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BOTTOM CREVASSES

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Abstract. An approximate calculation is made of the rate at which a bottom crevasse in a cold ice shelf or tabular iceberg can close shut by freezing of water and can creep open through the creep deformation of ice. In all but the thickest ice shelves and icebergs, those with a thickness greater than about 400 m, the freezing process is the more important mechanism if the ice is cold (< −10°C). Consequently in a cold iceberg or ice shelf a bottom crevasse, once formed, will freeze shut.

Résumé. *Crevasses de fond.* On fait un calcul approximatif de la valeur à laquelle une crevasse de fond dans une plateforme de glace froide ou un iceberg tubulaire peut se refermer par regel de l'eau dans et peut rester ouverte par le glissement lors de la déformation de la glace par fluage. Sauf pour les plateformes de glace et les icebergs les plus épais, ceux dont l'épaisseur excède 400 m, le processus de regel l'emporte si la glace est froide (< −10°C). Par conséquent, dans une plateforme de glace ou un iceberg froid, une crevasse de fond, une fois formée, se refermera par regel.

Zusammenfassung. *Spalten im Untergrund.* Die Geschwindigkeit, mit der eine Spalte am Untergrund eines kalten Schelfeises oder Tafeleisbergs durch Anfrieren von Wasser sich schliessen bzw. durch die Kriechdeformation des Eises sich öffnen kann, wird näherungsweise berechnet. In allen Schelfeisen und Eisbergen, ausser den dicksten mit einer Dicke von mehr als etwa 400 m überwiegt der Gefrierprozess, wenn das Eis kalt (< −10°C) ist. Folglich wird eine Spalte am Untergrund eines kalten Eisberges oder Schelfeis, die sich einmal gebildet hat, wieder zufrieren.

Introduction

In an earlier paper (Weertman, 1973[a]), the existence of bottom crevasses in floating ice shelves was proposed on theoretical grounds. Bottom crevasses have subsequently been "seen" with the aid of radar within the Ross Ice Shelf by Clough (1974) and within the Larsen Ice Shelf by Swithinbank ([c1978]). Bottom crevasses that extend above the water-line of a tabular iceberg are shown in a figure of a paper of Weeks and Mellor ([c1978]).

Because floating ice shelves and the tabular icebergs that break off of them are cold it might be expected that bottom crevasses within them are short-lived features. A bottom crevasse is, of course, filled with water. This water must freeze continuously to the walls of a bottom crevasse within a cold ice mass if there is no appreciable circulation of water into and out of the crevasse. But creep deformation can cause continuous opening of a crevasse. Thus whether a bottom crevasse remains open for an appreciable length of time depends upon the relative importance of two competing processes. The purpose of this short paper is to make an approximate calculation of the rate at which a bottom crevasse closes or opens. Such a calculation should be of interest to the problem of towing tabular icebergs. Bottom crevasses might lead to rapid disintegration of towed tabular icebergs. Virtually no attention was given to bottom crevasses in the papers in the very recently published book on iceberg utilization (Husseiny, [c1978]).

Theory

It can be shown (Weertman, 1973[a]) that an isolated crevasse which contains no water at the upper surface of an ice mass (a top crevasse) penetrates downwards to a depth L given by

$$L = \pi\sigma/2\rho g, \qquad (1)$$

where ρ is the density of ice, g is the gravitational acceleration, and σ is the tensile stress that exists within the ice mass in addition to the hydrostatic stress component. In a floating ice shelf the average value of σ is equal to

$$\sigma = \Delta\rho g h/2, \qquad (2)$$

where h is the ice thickness and $\Delta\rho$ is the difference in the density between sea-water and the average ice density of the ice shelf. Thus an isolated, water-free top crevasse in an ice shelf has a depth L equal to

$$L = \pi h\Delta\rho/4\rho. \qquad (3)$$

For high-density ice the crevasse depth is about 7% of the ice thickness.

It can also be shown (Weertman, 1973[a]) that a bottom crevasse in an ice shelf, which of course is filled with water, penetrates upwards to a height that is approximately a factor $\rho/\Delta\rho$ larger than the value of L that is given by Equations (1) and (3). Thus a bottom crevasse has a length L equal to

$$L \approx \pi h/4, \tag{4}$$

or a length that is equal to 78% of the total thickness of an ice shelf. A pair of top and bottom crevasses, one above the other, could account for about 85% of the cross section of an ice sheet or tabular iceberg. (The length L that is given by Equation (4) is probably an underestimate of a bottom-crevasse length. It was found under the assumption that a crevasse moves into an infinite half-space. The upper surface of a floating ice shelf will cause the actual value of L to be larger than that given by Equation (4).)

The calculations that lead to the equations just given are made under the assumption that ice is an elastic solid with essentially zero fracture strength. Since the fracture process that occurs during the initial cracking takes place rapidly, ice indeed does act in this short time period as an elastic solid. The correction to the value of L because ice has a finite fracture strength is very small when the value of σ is within several orders of magnitude of the value $\sigma = 0.1$ MPa (1 bar). One need only use the experimental values of the critical stress intensity factor K_c that are collected from the literature and tabulated by Smith ([c1978]) to demonstrate this fact.

The critical value L_c that a pre-existing crack must have before it can grow in a catastrophic fashion into a crevasse under a stress σ is not a small length. The values of K_c for ice that are listed by Smith (1978) are of the order of 0.15 MPa m$^{\frac{1}{2}}$. Since $\sigma(\pi L_c)^{\frac{1}{2}} = K_c$ the value of L_c is equal to 0.7 m when $\sigma = 0.1$ MPa. This value of σ would exist in an ice shelf 200 m thick. However, the critical crack length is probably considerably smaller than the value just calculated. Johnston and Parker (1957) showed that the fracture strength of ice is reduced by a factor of three in the presence of surface-active agents. Salt water was one of the effective surface-active agents they used in their experiments. Thus the value of K_c should be reduced by a factor of three in the presence of sea-water and the critical crack length L_c will be reduced by one order of magnitude.

Cracks at the bottom of an ice shelf might be formed by the etching by sea-water of grain boundaries within the ice. If sea-water freezes to the bottom, surface cracks could form through brine entrapment. Of course, if processes such as bottom melting prevent the formation of surface cracks, bottom crevasses will not form because they cannot be nucleated.

The crevasse length given by Equations (1), (3), and (4) apply to isolated crevasses. In a field of closely spaced crevasses, ones whose spacing is appreciably smaller than their lengths, the crevasse length is reduced by a factor of $2/\pi$. (When the fracture strength of ice is finite, crevasse spacing must satisfy the stability criterion recently investigated by Nemat-Nasser and others (1979).)

Freezing shut

Bottom crevasses in a cold ice mass might heal themselves shut after their formation through the freezing of water. The average displacement λ with which an air-filled top crevasse or a water-filled bottom crevasse is opened elastically is approximately equal to (Weertman, 1973[a])

$$\lambda \approx L(\sigma/\mu), \tag{5}$$

where μ is the shear modulus of ice. This thickness of water must be frozen in order to close a bottom crevasse.

Inserting Equation (2) and (4) into Equation (5) gives

$$\lambda \approx \pi\Delta\rho g h^2/8\mu \tag{6}$$

as an estimate of the average displacement within a bottom crevasse immediately after its formation. For $h = 200$ m and $\Delta\rho/\rho \approx \frac{1}{10}$ with $\mu = 3$ GPa the value of λ is $\lambda \approx 5$ mm. Thus only a small amount of water need be frozen to close the crevasse.

In a time period equal to t cold ice out to a distance r on either side of a crevasse is warmed to a temperature close to the melting point where r is given by

$$r \approx (4Dt)^{\frac{1}{2}}, \tag{7}$$

where D is the thermal diffusion coefficient. (Equation (7) is the usual estimate of the diffusion distance in diffusion problems.) If $\Delta\theta$ is the change in temperature of the ice within the distance r, then for a unit area of crevasse an amount of heat equal to $2C\Delta\theta r$ is absorbed by the ice and removed from the sea-water. The rate of closing of the crevasse, $-\mathrm{d}\lambda/\mathrm{d}t$, is thus given by

$$\mathrm{d}\lambda/\mathrm{d}t = -(2C\Delta\theta/H)(D/t)^{\frac{1}{2}}, \tag{8}$$

where H is the heat of fusion of ice and C is the specific heat capacity of ice.

The crack displacement λ is found by integrating Equation (8) and is equal to

$$\lambda = \lambda_0 - (4C\Delta\theta/H)(Dt)^{\frac{1}{2}}, \tag{9}$$

where $\lambda_0 = \pi\Delta\rho g h^2/8\mu$.

The crevasse closure time is found by setting $\lambda = 0$ in Equation (9). (The actual displacement on crevasse opening varies from a value equal to zero at the crevasse tip to a maximum displacement at the bottom surface. Thus, if the ice-shelf temperature is a constant, crevasse closure occurs first near the tip and last at the bottom surface.) The average time for a crevasse with a 5 mm opening displacement to freeze shut is about 2 min when $\Delta\theta = 15$ K (using $D = 1.5\times10^{-6}$ m^2 s^{-1}, $C = 2$ MJ m^{-3} K^{-1}, and $H = 0.34$ GJ m^{-3}). Thus bottom crevasses should freeze shut very quickly if their opening displacement is only an elastic one.

Creeping open

Immediately after a crevasse is created, its opening displacement is primarily an elastic one. But at later times the opening displacement can be the result of creep deformation. The creep-deformation displacement can be orders of magnitude larger than the elastic displacement.

If ice obeyed a Newtonian (that is, linear) creep-deformation equation the crevasse displacement velocity $\mathrm{d}\lambda/\mathrm{d}t$ would be very simple to find. The creep problem is formally the same as the elastic problem in this situation. It is only necessary to substitute the term $\mathrm{d}\lambda/\mathrm{d}t$ for λ and the viscosity η for the shear modulus μ in the equations. Thus Equation (5) is replaced by the equation

$$\mathrm{d}\lambda/\mathrm{d}t \approx L(\sigma/\eta) \tag{10}$$

and Equation (6) by

$$\mathrm{d}\lambda/\mathrm{d}t \approx \pi\Delta\rho g h^2/8\eta. \tag{11}$$

Equation (9) becomes

$$\lambda = \lambda_0 + \lambda_0(\mu/\eta)\, t - (4C\Delta\theta/H)(Dt)^{\frac{1}{2}}. \tag{12}$$

According to Equation (12), the crevasse never freezes shut if the following inequality is satisfied

$$(2C\Delta\theta/\lambda_0 H)(D\eta/\mu)^{\frac{1}{2}} = \gamma(\Delta\theta\rho/h^2\Delta\rho)(\eta/\mu)^{\frac{1}{2}} < 1, \tag{13}$$

where $\gamma = 16C\mu D^{\frac{1}{2}}/\pi\rho g H = 11$ m^2 K^{-1} s$^{-\frac{1}{2}}$.

But the creep law of ice is non-Newtonian. For a uniaxial tension test it is given by the power-law creep equation (except at very high stresses and very low stresses)

$$\dot{\epsilon} = \dot{\epsilon}_0(\sigma/\sigma_0)^n \exp(-Q/R\theta) \exp(Q/B\theta_0), \tag{14}$$

where $\dot{\epsilon}$ is the creep-rate, $\dot{\epsilon}_0$ is the creep-rate at a reference temperature θ_0 and reference stress σ_0, R is the gas content, Q is the creep activation energy, and $n \approx 3$. At any stress level σ an effective viscosity η can be defined by the equation

$$\eta = \sigma/2\dot{\epsilon}. \tag{15}$$

Thus

$$\eta = \eta_0(\sigma_0/\sigma)^{n-1} \exp(Q/R\theta) \exp(-Q/R\theta_0), \tag{16}$$

where $\eta_0 = \sigma_0/2\dot{\epsilon}_0$ is the effective viscosity at the reference temperature and stress.

The stress (that is, the non-hydrostatic stress) around a crevasse will range in value from a magnitude of that of σ away from the crevasse tip to very high values right at the crevasse tip. Thus the effective viscosity will take on a wide range of values near a crevasse. A similar situation exists for a crack in a solid that is elastic until a yield stress is exceeded. Above the yield stress the material deforms plastically. In this situation if σ does not exceed the yield stress, intense non-linear deformation occurs only very close to the crack tip. The elastic displacement of the crack faces is almost the same as for a crack in an elastic solid. Thus for a crevasse in a power-law-creep material it is reasonable to use an effective viscosity where the effective viscosity is calculated using the stress level σ. The crack-opening displacement velocity calculated using this particular effective viscosity will be approximately equal to but somewhat smaller than the actual displacement velocity.

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At a temperature of $-10°C$ the creep-rate of ice under a uniaxial stress of 0.1 MPa is about 2×10^{-10} s^{-1} (see fig. 4 in Weertman, 1973[b]). If this stress and this temperature are used as reference ones then $\eta_0 = 2.5 \times 10^{14}$ Pa s.

By combining Equations (2), (13), and (16) the following relationship is found for the condition required to have a bottom crevasse remain open

$$(\gamma \Delta\theta \rho / h^2 \Delta\rho)^2 (\eta_0/\mu)(2\sigma_0/\Delta\rho g h)^{n-1} \exp(Q/R\theta) \exp(-Q/R\theta_0) < 1. \qquad (17)$$

For $n = 3$, $\theta = -10°C$, and $\Delta\theta = 10$ K, Equation (17) predicts that a bottom crevasse in an ice shelf or tabular iceberg that is thicker than about $h \geqslant 400$ m will remain open. (Of course, bottom crevasses in thinner ice shelves can remain open if there is sufficient circulation of water whose temperature is above the melting point into and out of the crevasses. Equation (17) is found under the assumption that such circulation is negligible.)

Partial check

One partial check can be made of the equation for crevasse opening velocity. Meier (1958) measured the velocity with which a water free crevasse opened up in blue ice at the top surface of the Greenland ice sheet. He found that a 25 m deep crevasse opened up at a rate of 1 to 2 mm d^{-1}. The crevasse was in ice of a temperature of $-1°C$ to $-6°C$. The stress σ was not measured but the crevasse was in ice that evidently was extending at a strain-rate of the order of 0.01 $year^{-1}$. For a temperature of $-3°C$ this strain-rate corresponds to a stress of about 0.084 MPa (using $Q = 63$ kJ mol^{-1}). (A crevasse depth of $L = 25$ m according to Equation (1) requires a stress of 0.16 MPa, a value in approximate agreement with this estimate.) For this stress (0.084 MPa) and temperature the effective viscosity η is equal to $\eta = 1.8 \times 10^{14}$ Pa s. Inserting the values $\eta = 1.8 \times 10^{14}$ Pa s, $\sigma = 0.084$ MPa, and $L = 25$ m into Equation (10) gives the result that $d\lambda/dt = 1.2 \times 10^{-8}$ m s^{-1} = 1 mm d^{-1}, a result that is in reasonable agreement with the measured opening velocity.

Conclusion

The approximate analysis given in this paper shows that bottom crevasses, once formed, are likely to freeze shut in all but the thickest cold ice shelves and cold tabular icebergs.

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