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Singular measures with absolutely continuous convolution squares.† (Corrigendum)

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Mr Denis Lichtman has kindly pointed out to us that an added hypothesis is needed in Theorem (4·1). The revised theorem follows.

4·1. THEOREM. *Let G be an infinite compact Abelian group with character group* X. *Let* Δ *be a countably infinite dissociate set in* X, *enumerated in any order as* $(\chi_k)_{k=1}^{\infty}$. *Let* $(\beta_k)_{k=1}^{\infty}$ *be any sequence of complex numbers such that*: $|\beta_k| \leqslant \frac{1}{2}$; β_k *is real if* $\chi_k^2 = 1$; *and* $\sum_{k=1}^{\infty} |\beta_k|^2 = \infty$. *For each positive integer m, let*

(i) $p_m(x) = \prod_{k=1}^{m} (1+\beta_k\chi_k(x)+\bar{\beta}_k\chi_k^{-1}(x)).$

Then we have

(ii) $\varliminf_{m\to\infty} p_m(x) = 0$

for almost all $x \in G$.

We describe the changes needed in the proof as printed loc. cit.

The first three lines of the proof are changed as follows. We define χ_{-k} and χ_0 as before. For all positive integers k, we define: $\gamma_k = \beta_k$ if $\chi_k^2 \neq 1$; $\gamma_k = 2\beta_k$ if $\chi_k^2 = 1$; $\gamma_{-k} = \bar{\beta}_k$ if $\chi_k^2 \neq 1$; $\gamma_{-k} = 0$ if $\chi_k^2 = 1$; and $\gamma_0 = 0$. For all positive integers m, let

$$f_m = \sum_{k=-m}^{m} \gamma_k\chi_k \quad \text{and let} \quad B_m = (|\gamma_1|^2+\cdots+|\gamma_m|^2)^{\frac{1}{2}}.$$

For the proof, we need the following counting argument.

(A) Consider a fixed pair (p,q) of integers, positive or negative, such that $\chi_p^2 \neq 1$ if $p < 0$ and $\chi_q^2 \neq 1$ if $q < 0$. Consider also the pairs of integers (j,k) such that $\chi_j^2 \neq 1$ if $j < 0$ and $\chi_k^2 \neq 1$ if $k < 0$, for which $\chi_p\bar{\chi}_q = \chi_j\bar{\chi}_k$. It can be proved from the dissociation of $(\chi_k)_{k=1}^{\infty}$ that the number of these pairs never exceeds nine. The computation is routine although long, and we omit it.

Now the proof, beginning with the fourth line, can be read as originally printed, with the following changes.

Replace β_k by γ_k everywhere.

Replace (4·1·6) to the end of the paragraph by

$$\text{if} \quad |k| > m_0, \quad |j| > m_0, \quad j \neq k, \quad \text{and} \quad \gamma_k\gamma_j \neq 0, \quad \text{then} \quad \chi_k\bar{\chi}_j \notin \Gamma. \qquad (4·1·6)$$

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Condition (4·1·5) is easily met, since $\Gamma \cap \Delta$ is finite (possibly void). The possibility of fulfilling (4·1·6) follows at once from (A) applied to the set of $\chi_p \bar{\chi}_q \in \Gamma$ such that $\gamma_p \gamma_q \neq 0$.

Next, in (4·1·8), replace ϵ_0 by $2^{\frac{1}{2}}\epsilon_0$.

In (4·1·9), replace $2^{\frac{1}{2}}$ by 2.

In (4·1·12), replace ϵ_0^2 by $9\epsilon_0^2$. The reason for this is given by (A).

In (4·1·13), replace $2\epsilon_0$ by $6\epsilon_0$.

After (4·1·16), replace the reference (4·1·19) by (4·1·9).

Replace (4·1·17) by

$$\lambda(D) \leqslant 3\epsilon_0 + 6^{\frac{1}{3}} \left(\epsilon_1 \left(1 - \frac{B_{m_0}^2}{B_m^2}\right)^{-\frac{1}{2}} + \epsilon_0\right)^{\frac{2}{3}}. \tag{4·1·17}$$