parameter ξ and the optimal enforcement level: $\epsilon^* = \epsilon(D^*(\xi); \xi)$:

$$\frac{d\epsilon(D^*(\xi); \xi)}{d\xi} = \frac{d\epsilon}{dD^*}\frac{dD^*}{d\xi} + \frac{d\epsilon}{d\xi}, \tag{18}$$

where the first term is negative and captures an indirect effect of ξ on ϵ^*: when ξ increases, the optimal credit limit decreases, so the enforcement rate needed to implement a lower credit limit also decreases. However, the second term is positive and captures a direct effect: as ξ increases, credit fees increase and agents are less likely to use credit. So for a given credit limit, a higher enforcement rate is required to implement it.

Therefore, the relationship between ϵ^* and ξ and also ϵ^* and D^* can be non-monotonic in some cases. In Figure 5, we depict such an example. Here we have $u(y) = \frac{(y+b)^\alpha - b^\alpha}{\alpha}$ with $\alpha = 0.25, \sigma = 0.18, r = 0.15, b = 0.05$, and the cost of enforcement is given by $\xi\epsilon^m$ with $m = 1.27$. When $0 < \xi < 0.036$, the optimal credit limit increases with the enforcement rate. When $0.036 \leqslant \xi < 0.04$, the optimal credit limit decreases with the enforcement rate. When $\xi \geqslant 0.04$, the social welfare in a credit equilibrium is lower than the welfare in a pure monetary equilibrium.

4.2. Monetary policy

We show here that our main results hold when the money supply grows at a constant rate $\gamma \geqslant 1$ and $M_{t+1} = \gamma M_t$. Note that the underlying friction that the government has no coercion power to tax agents implies that any deflationary monetary policy is not feasible. The monetary policy sets the lower bound of welfare that can be achieved in the economy, but it does not influence either the implementability or the optimal policy when a nonmonetary equilibrium is optimal, which is when the cost parameter ξ is sufficiently low. The next proposition gives a sufficient condition such that the main results hold, where i denotes the nominal interest rate with

$i = \gamma(1 + r) - 1 \geqslant r$, and $\bar{z}(i)$ and $\Psi^m(i)$ denote the equilibrium real balances and value of using money given monetary policy i respectively:

Proposition 4.3. *When* $\xi \leqslant \min\{\frac{\sigma}{\sigma+r}\Psi^m(i), i\bar{z}(i)\}$, *for any* $\epsilon \in [0, 1]$, $D \in [\tilde{D}(\epsilon; \xi, i), \bar{D}(\epsilon; \xi)]$ *is implementable and is not empty, with* $\frac{d\bar{D}(\epsilon;\xi)}{d\epsilon} > 0$. *The optimal welfare in the pure credit equilibrium and optimal credit limit are* $\mathcal{W}^c(\xi)$ *and* $D^*(\xi)$ *and are independent with i.*

Note that monetary policy i does not affect the upper bound of the implementable credit limit $\bar{D}(\epsilon; \xi)$ in a pure credit equilibrium, and therefore does not affect the results in the welfare analysis when a pure credit equilibrium is optimal. The monetary policy i does, however, influence the implementable credit limit in a coexistence equilibrium, but this equilibrium is never a socially optimal choice. Moreover, the monetary policy i also influences the threshold of credit limit below which the buyer prefers money, even though credit is provided, $\tilde{D}(\epsilon; \xi, i)$. However in the welfare analysis, we only consider the highest implementable credit limit that would maximize social welfare.

5. Concluding remarks

We have developed a model of endogenous use of unsecured credit and money, but under costly enforcement. We obtained three main results. First, the use of credit can be sustained in an equilibrium that is both incentive compatible and fiscally feasible only if the marginal cost of enforcement is not overly high. Second, even when it is sustainable, a high enforcement rate may not necessarily be the optimal one; moreover, for a range of marginal cost of enforcement, it is optimal to use money alone while it is incentive compatible and fiscally feasible to sustain the use of credit. Third, there can be a non-monotonic relationship between the optimal credit limit and enforcement levels. These results suggest that looking at the debt-GDP ratio alone or at indexes for institution qualities, which are typically proxies for efficiency of enforcement, as the main guidance for policy recommendations on different use of means-of-payments, can be misleading.

Although the optimal arrangement in our model is either a pure-currency economy or a pure-credit economy, other features could potentially be included in our baseline model to obtain coexistence, such as a two-stage DM structure, as in Araujo and Hu (2018), with different costs of using the monitoring technology across meetings between the two DM stages. Our results can be useful as they point out the basic trade-offs, both for individuals and for the society as a whole.

Notes

1 See The 2020 McKinsey Global Payment Report for the cash usages by country, where in emerging economies, such as Argentina, India, Indonesia, Malaysia and Mexico, cash accounts for an average 80 percent of total transaction volume. See also the Fifth Global Payment System Survey by the World Bank for the cashless transaction per capita according to country income levels.

2 Bethune et al., (2018) show this by assuming a fixed bargaining protocol across the pure currency and the pure credit economies.

3 This result is a natural consequence of the result in Gu et al. (2016) when we add cost to the record-keeping system.

4 Of course we can let sellers pay the cost. But this will cause holdup problem and complementary, as discussed in Lotz and Zhang (2016). We assume that the credit fee χ is fixed but an alternative assumption could be that the fee is proportional to the credit usage. However, this design may not be optimal for welfare. The reason is to finance a specific level of record-keeping, a higher credit fee would be required. If this fee is lump-sum, it would not affect the buyer's credit usage, but will decreases the buyer's use of credit if it is proportional. When this fee is set high, such distortion results in lower implementable credit limit, consequently, lower welfare.

5 Alternatively, we can let buyers pay the credit fee ex ante before accessing credit; by doing so, the record-updating has to be done also ex ante, which means the technology labels a buyer who does not pay the fee directly as *bad*, before the buyer enters DM and uses credit. Because otherwise, buyers could always avoid the tax without leaving any record and being punished. The two versions, paying credit fee ex post or ex ante, do not influence the incentive constraint and the main results.

6 This formulation is commonly used in literature to measure the quality of the record-updating, but it has other interpretations. Kocherlakota and Wallace (1998) interpret the probability as a measure of the average lag between transaction updating. In Bethune et al. (2015), it is a measure of the sophistication of the financial system. In Sanches and Williamson (2010), it represents the fraction of sellers that have monitoring potential.

7 Of course, this can be done through a private sector as well; the main point here is that the monitoring is costly and there is a need for budget-balancing. We assume that this is done by the benevolent government to avoid any further agency problem.

8 We follow Araujo and Hu (2018) and interpret the cost of credit as the social resources invested to the record-keeping system, including to operate the credit card system and to screen, monitor and reveal information. This social cost is financed by taxation called credit fee and agents can always default on it. There are other ways to model costly credit. For example, in Wang et al. (2020) and Dong and Huangfu (2021), buyers incur a fixed or proportional utility cost in the decentralized market while using credit. In Lotz and Zhang (2016), sellers randomly incur heterogeneous utility cost and the cost is zero for a fraction of sellers. Such utility cost allows for heterogeneity but avoid the payment issue that could indirectly impact the incentive constraint and therefore credit limit, which is our main focus in this paper.

9 In Araujo and Hu (2018), they show that such punishment in fact a feature of the optimal trading protocol to enhance incentive-compatibility.

10 In this special case with costless credit fee, the welfare implication is also straightforward and well-known, money and credit is not co-essential, and it is never the case that an increase in the debt limit hurts welfare since credit is costless.

11 See, for example, Bethune et al. (2018) for a general discussion on this point.

12 In the Section 4, we show that under convex record-keeping technology, there can be an non-monotonic relationship between ϵ^* and $\bar{D}$, as the effects are reversed when the enforcement rate becomes excessively large.

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A. Proofs of Lemmas and Propositions

Proof of Lemma 3.1.

We prove Lemma 3.1 by three steps:

(1) There exists $\underline{z} \geqslant 0$ such that, if $\underline{z} > 0$, then $V^m(z) > V^c(z)$ if and only if $z > \underline{z}$; if $\underline{z} = 0$, then $V^m(z) \geqslant V^c(z)$ for all z with strict inequality for all $z > 0$.

(2) $\Psi^c(D) \geqslant \Psi^m$ if and only if $\chi \leqslant \bar{\chi}$ and $D \geqslant \tilde{D}$, where $\tilde{D}$ is the unique solution to $\Psi^c(D) = \Psi^m$.

(3) The buyer chooses to access the financial system if and only if $\Psi^c(D) \geqslant \Psi^m$.

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(1) Let

$$f(z;D,\chi) = V^c(z) - V^m(z) = \sigma[u(y^c(z,D) - y^c(z,D)] - \sigma\chi - \sigma[u(y^m(z)) - y^m(z)]$$

denote the difference between the buyer's DM continuation value when either accessing the financial system or not. It then follows that $f(z;D,\chi)$ strictly decreases with z for $z \leqslant y^*$, with $f(0;D,\chi) = \sigma[u(D) - D] - \sigma\chi$ and $f(y^*;D,\chi) = -\sigma\chi < 0$. We consider two cases. First, suppose that $\chi \leqslant u(D) - D$. Then, there exists a unique $\underline{z}(D,\chi) \in [0, y^*]$ such that $f(\underline{z};D,\chi) = 0$. Second, suppose that $\chi > u(D) - D$. Then, let $\underline{z} = 0$. It follows that $V^m(z) > V^c(z)$ for all $z > \underline{z}(D,\chi)$, and, for the first case, $V^m(z) < V^c(z)$ for all $z < \underline{z}$. Moreover, $\underline{z}(D,\chi)$ increases in D and decreases in χ as f increases in D and decreases in χ.

(2) From equation (7), $\Psi^c(D) = -r(\bar{z} - D) + \sigma[u(\bar{z}) - \bar{z}]$ if $D < \bar{z}$, and $\Psi^c(D) = \sigma[u(D) - D]$ if $\bar{z} \leqslant D \leqslant y^*$. Hence, $\Psi^c(D)$ strictly increases with D for all $0 \leqslant D \leqslant y^*$. From equation (6), $\Psi^m = -r\bar{z} + \sigma[u(\bar{z}) - \bar{z}]$, which does not depend on D. Therefore, $\Psi^c(D) - \Psi^m$ strictly increases with D, with the maximum value $\Psi^c(y^*) - \Psi^m = \sigma[u(y^*) - y^*] - \sigma\chi - \Psi^m$ and the minimum value $\Psi^c(0) - \Psi^m = -\sigma\chi < 0$.

By the intermediate value theorem, when $\Psi^c(y^*) - \Psi^m \geqslant 0$, or equivalently

$$\chi \leqslant \frac{\sigma[u(y^*) - y^*] - \Psi^m}{\sigma} \equiv \bar{\chi},$$

there exists a unique $\tilde{D} \in [0, y^*]$ that solves $\Psi^c(D) = \Psi^m$, and $\Psi^c(D) \geqslant \Psi^m$ if and only if $D \geqslant \tilde{D}$. Otherwise, $\Psi^c(D) < \Psi^m$ for any $0 \leqslant D \leqslant y^*$.

(3) Combining equation (2) and (5), we can rewrite the buyer's CM decision as

$$\max_z \{-rz + \sigma \max\{V^m(z), V^c(z)\}\}. \tag{19}$$

We show that problem (19) has the same solution as

$$\max\{\Psi^m, \Psi^c(D)\} = \max\{\max_z\{-rz + \sigma V^m(z)\}, \max_z\{-rz + \sigma V^c(z)\}\}, \tag{20}$$

by considering three cases.

(3.a) $D < \tilde{D}$. In this case the solution to (20) is such that $\Psi^m > \Psi^c(D)$ and $z = \bar{z}$. Note that $\bar{z} > \underline{z}(D,\chi)$ because

$$f(\bar{z};D,\chi) = -r\bar{z} + \sigma[u(y^c(\bar{z},D)) - y^c(\bar{z},D)] - \sigma\chi - \Psi^m < \Psi^c(D) - \Psi^m < 0.$$

The solution to (19) is also $z = \bar{z}$: if $z > \underline{z}(D,\chi)$, then $V^m(z) > V^c(z)$ and $\max\{-rz + \sigma V^m(z)\} = \Psi^m$, which has solution $\bar{z} > \underline{z}(D,\chi)$; if $z \leqslant \underline{z}(D,\chi)$, then $V^m(z) \leqslant V^c(z)$ and $\max\{-rz + \sigma V^c(z)\} \leqslant \Psi^c(D) < \Psi^m$. Therefore, $\bar{z}$ is also the unique solution of (19).

(3.b) $D \in [\tilde{D}, \max\{\bar{z}, \tilde{D}\})$. Then, the solution to (20) is such that $\Psi^m \leqslant \Psi^c(D)$ and $z = \bar{z} - D$. Note that $\bar{z} - D \leqslant \underline{z}(D,\chi)$ because

$$f(\bar{z} - D;D,\chi) = \Psi^c(D) - [-r(\bar{z} - D) + \sigma(u(\bar{z} - D) - (\bar{z} - D)] \geqslant \Psi^c(D) - \Psi^m > 0.$$

The solution to (19) is also $z = \bar{z} - D$: if $z \leqslant \underline{z}(D,\chi)$, then $V^c(z) \geqslant V^m(z)$, and $\max\{-rz + \sigma V^c(z)\} \leqslant \Psi^c(D)$ which has solution $z = \bar{z} - D \leqslant \underline{z}(D,\chi)$; if $z > \underline{z}(D,\chi)$, then $V^c(z) < V^m(z)$, and $\max\{-rz + \sigma V^m(z)\} = \Psi^m \leqslant \Psi^c(D)$. So the unique solution to (19) is also $\bar{z} - D$.

(3.c) $D \geqslant \max\{\bar{z}, \tilde{D}\}$. The solution to (20) is such that $\Psi^m \leqslant \Psi^c(D)$ and $z = 0$. The solution to (19) is also $z = 0$: if $z \leqslant \underline{z}(D,\chi)$, then $V^m(z) \leqslant V^c(z)$ and $\max\{-rz + \sigma V^c(z)\} \leqslant \Psi^c(D)$ which has solution $z = 0 \leqslant \underline{z}(D,\chi)$; if $z > \underline{z}(D,\chi)$, then $V^m(z) \geqslant V^c(z)$ and $\max\{-rz + \sigma V^m(z)\} = \Psi^m \leqslant \Psi^c(D)$. So the unique solution to equation (19) is the unique solution to equation (20).

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Proof of Lemma 3.2.

Combine equation (9) and (10), we get equation (11) in Lemma 3.2. Rewrite equation (11) by a function $g(D;\epsilon,\xi)$ as

$$g(D;\epsilon,\xi)\equiv\Psi^c(D)-\frac{r}{\epsilon}D-\frac{r\xi}{\sigma}=\begin{cases}\Psi^m+rD-\xi\epsilon-\frac{r}{\epsilon}D-\frac{r\xi}{\sigma} & \text{if} \quad 0\leqslant D<\bar{z};\\ \sigma(u(D)-D)-\xi\epsilon-\frac{r}{\epsilon}D-\frac{r\xi}{\sigma} & \text{if} \quad \bar{z}\leqslant D<y^*;\\ \sigma[u(y^*)-y^*]-\xi\epsilon-\frac{r}{\epsilon}D-\frac{r\xi}{\sigma} & \text{if} \quad D\geqslant y^*,\end{cases} \tag{21}$$

so the incentive constraint is equivalent as $g(D;\epsilon,\xi)\geqslant 0$. Notice that $g(D;\epsilon,\xi)$ is a decreasing function of D for any (ϵ,c), with the maximum value $g(0;\epsilon,\xi)=\Psi^m-\xi\epsilon-\frac{r\xi}{\sigma}$ and the minimum value $g(\infty;\epsilon,\xi)=-\infty$. So there exists a unique $\bar{D}=\bar{D}(\epsilon;\xi)$ such that $g(\bar{D};\epsilon,\xi)=0$ if $\Psi^m-\xi\epsilon-\frac{r\xi}{\sigma}\geqslant 0$; otherwise let $\bar{D}<0$. Then we have $g(D;\epsilon,\xi)\geqslant 0$ for all $D\leqslant\bar{D}$.

Proof of Proposition 3.1.

We prove in three steps:

(1) For $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$, there exists $\bar{\epsilon}(\xi)$ such that $\tilde{D}\leqslant\bar{D}$ if and only if $\epsilon\in[0,\bar{\epsilon}(\xi)]$.

(2) For any $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$ and any $D\leqslant\bar{\bar{D}}\equiv\bar{D}(\bar{\epsilon}(\xi);\xi)$, there is a well-defined and continuously differentiable function $\epsilon(D;\xi)$, as the inverse of $\bar{D}(\epsilon;\xi)$, such that D is implementable if and only if $\epsilon(D;\xi)\leqslant\epsilon\leqslant\bar{\epsilon}(\xi)$.

(3) Properties of $\bar{D}(\epsilon;\xi)$ in Proposition 3.1(2.a) and (2.b).

(1) From equation (21), $\tilde{D}\leqslant\bar{D}$ if and only if $g(\tilde{D};\epsilon,\xi)\geqslant g(\bar{D};\epsilon,\xi)=0$. So we only need to show that there exists $\bar{\epsilon}(\xi)$ such that $g(\tilde{D};\epsilon,\xi)\geqslant 0$ if and only if $\epsilon\in[0,\bar{\epsilon}(\xi)]$ for $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$. For $\xi>\frac{\sigma}{\sigma+r}\Psi^m$, we show that $g(\tilde{D};\epsilon,\xi)<0$ for all $\epsilon\in[0,1]$.

First note that $\tilde{D}$ increases in both ξ and ϵ. To see this, let $\Psi^m=\Psi^c(\tilde{D})$ as in equation (6) and (7): if $\xi\epsilon\leqslant r\bar{z}$, $\tilde{D}=\frac{\xi\epsilon}{r}$ and $\tilde{D}$ increases with ϵ and ξ. If $\xi\epsilon>r\bar{z}$, $\tilde{D}$ is determined by $\sigma[u(\tilde{D})-\tilde{D}]=\Psi^m+\xi\epsilon$, and $\tilde{D}$ increases with ϵ and ξ because $\sigma[u(D)-D]$ increases with D. Moreover, when $\xi\epsilon>r\bar{z}$, $\frac{\tilde{D}(\epsilon;\xi)}{\epsilon}$ is also increasing in ϵ because

$$\frac{\partial\frac{\tilde{D}(\epsilon;\xi)}{\epsilon}}{\partial\epsilon}=\frac{\sigma[u(\tilde{D})-\tilde{D}]-\Psi^m-\sigma\tilde{D}(u'(\tilde{D})-1)}{\epsilon^2\sigma[u'(\tilde{D})-1]}\geqslant\frac{\sigma[u(\bar{z})-\bar{z}]-\Psi^m-r\bar{z}}{\epsilon^2\sigma[u'(\tilde{D})-1]}=0, \tag{22}$$

where the inequality uses the fact that $\tilde{D}\geqslant\bar{z}$ and $\sigma[u(D)-D]-\Psi^m-\sigma D(u'(D)-1)$ is an increasing function of D.

Plug $\tilde{D}$ into $g(D;\epsilon,\xi)$,

$$g(\tilde{D};\epsilon,\xi)=\begin{cases}\Psi^m-\frac{\sigma+r}{\sigma}\xi & \text{if } \xi\epsilon\leqslant r\bar{z};\\ \Psi^m-\frac{r}{\epsilon}\tilde{D}(\epsilon;\xi)-\frac{r\xi}{\sigma} & \text{if } \xi\epsilon>r\bar{z},\end{cases} \tag{23}$$

where in both cases $g(\tilde{D};\epsilon,\xi)$ decreases in ξ and decreases in ϵ. Moreover, if $\xi\epsilon>r\bar{z}$, $g(\tilde{D};\epsilon,\xi)<g(\tilde{D};\frac{r\bar{z}}{\xi},\xi)=\Psi^m-\frac{\sigma+r}{\sigma}\xi$. So for any $\epsilon\in[0,1]$, $g(\tilde{D};\epsilon,\xi)\leqslant\Psi^m-\frac{\sigma+r}{\sigma}\xi$.

When $\xi>\frac{\sigma}{\sigma+r}\Psi^m$, it is straightforward to verify that $g(\tilde{D};\epsilon,\xi)\leqslant\Psi^m-\frac{\sigma+r}{\sigma}\xi<0$, so there is no implementable ϵ and $\bar{\epsilon}(\xi)$ is empty. Now we discuss $\bar{\epsilon}(\xi)$ when $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$ in two cases:

(1.a) $r\bar{z}\geqslant\frac{\sigma}{\sigma+r}\Psi^m$. Given that $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$, $\xi\epsilon\leqslant\frac{\sigma}{\sigma+r}\Psi^m\leqslant r\bar{z}$, and hence $g(\tilde{D};\epsilon,\xi)=\Psi^m-\frac{\sigma+r}{\sigma}\xi\geqslant 0$ for any $\epsilon\in[0,1]$. Thus $\bar{\epsilon}(\xi)=1$.

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(1.b) $r\bar{z} < \frac{\sigma}{\sigma+r}\Psi^m$. We consider two subcases. If $\xi < r\bar{z}$, $\xi\epsilon < r\bar{z}$ for all $\epsilon \in [0,1]$, and hence $g(\tilde{D};\epsilon,\xi) = \Psi^m - \frac{\sigma+r}{\sigma}\xi > \Psi^m - \frac{\sigma+r}{\sigma}r\bar{z} > \Psi^m - \frac{\sigma+r}{\sigma}\frac{\sigma}{\sigma+r}\Psi^m = 0$. Thus, $\bar{\epsilon}(\xi) = 1$. If $r\bar{z} \leqslant \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$, notice that when $\epsilon = 1$, $g(\tilde{D};1,\xi)$ decreases in ξ, with the maximum value $g(\tilde{D};1,r\bar{z}) = \Psi^m - r\bar{z} - \frac{r}{\sigma}r\bar{z} \geqslant 0$ and the minimum value $g(\tilde{D};1,\frac{\sigma}{\sigma+r}\Psi^m) = \Psi^m - r\tilde{D} - \frac{r}{\sigma}\Psi^m = -r\tilde{D} + \sigma[u(\tilde{D}) - \tilde{D}] - \Psi^m \leqslant 0$. So there exists a unique threshold $\dot{\xi} \in [r\bar{z}, \frac{\sigma}{\sigma+r}\Psi^m]$, determined by $g(\tilde{D};1,\dot{\xi}) = 0$, such that $g(\tilde{D};1,\xi) \geqslant 0$ if $\xi \leqslant \dot{\xi}$, and $g(\tilde{D};1,\xi) < 0$ if $\xi > \dot{\xi}$. Then when $r\bar{z} \leqslant \xi \leqslant \dot{\xi}$, if $0 \leqslant \epsilon < \frac{r\bar{z}}{\xi}$, $g(\tilde{D};\epsilon,\xi) = \Psi^m - \frac{\sigma+r}{\sigma}\xi \geqslant 0$, and if $\frac{r\bar{z}}{\xi} \leqslant \epsilon \leqslant 1$, $g(\tilde{D};\epsilon,\xi) \geqslant g(\tilde{D};1,\xi) \geqslant 0$. So in the range $r\bar{z} \leqslant \xi \leqslant \dot{\xi}$, $\bar{\epsilon}(\xi) = 1$, and all $\epsilon \in [0,1]$ is implementable. When $\dot{\xi} < \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$, if $0 \leqslant \epsilon < \frac{r\bar{z}}{\xi}$, $g(\tilde{D};\epsilon,\xi) = \Psi^m - \frac{\sigma+r}{\sigma}\xi \geqslant 0$. If $\frac{r\bar{z}}{\xi} \leqslant \epsilon \leqslant 1$, $g(\tilde{D};\epsilon,\xi)$ decreases in ϵ, with the maximum value $g(\tilde{D};\frac{r\bar{z}}{\xi},\xi) = \Psi^m - \frac{\sigma+r}{\sigma}\xi \geqslant 0$, and the minimum value $g(\tilde{D};1,\xi) < 0$. So there exists a unique $\bar{\epsilon}(\xi) \in [\frac{r\bar{z}}{\xi}, 1)$ which solves $g(\tilde{D};\bar{\epsilon},\xi) = 0$, and $g(\tilde{D};\epsilon,\xi) \geqslant 0$ for all $\epsilon \in [0,\bar{\epsilon}(\xi)]$. So in the range $\dot{\xi} < \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$, only $\epsilon \in [0,\bar{\epsilon}(\xi)]$ is implementable.

(2) First we show that $r\tilde{D}(\epsilon;\xi) \geqslant \xi\epsilon^2$ for any $\epsilon \in [0,\bar{\epsilon}]$. We consider two cases. If $\xi\epsilon < r\bar{z}$, $\tilde{D} = \frac{\xi\epsilon}{r}$ and $r\tilde{D} - \xi\epsilon^2 = \xi\epsilon(1-\epsilon) \geqslant 0$. If $\xi\epsilon \geqslant r\bar{z}$, $\tilde{D}$ is determined by $\sigma[u(\tilde{D}) - \tilde{D}] - \xi\epsilon = \Psi^m$. Thus $r\tilde{D} - \xi\epsilon^2 > r\tilde{D} - \xi\epsilon = r\tilde{D} - \sigma[u(\tilde{D}) - \tilde{D}] + \Psi^m \geqslant r\bar{z} - \sigma[u(\bar{z}) - \bar{z}] + \Psi^m = 0$. It then follows that for any $\epsilon \in [0,\bar{\epsilon}]$, $\frac{\partial g(D;\epsilon,\xi)}{\partial\epsilon} = -\xi + \frac{rD}{\epsilon^2} \geqslant -\xi + \frac{r\tilde{D}}{\epsilon^2} \geqslant 0$.

Given $\epsilon = 1$, first-best is implementable if $g(y^*;1,\xi) \geqslant 0$, which has solution $\xi < \xi^* \equiv \frac{\sigma}{\sigma+r}[\sigma(u(y^*) - y^*) - ry^*]$. Note that $\xi^* \geqslant 0$ if and only if $r \leqslant \frac{\sigma(u(y^*)-y^*)}{y^*}$; otherwise if $r > \frac{\sigma(u(y^*)-y^*)}{y^*}$, y^* is never implementable even for $\xi = 0$. Given any $r \leqslant \frac{\sigma(u(y^*)-y^*)}{y^*}$ and $\xi < \xi^*$, y^* is implementable if $g(y^*;\epsilon,\xi) \geqslant 0$ which has solution $\epsilon \geqslant \epsilon^*(\xi)$. Moreover, $\epsilon^*(\xi)$ increases in ξ. We define

$$\bar{\bar{\epsilon}}(\xi) = \begin{cases} \epsilon^*(\xi) & \text{if } \xi \leqslant \xi^*; \\ \bar{\epsilon}(\xi) & \text{if } \xi^* < \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m. \end{cases} \tag{24}$$

Note that $\bar{\bar{D}}(\xi) = \bar{D}(\bar{\bar{\epsilon}}(\xi);\xi) = \bar{D}(\bar{\epsilon}(\xi);\xi)$ as the highest implementable credit limit for given ξ. Thus,

$$\bar{\bar{D}}(\xi) = \begin{cases} y^* & \text{if } \xi \leqslant \xi^*; \\ \bar{D}(\bar{\epsilon}(\xi);\xi) < y^* & \text{if } \xi^* < \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m. \end{cases} \tag{25}$$

As when $\epsilon = \bar{\bar{\epsilon}}$, $\bar{\bar{D}}$ is implementable, so $g(\bar{\bar{D}};\bar{\bar{\epsilon}},\xi) \geqslant 0$. For any $D \leqslant \bar{\bar{D}}(\xi)$, $g(D;\epsilon,\xi)$ strictly increases in ϵ for $\epsilon \leqslant \bar{\bar{\epsilon}}$, with the minimum value $g(D;0,\xi) = -\infty$ and the maximum value $g(D;\bar{\bar{\epsilon}},\xi) \geqslant g(\bar{\bar{D}};\bar{\bar{\epsilon}},\xi) \geqslant 0$. By the Intermediate Value Theorem, there exists a unique $\epsilon(D;\xi) \in [0,\bar{\bar{\epsilon}}]$ such that $g(D;\epsilon(D;\xi),\xi) = 0$. When $\epsilon \geqslant \epsilon(D;\xi)$, $g(D;\epsilon,\xi) \geqslant 0$. As for $D \in [0,\bar{\bar{D}}]$, $g(D;\epsilon,\xi)$ is a continuously differentiable function on ϵ with positive partial derivative w.r.t ϵ, hence $\epsilon(D;\xi)$ is continuously differentiable.

(3) From equation (21), we define the enforcement rate when $\bar{D} = \bar{z}$ as $\underline{\epsilon}(\xi)$, which is determined by $g(\bar{z};\underline{\epsilon}(\xi),\xi) = \Psi^m + r\bar{z} - \xi\underline{\epsilon} - \frac{r\bar{z}}{\underline{\epsilon}} - \frac{r\xi}{\sigma} = 0$ with $\underline{\epsilon}'(\xi) = \frac{\underline{\epsilon}^2(r/\sigma+\underline{\epsilon})}{r\bar{z}-\xi\underline{\epsilon}^2} > 0$, as $r\bar{z} \geqslant r\tilde{D} \geqslant \xi\epsilon$. To compute the derivatives, we use the implicit function theorem and solve it by taking first-order derivative of $g(\bar{D}(\epsilon;\xi);\epsilon,\xi) = 0$ with respect to ϵ, which implies

$$\left.\frac{d\bar{D}}{d\epsilon}\right|_{0\leqslant\epsilon<\underline{\epsilon}} = \frac{\frac{r}{\epsilon}\bar{D} - \xi\epsilon}{r(1-\epsilon)} > 0 \text{ and } \left.\frac{d\bar{D}}{d\epsilon}\right|_{\underline{\epsilon}\leqslant\epsilon<\bar{\bar{\epsilon}}} = \frac{\frac{r}{\epsilon}\bar{D} - \xi\epsilon}{r - \epsilon\sigma[u'(\bar{D}) - 1]} > 0; \tag{26}$$

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where the inequality comes from $\frac{r}{\epsilon}\bar{D}-\xi\epsilon > \frac{r}{\epsilon}\tilde{D}-\xi\epsilon$ and $r\tilde{D}\geqslant\xi\epsilon$. Similarly, we take first-order derivative of $g(\bar{D}(\epsilon;\xi);\epsilon,\xi)=0$ with respect to ξ to obtain

$$\left.\frac{d\bar{D}}{d\xi}\right|_{0\leqslant\epsilon<\underline{\epsilon}} = \frac{-\frac{r}{\sigma}-\epsilon}{r(1-\epsilon)} < 0; \text{ and } \left.\frac{d\bar{D}}{d\xi}\right|_{\underline{\epsilon}\leqslant\epsilon<\bar{\epsilon}} = \frac{-\frac{r}{\sigma}-\epsilon}{r-\epsilon\sigma[u'(\bar{D})-1]} < 0. \tag{27}$$

Finally, when $\epsilon=0$, it is obvious that $\bar{D}(0;\xi)=0$. When $\epsilon=\bar{\epsilon}(\xi)$, given $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$, by some algebra we can show that $\underline{\epsilon}(\xi)\leqslant\underline{\epsilon}(\frac{\sigma}{\sigma+r}\Psi^m)=\bar{\epsilon}(\xi)$, so $\bar{D}(\bar{\epsilon}(\xi);\xi)\geqslant\bar{D}(\underline{\epsilon}(\xi);\xi)=\bar{z}$.

Proof of Proposition 3.2.

We first prove that the social planner's problem can be reduced to equation (14). For $\xi\leqslant\frac{\sigma}{\sigma+r}\Psi^m$, from Proposition 3.1, a pair (ϵ,D) is incentive compatible if and only if $\epsilon\leqslant\bar{\epsilon}(\xi)$ and $D\in[\tilde{D}(\epsilon;\xi),\bar{D}(\epsilon;\xi)]$. Given an implementable pair, the equilibrium DM trade is given by $y^c=y^c(z+D)$ with z the solution to (7); that is, $y^c=\max\{\bar{z},D\}$. Thus, to maximize welfare among credit equilibria, the problem becomes

$$\max_{(D,\epsilon)} \mathcal{W}^c(D,\epsilon)\equiv\sigma\left[u\left(\max\{\bar{z},D\}\right)-\max\{\bar{z},D\}\right]-\epsilon\xi,$$
$$\text{s.t. } \tilde{D}(\epsilon;\xi)\leqslant D\leqslant\bar{D}(\epsilon;\xi). \tag{28}$$

Now, since y^c increases in D and the welfare increases in y^c (note that D and hence y^c is always below or equal to y^*), it is optimal to choose $D=\bar{D}$ for any given ϵ. As a result, we can reduce the problem to the choice of ϵ. However, Proposition 3.1 (2.a) implies that there is a one-to-one relationship between $\bar{D}$ and ϵ as long as $\bar{D}<y^*$. Now, once $\bar{D}$ reaches y^*, a higher ϵ does not increase the first term in (28) but only decreases the total welfare by increasing the cost in the second term. Thus we may restrict our choice of ϵ so that there is always one-for-one correspondence between $\bar{D}$ and ϵ (so, if $\bar{D}=y^*$ is considered, we just choose the lowest ϵ so that $\bar{D}(\epsilon;\xi)=y^*$).

Now let $\bar{\bar{D}}(\xi)\equiv\bar{D}(\bar{\epsilon}(\xi);\xi)$, the highest $\bar{D}$ implementable under the marginal cost of enforcement ξ. We can then define the inverse of $\bar{D}(\cdot;\xi)$ for the given ξ, and take $\epsilon(D;\xi)$ to be the smallest ϵ so that $\bar{D}(\epsilon;\xi)=D$. Note that the function $\epsilon(D;\xi)$ in $[0,\bar{\bar{D}}]$ is well-defined and continuous as $\bar{D}$ is strictly increasing up to $\bar{D}=y^*$ by Proposition 3.1 (2.a) and (2.b).

Then we show Proposition 3.2(1) and 3.2(2).

(1): When $D<\bar{z}$, $y^c=\bar{z}$ and $\mathcal{W}^c=\sigma[u(\bar{z})-\bar{z}]-\xi\epsilon(D;\xi)$ where $\epsilon(D;\xi)$ is a strictly increasing function in D, (see Proposition 3.1(2.a)). Thus $\frac{d\mathcal{W}^c}{dD}=-\xi\frac{d\epsilon}{dD}<0$.

(2): When $D\geqslant\bar{z}$, $y^c=D$. Rewrite equation (15) as

$$\frac{d\mathcal{W}^c}{dD}=\frac{r}{rD-\xi\epsilon^2}[\sigma D(u'(D)-1)-\xi\epsilon(D;\xi)], \tag{29}$$

where $rD-\xi\epsilon^2>0$ (see the proof of Proposition 3.1(2)), and the sign of $\frac{d\mathcal{W}^c}{dD}$ depends on $\sigma D(u'(D)-1)-\xi\epsilon(D;\xi)$. Notice that since $u(D)=\frac{D^\alpha}{\alpha}$, $\sigma D(u'(D)-1)$ strictly decreases in D and $\epsilon(D;\xi)$ strictly increases in D. So $\sigma D(u'(D)-1)-\xi\epsilon(D;\xi)$ decreases in $D\in[\bar{z},\bar{\bar{D}}(\xi)]$, with maximum value $\sigma\bar{z}(u'(\bar{z})-1)-\xi\epsilon(\bar{z};\xi)=r\bar{z}-\xi\epsilon(\bar{z};\xi)>0$ (see the proof of Proposition 3.1(2)), and the minimum value $\sigma\bar{\bar{D}}(\xi)(u'(\bar{\bar{D}}(\xi))-1)-\xi\epsilon(\bar{\bar{D}}(\xi);\xi)$. Now we show that $\sigma\bar{\bar{D}}(\xi)(u'(\bar{\bar{D}}(\xi))-1)-\xi\epsilon(\bar{\bar{D}}(\xi);\xi)<0$ in three subcases:

(2.a) $\xi\leqslant\xi^*$. From equation (25), $\bar{\bar{D}}(\xi)=y^*$, so $\sigma\bar{\bar{D}}(u'(\bar{\bar{D}})-1)-\xi\epsilon(\bar{\bar{D}};\xi)=-\xi\epsilon(y^*;\xi)<0$.

(2.b) $\xi^*<\xi\leqslant\dot{\xi}$. From equations (24) and (25), $\bar{\bar{D}}<y^*$ and $\epsilon(\bar{\bar{D}},\xi)=1$. Together with the incentive constraint $\sigma[u(\bar{\bar{D}})-\bar{\bar{D}}]-\xi=r\bar{\bar{D}}+\frac{r\xi}{\sigma}$, we rewrite $\sigma\bar{\bar{D}}(u'(\bar{\bar{D}})-1)-\xi\epsilon(\bar{\bar{D}};\xi)=\frac{\sigma}{\sigma+r}[(\sigma+$

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$r)\bar{\bar{D}}u'(\bar{\bar{D}}) - \sigma u(\bar{\bar{D}})]$, which decreases in $\bar{\bar{D}} \in [\bar{z}, y^*)$, so $\sigma\bar{\bar{D}}(u'(\bar{\bar{D}}) - 1) - \xi\epsilon(\bar{\bar{D}}; \xi) \leqslant \frac{\sigma}{\sigma+r}[(\sigma + r)\bar{z}u'(\bar{z}) - \sigma u(\bar{z})] < 0$.

(2.c) $\dot{\xi} < \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$. From the proof of Proposition 3.1(1.b), $\xi\epsilon(\bar{\bar{D}}; \xi) > r\bar{z}$, so $\sigma\bar{\bar{D}}(u'(\bar{\bar{D}}) - 1) - \xi\epsilon(\bar{\bar{D}}; \xi) < \sigma\bar{\bar{D}}(u'(\bar{\bar{D}}) - 1) - r\bar{z} < \sigma\bar{z}(u'(\bar{z}) - 1) - r\bar{z} = 0$.

Therefore, by the Intermediate Value Theorem, there exists $D \in [\bar{z}, \bar{\bar{D}}(\xi)]$, below which $\frac{d\mathcal{W}^c}{dD} > 0$, and above which $\frac{d\mathcal{W}^c}{dD} < 0$.

Proof of Proposition 3.3.

By Proposition 3.2(1), $\mathcal{W}^c[D, \epsilon(D; \xi)]$ strictly decreases in D for $D < \bar{z}$. Thus, we consider only $D \in [\bar{z}, \bar{\bar{D}}(\xi)]$. Moreover, for $\xi > \frac{\sigma}{\sigma+r}\Psi^m$, there is no implementable D. So we only consider $\xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$. In this range, $\mathcal{W}^c[D, \epsilon(D; \xi)] = \sigma[u(D) - D] - \xi\epsilon(D; \xi)$. Proposition 3.1(2) implies that $\mathcal{W}^c[D, \epsilon(D; \xi)]$ is continuously differentiable on (D, ξ) for $\epsilon \in [\underline{\epsilon}(\xi), \bar{\epsilon}(\xi)]$. By the Theorem of Maximum, $\mathcal{W}^c(\xi)$ is continuous in ξ. Moreover, $\bar{\bar{D}}(\xi)$ is continuously differentiable for $\xi \in [0, \xi^*)$ and $\xi \in (\xi^*, \frac{\sigma}{\sigma+r}\Psi^m]$. Now, by the Envelope Theorem,

$$\frac{\mathcal{W}^c(\xi)}{d\xi} = -\xi\frac{d\epsilon}{d\xi} - \epsilon + \lambda\frac{d\bar{\bar{D}}}{d\xi} < 0, \tag{30}$$

where $\lambda > 0$, for $\xi < \xi^*$ and $\xi^* < \xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m$. This, together with the continuity of $\mathcal{W}^c$, implies that $\mathcal{W}^c(\xi)$ strictly decreases in ξ for all $\xi \in [0, \frac{\sigma}{\sigma+r}\Psi^m]$.

Now, when $\xi = 0$, $D^*(0) = y^* > \bar{z}$ and hence $\mathcal{W}^c(0) > \mathcal{W}^m$. When $\xi = \frac{\sigma}{\sigma+r}\Psi^m$, the only implementable $D = \bar{\bar{D}}(\frac{\sigma}{\sigma+r}\Psi^m) = \bar{z}$, so at optimal $D^*(\frac{\sigma}{\sigma+r}\Psi^m) = \bar{z}$ and

$$\mathcal{W}^c\left(\frac{\sigma}{\sigma+r}\Psi^m\right) = \sigma[u(\bar{z}) - \bar{z}] - \frac{\sigma}{\sigma+r}\Psi^m\bar{\epsilon}\left(\bar{z}; \frac{\sigma}{\sigma+r}\Psi^m\right) < \mathcal{W}^m.$$

Thus there exists a unique $\hat{\xi} \in [0, \frac{\sigma}{\sigma+r}\Psi^m]$ such that $\mathcal{W}^c(\hat{\xi}) = \mathcal{W}^m$.

Proof of Proposition 3.4.

From Proposition 3.3, when $\xi \leqslant \hat{\xi}$, $\mathcal{W}^c(\xi) = \sigma[u(D) - D] - \xi\epsilon(D; \xi)$ and is twice continuously differentiable. The ranges of the choice variable $D \in [\bar{z}, \bar{\bar{D}}(\xi)]$ are both weakly decreasing in ξ. Moreover, there exists $\tilde{\xi}$ such that $\frac{\partial^2\mathcal{W}^c(\xi)}{\partial D\partial\xi} < 0$ for $\xi < \tilde{\xi}$, because $\frac{\partial^2\mathcal{W}^c(\xi)}{\partial D\partial\xi}$ is continuous on ξ and when $\xi = 0$:

$$\frac{\partial^2\mathcal{W}^c(\xi)}{\partial D\partial\xi}|_{\xi=0} = \left(-\frac{\partial\epsilon}{\partial D} - \xi\frac{\partial^2\epsilon}{\partial D\partial\xi}\right)|_{\xi=0} = -\frac{\partial\epsilon}{\partial D}|_{\xi=0} = -\frac{r\epsilon - \epsilon^2\sigma[u'(D) - 1]}{rD} < 0. \tag{31}$$

Then by supermodularity, the optimal credit limit $D^*(\xi)$ decreases in ξ when $\xi \leqslant \tilde{\xi}$.

Given functional form $u(y) = \frac{y^\alpha}{\alpha}$ $(0 \leqslant \alpha \leqslant 1)$ and $\xi \leqslant \hat{\xi}$, we discuss the optimal credit limit $D^*(\xi)$ in two cases:

1. $r < \sigma\frac{1-\alpha}{\alpha}$. From Proposition 3.2(2), social welfare first increases in D and then decreases in D, so for any $\xi \in [0, \hat{\xi}]$, the optimal credit limit is uniquely determined by FOC, $\sigma D^*[u'(D^*) - 1] = \xi\epsilon(D^*, \xi)$, with $\frac{dD^*}{d\xi} = \frac{\epsilon + c\frac{d\epsilon}{d\xi}}{\sigma[u'(D^*) - 1 + D^*u''(D^*)] - \xi\frac{d\epsilon}{dD^*}} < 0$.

2. $r \geqslant \sigma\frac{1-\alpha}{\alpha}$. From Proposition 3.1, for any $\xi \leqslant \hat{\xi}$, $\bar{\epsilon}(\xi) = 1$ because $r\bar{z} \geqslant \frac{\sigma}{\sigma+r}\Psi^m$. Then by the similar method used in the proof of Proposition 3.2(2.b), social welfare in a pure credit

equilibrium now increases in D as $\frac{d\mathcal{W}^c[D,\epsilon(D;\xi)]}{dD} > 0$ for all $D \in [\bar{z}, \bar{\bar{D}}(\xi)]$. So the optimal credit limit is $D^*(\xi) = \bar{\bar{D}}(\xi)$, which decreases with ξ as $\frac{d\bar{\bar{D}}}{d\xi} = \frac{d\bar{D}(1;\xi)}{d\xi} < 0$ as in equation (27).

Proof of Proposition 4.1.

Rewrite the incentive constraint as in equation (21): $g(D;\epsilon,\xi) = \Psi^c(D) - \frac{r}{\epsilon}D - \frac{r}{\sigma\epsilon}\xi v(\epsilon)$, which is a decreasing function on D with minimum value negative. We prove that $\xi \leqslant \min\{\frac{1}{v'(1)}\frac{\sigma}{\sigma+r}\Psi^m, r\bar{z}\}$ is a sufficient condition for implementability by showing that $g(\tilde{D};\epsilon,\xi) \geqslant 0$ for any $\epsilon \in [0,1]$. Note that given $\xi \leqslant r\bar{z}$ and the assumption $v(1) = 1$, $\xi v(\epsilon) \leqslant r\bar{z}$ for any ϵ. So $\tilde{D} = \frac{\xi v(\epsilon)}{r}$ and $g(\tilde{D};\epsilon,\xi) = \Psi^m - \frac{\sigma+r}{\sigma}\frac{\xi v(\epsilon)}{\epsilon} > \Psi^m(1 - \frac{v'(\epsilon)}{v'(1)}) \geqslant 0$.

The highest implementable credit limit $\bar{D}$ increases with ϵ as: when $\bar{D} < \bar{z}$, $\frac{d\bar{D}}{d\epsilon} = \frac{\Psi^c(\bar{D})-\xi v'(\epsilon)(\epsilon+\frac{r}{\sigma})}{r(1-\epsilon)} \geqslant \frac{\Psi^c(\tilde{D})-\xi v'(\epsilon)(\epsilon+\frac{r}{\sigma})}{r(1-\epsilon)} = \frac{\Psi^m-\xi v'(\epsilon)(\epsilon+\frac{r}{\sigma})}{r(1-\epsilon)} \geqslant \frac{\Psi^m(1-\frac{v'(\epsilon)}{v'(1)})}{r(1-\epsilon)} \geqslant 0$; and when $D \geqslant \bar{z}$, $\frac{d\bar{D}}{d\epsilon} = \frac{\Psi^c(\bar{D})-\xi v'(\epsilon)(\epsilon+\frac{r}{\sigma})}{r-\sigma\epsilon(u'(\bar{D})-1)} \geqslant \frac{\Psi^m(1-\frac{v'(\epsilon)}{v'(1)})}{r-\sigma\epsilon(u'(\bar{D})-1)} \geqslant 0$.

Proof of Proposition 4.2.

The proof follows the same path as under linear cost. Optimal social welfare strictly decreases with ξ is proved by the Envelope Theorem as in equation (30), and with convex cost, $\frac{d\mathcal{W}^c(\xi)}{d\xi} = -\epsilon v'(\epsilon)\frac{d\epsilon}{d\xi} + \lambda\frac{\bar{\bar{D}}}{d\xi} - v(\epsilon) < 0$ with $\lambda > 0$. The optimal credit limit decreases with ξ when ξ is sufficiently small is proved by supermodularity as in equation (31), and with convex cost:

$$\frac{\partial^2\mathcal{W}^c(\xi)}{\partial D\partial\xi}|_{\xi=0} = \left(-v'(\epsilon)\frac{\partial\epsilon}{\partial D} - \xi v'(\epsilon)\frac{\partial^2\epsilon}{\partial D\partial\xi}\right)|_{\xi=0} = -v'(\epsilon)\frac{\partial\epsilon}{\partial D}|_{\xi=0} < 0.$$

Proof of Proposition 4.3.

Given $i \geqslant r$, the equilibrium real balances is $\bar{z}(i) \leqslant \bar{z}$ which solves $\sigma[u'(\bar{z}(i)) - 1] = i$, and the continuation value of using money only is $\Psi^m(i) = -i\bar{z}(i) + \sigma[u(\bar{z}(i)) - \bar{z}(i)] \leqslant \Psi^m$. The continuation value of using credit when $D < \bar{z}(i)$ is $\Psi^c(D;i) = \Psi^m(i) + iD - \xi\epsilon$; otherwise it is the same as described in Lemma 3.2.

The incentive constraint is now: $g(D;\epsilon,\xi,i) = \Psi^c(D;i) - \frac{r}{\epsilon}D - \frac{r}{\sigma}\xi \geqslant 0$, which is either a strictly decreasing function on D, or a first increasing then decreasing function on D, depending on the value of ϵ and i. We will show that in both cases, given $\xi \leqslant \min\{\frac{\sigma}{\sigma+r}\Psi^m(i), i\bar{z}(i)\}$, $g(\tilde{D}(\epsilon;\xi,i);\epsilon,\xi,i) \geqslant 0$ for any $\epsilon \in [0,1]$, therefore, there exists $\bar{D} > \tilde{D}$ such that $g(\bar{D};\epsilon,\xi,i) = 0$. To prove it, note that if $\xi \leqslant i\bar{z}(i)$, $\tilde{D}(\epsilon;\xi,i) = \frac{\xi\epsilon}{i} \leqslant \bar{z}(i)$. So $g(\tilde{D}(\epsilon;\xi,i);\epsilon,\xi,i) = \Psi^m(i) - \frac{r}{i}\xi - \frac{r}{\sigma}\xi \geqslant \Psi^m(i) - \xi - \frac{r}{\sigma}\xi \geqslant 0$ as $\xi \leqslant \frac{\sigma}{\sigma+r}\Psi^m(i)$.

To show that the welfare implication is the same as it is without inflation, we need to prove that the highest implementable credit limit $\bar{\bar{D}}(\xi) = \bar{D}(1;\xi)$ satisfies $\bar{D}(1;\xi) \geqslant \bar{z}(i)$ when $\xi \leqslant \min\{\frac{\sigma}{\sigma+r}\Psi^m(i), i\bar{z}(i)\}$, which is shown by $g(\bar{z}(i);1,\xi,i) = -r\bar{z}(i) + \sigma[u(\bar{z}(i)) - \bar{z}(i)] - \frac{\sigma+r}{\sigma}\xi \geqslant \Psi^m(i) - \frac{\sigma+r}{\sigma}\xi \geqslant 0$.

B. Nonmonetary equilibrium

For given credit limit D and credit fee χ, there always exists a nonmonetary equilibrium when money has no value and credit is the only means-of-payment. We denote the value function for a buyer who accessed the financial system in the previous DM and enters CM with debt d by $\hat{W}(d)$.

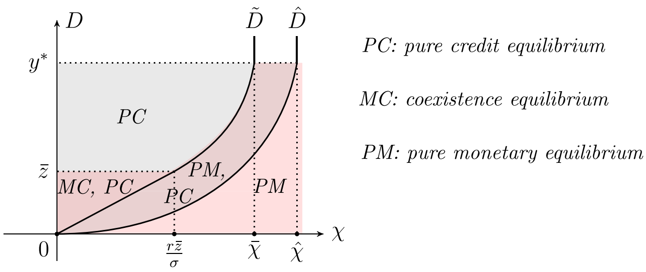

Figure 6. Stationary monetary and nonmonetary equilibrium for given credit limit and credit fee.

The standard Lagos-Wright argument implies that the CM value function is linear on liquidity condition d, that is, $\hat{W}(d) = -d - \chi + \hat{W}(0)$. We assume that in the equilibrium buyers can promise to repay the debt d up to the threshold D and the credit fee χ and remains a *good*-type in the record-keeping system. Denote the buyer's continuation value in the DM as $\hat{V}$ that solves:

$$\hat{V} = \max_{d \leq D} \{\sigma [u(\hat{y}) + \hat{W}(d)] + (1-\sigma)\hat{W}(0)\}, \tag{32}$$

subject to seller's participation constraint $\hat{y} \leq d$. With probability σ the buyer meets the seller the buyer makes an take-it-or-leave-it offer $(\hat{y}, d)$. The seller, if agrees, produces goods $\hat{y}$ with linear cost $\hat{y}$ and the buyer consumes the goods in the DM. Then the match breaks and the buyer and the seller enter the CM to settle the debt. It is straightforward that the buyer uses all the liquidity $d = D$ to consume DM goods and hence $\hat{y} = D$. When there is no meeting between the buyer and the seller, no trade happens in the DM.

We denote $\hat{\Psi}$ as the buyer's value of accessing the record-keeping technology which is $\hat{\Psi} = \sigma[u(D) - D - \chi]$. Note that if the buyer chooses to not access finance, the equilibrium is in autarky.

For the given credit limit $D \leqslant y^*$ and credit fee χ, the next lemma describes the buyer's decision to access the financial system and the stationary equilibrium allocations.

Lemma B1. *Let $\hat{\chi} \equiv u(y^*) - y^*$. A nonmonetary equilibrium always exists with $z = 0$, and we characterize the stationary equilibrium below with $y = d$.*

(1) If $\chi > \hat{\chi}$, buyers do not use credit in DM with $d = 0$ and the economy is in autarky.

(2) Otherwise, there exists $\hat{D}(\chi)$, determined by $\sigma[u(\hat{D}) - \hat{D} - \chi] = 0$, such that

(2.1) if $D \geqslant \hat{D}(\chi)$, in equilibrium buyers only use credit $d = D$ in the DM;

(2.2) if $D < \hat{D}(\chi)$, $d = 0$ in the DM and the economy is in autarky.

In Lemma B1, when the credit fee is higher than the first-best gain from trade ($\chi > \hat{\chi}$), buyers do not access the financial system for any credit limit. As money has no value, the DM trade is not active.

When the credit fee is lower than that threshold, the buyer's decision to access the financial system depends on the credit limit. In Lemma B1 (2.1), when the credit limit is higher than a threshold $\hat{D}$, buyers use credit to trade in the DM, and a pure credit equilibrium exists. Otherwise in Lemma B1 (2.2), when the credit limit is low, the gain from trade cannot compensate for the cost of using credit, so buyers do not access the financial system and do not trade in the DM, regardless of the credit limit. Figure 6 shows the range of credit fee and credit limit such that monetary equilibrium and nonmonetary equilibrium coexist.

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Note that there is a range of credit fees such that a money and credit equilibrium (MC) coexists with a pure credit equilibrium (PC), a finding similar to what is found in Wang et al. (2020) under fixed cost, except that the credit is unconstrained in their model due to perfect commitment, which could be seen as a special case where D approaches y^*.

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