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OPTIMAL PROPORTIONAL REINSURANCE AND INVESTMENT PROBLEM WITH CONSTRAINTS ON RISK CONTROL IN A GENERAL JUMP-DIFFUSION FINANCIAL MARKET

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Abstract

We study the optimal proportional reinsurance and investment problem in a general jump-diffusion financial market. Assuming that the insurer's surplus process follows a jump-diffusion process, the insurer can purchase proportional reinsurance from the reinsurer and invest in a risk-free asset and a risky asset, whose price is modelled by a general jump-diffusion process. The insurance company wishes to maximize the expected exponential utility of the terminal wealth. By using techniques of stochastic control theory, closed-form expressions for the value function and optimal strategy are obtained. A Monte Carlo simulation is conducted to illustrate that the closed-form expressions we derived are indeed the optimal strategies, and some numerical examples are presented to analyse the impact of model parameters on the optimal strategies.

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1. Introduction

Since insurers can control their risk by investing their surpluses in financial markets and purchasing reinsurance, optimal investment or optimal reinsurance problems with various objectives have become more important in insurance risk management, and they have gained much interest in the actuarial literature. For example, Azcue and Muler [1], Hipp and Plum [13], Schmidli [22, 23], Liu and Yang [17], Promislow and

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Young [20] and Luo et al. [18] studied the optimal investment or reinsurance problem and the investment problem of minimizing the ruin probability.

Besides ruin probability minimization, some scholars focused on maximizing the expected utility of insurers' terminal wealth. Browne [6] obtained an optimal investment strategy for an insurer whose surplus process is modelled by a drifted Brownian motion. Besides, the insurer is allowed to invest his (her) surplus in a risky asset with the price governed by a geometric Brownian motion (GBM). Yang and Zhang [26] studied the optimal investment problem for an insurer with a jump-diffusion surplus process, in which the price of the risky asset is described by a GBM. Wang [24] extended the results of Yang and Zhang [26] to the case of multiple risky assets. Later, Bai and Guo [2] discussed the optimal reinsurance and investment problem for an insurer with a diffusion approximation claim process. Moreover, the insurer can invest in multiple risky assets whose prices are driven by GBMs. Xu et al. [25] investigated a problem similar to Bai and Guo [2] for a perturbed classical risk model. Bai and Guo [3] studied the problem in their earlier paper [2] again with the assumption that the insurer purchases excess-of-loss reinsurance. Liang et al. [15] studied an investment and proportional reinsurance problem with constrained control variables. Other related works can be found in Cao and Wan [7], Chen et al. [8], Gu et al. [11], Zhou et al. [28] and so on.

Among most of the articles mentioned above, the risky assets' prices are assumed to be driven by GBMs. But much empirical evidence shows that the risky assets' price follows a jump-diffusion model which is more similar to the real financial markets than the GBM. The jump-diffusion financial market is a natural extension of the GBM, and has been extensively studied in other optimal problems. For maximizing the expected utility of the returns with jump-diffusion price processes, Bardhan and Chao [4] and Jeanblanc-Picqué and Pontier [14] studied the optimal investment-consumption problem. Guo and Xu [12] considered the portfolio selection problem in which the prices of the stocks follow jump-diffusion process. They obtain a closed form expression for the efficient frontier. Zeng and Li [27] investigated continuous-time asset-liability management under benchmark and mean-variance criteria in a jump-diffusion market. In the field of insurance actuarial science, only the following two papers explore the reinsurance-investment problem in the jump-diffusion financial market. Lin and Yang [16] studied an optimal investment and proportional reinsurance policy which maximizes the expected exponential utility of the terminal wealth. Bi and Guo [5] studied the optimal investment and reinsurance problem for an insurer under the criterion of mean-variance where the price of the risky asset follow a jump-diffusion process.

To our knowledge, there is little work in the literature on the common constraints of the control variables. Most of these literature results are obtained with no common constraints on the control variables. It means that the proportional reinsurance strategy may be larger than one. We argue that this result is unrealistic from an economic point of view. For this reason, Bai and Guo [2] placed a no short-selling constraint on the Brownian motion risk model, and considered the optimal reinsurance–investment problem with multiple risky assets. Under the constraint of no short-selling, the retention level belongs to the interval [0, 1].

Unlike most of the existing literature, in this paper we study the optimal proportional reinsurance and investment problem in a general jump-diffusion financial market. In our model, the company is allowed to invest all of its surplus in a financial market consisting of a risk-free asset and a risky asset. The price of the risky asset is assumed to follow a general jump-diffusion process. In the aspect of reinsurance, we adopt a proportional reinsurance strategy and limit the retention rate to [0, 1]. Aiming to maximize the expected exponential utility of the terminal wealth, we use the Hamilton–Jacobi–Bellman (HJB) approach and derive the corresponding HJB equation. The main contribution of this paper is two-fold: (1) we add a general jump in the price of the risky asset; then the financial market follows a general jump-diffusion model which is more similar to the real financial market than the models in the literature; (2) we consider a reasonable constraint on the proportional reinsurance strategy, which makes the model more reasonable and realistic. Through some calculations, closed-form expressions for the value function and optimal strategy are derived. Moreover, we find that with a constraint on the proportional reinsurance, the value function is still a classical solution to the corresponding HJB equation.

The rest of the paper is organized as follows. In Section 2, the model assumptions are formulated. The main results and explicit expressions for the optimal values are derived in Section 3. In Section 4, a Monte Carlo simulation is conducted to support that the results we derived are indeed the optimal strategies, and some numerical examples are presented to show the impact of model parameters on the optimal strategies. The paper concludes with some discussions in Section 5.

2. Model formulation

We consider a jump-diffusion risk model, in which the surplus X_t of the insurer at time t is

$$X_t = u + ct - \sum_{i=1}^{N_1(t)} Y_i + \beta B_t^1,$$

where $u \geq 0$ is the initial surplus; $c > 0$ is the premium rate; $\{Y_i, i \geq 1\}$ is a sequence of independent and identically distributed nonnegative random variables with common distribution $F(y)$, density function $f(y)$, mean value $\mu_1 = E(Y_i)$ and moment generating function $M_Y(s) = E[e^{sY_i}]$; Y_i denotes the amount of the ith claim; $\{N_1(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1 > 0$, representing the number of claims up to time t; B_t^1 is a standard Brownian motion; and β is a constant, representing the diffusion volatility parameter. We assume that $E[Ye^{sY}] = M_Y'(s)$ exists for $0 < s < \zeta$, and $\lim_{s\to\zeta} E[Ye^{sY}] = \infty$ for some $0 < \zeta \leq +\infty$. Further, we assume that $\{N_1(t), t \geq 0\}$, $\{Y_i, i \geq 1\}$ and $\{B_t^1, t \geq 0\}$ are mutually independent.

As an effective tool to reduce risk, we allow the insurance company to reinsure a fraction of its claim with the retention level $q \in [0, 1]$. That is, for a claim Y_i, the insurer pays qY_i and the reinsurer pays $(1-q)Y_i$. Let $\delta(q)$ be the premium rate for the reinsurance; then the premium rate remaining for the insurer is $c-\delta(q)$. Here we assume that the insurer can choose a dynamic strategy $q=\{q_t, t\geq 0\}$, that is, the retention level can be adjusted continuously. Throughout the paper, we assume that the reinsurance premium is calculated according to the expected value principle:

$$\delta(q) = (1+\eta)(1-q)\lambda_1\mu_1,$$

where $\eta > 0$ is the safety loading of the reinsurer. Let $\theta = c/(\lambda_1\mu_1) - 1 > 0$ be the safety loading of the insurer. Without loss of generality, we assume that $\eta > \theta$. In order that the net profit condition (see the book by Rolski et al. [21, p. 162]) is fulfilled, that is,

$$c - (1+\eta)(1-q)\lambda_1\mu_1 - q\lambda_1\mu_1 \geq 0,$$

we require

$$q \geq \overline{q} = 1 - \theta/\eta.$$

The corresponding surplus process of the insurance company after proportional reinsurance becomes

$$dX_t^q = [c-(1+\eta)(1-q)\lambda_1\mu_1]\,dt + \beta\,dB_t^1 - q\int_0^\infty yN_1(dt,dy),$$

where $N_1(dt,dy)$ is the Poisson random measure corresponding to the compound Poisson process $\sum_{i=1}^{N_1(t)} Y_i$.

Moreover, the company is allowed to invest all of its surplus in a financial market consisting of a risk-free asset (bond) and a risky asset (stock). The price of the risk-free asset is given by

$$dR_t = rR_t\,dt, \qquad r>0,$$

where r is the risk-free interest rate. The price process P_t of the risky asset is assumed to follow a general jump-diffusion process, that is, P_t satisfies the stochastic differential equation

$$dP_t = P_{t-}\Big[a\,dt + \sigma\,dB_t^2 + \int_R \gamma(t,z)\widetilde{N}_2(dt,dz)\Big], \tag{2.1}$$

where $a\,(>r)$, σ are positive constants and $\gamma(t,z)\geq -1$; B_t^2 is another standard Brownian motion; and $\widetilde{N}_2(dt,dz)$ is a compensated Poisson random measure, that is,

$$\widetilde{N}_2(dt,dz) = N_2(dt,dz) - \lambda_2 g(z)\,dt\,dz.$$

Here $N_2(dt,dz)$ is another Poisson random measure corresponding to the compound Poisson process $\sum_{i=1}^{N_2(t)} Z_i$, $\{N_2(t), t\geq 0\}$ is a Poisson process with rate $\lambda_2 > 0$ and $\{Z_i, i\geq 1\}$ are independent identically distributed random variables with a common distribution $G(z)$ and density function $g(z)$. We assume that $\{Y_i, i\geq 1\}$, $\{N_1(t), t\geq 0\}$,

$\{Z_i, i \geq 1\}$ and $\{N_2(t), t \geq 0\}$ are mutually independent. Let ρ denote the correlation coefficient of the two standard Brownian motions, that is, $E[B_t^1 B_t^2] = \rho t$.

Let A_t denote the total amount of money invested in the risky asset at time t. The remaining portion of the surplus is invested in the risk-free asset. At any time $t \geq 0$, $q = q_t$ and $A = A_t$ are chosen by the insurance company; we denote $\pi_t = (A_t, q_t)$. Once the strategy π_t is chosen, the dynamics of the wealth process of the insurance company becomes

$$\begin{aligned} dX_t^\pi &= [(a-r)A_t + rX_t^\pi + c - (1+\eta)(1-q_t)\lambda_1\mu_1]\,dt + \beta\,dB_t^1 \\ &\quad + A_t\sigma\,dB_t^2 - q_t\int_0^\infty yN_1(dt,dy) + A_t\int_R \gamma(t,z)\widetilde{N}_2(dt,dz), \qquad (2.2) \\ X_0^\pi &= x. \end{aligned}$$

REMARK 2.1. In this paper, we assume that continuous trading is allowed, no transaction cost or tax is involved in trading and all assets are infinitely divisible. We also assume that all processes and random variables are defined on a filtered probability space $(\Omega, \mathscr{F}, \mathscr{F}_t, P)$ satisfying the usual conditions.

A control strategy $\pi_t = (A_t, q_t)$ is said to be admissible if A_t and q_t are predictable with respect to $\mathscr{F}_t$ and satisfy the conditions

$$\overline{q} \leq q_t \leq 1 \quad \text{and} \quad E\bigg[\int_0^t A_s^2\,ds\bigg] < \infty$$

for all $t \geq 0$. We write Π for the set of all admissible strategies for the insurance company.

Assume now that the insurer is interested in maximizing the expected utility from terminal wealth, say at time T. The utility function is $u(x)$, which satisfies $u' > 0$ and $u'' < 0$. Then the objective function is

$$J^\pi(t,x) = E[u(X_T^\pi)|X_t^\pi = x]$$

and the corresponding value function is

$$V(t,x) = \sup_{\pi\in\Pi} J^\pi(t,x). \qquad (2.3)$$

Here we assume that the insurer has an exponential utility function

$$u(x) = m - \frac{\kappa}{\nu}e^{-\nu x}$$

with $\kappa > 0$ and $\nu > 0$. This utility has constant absolute risk aversion parameter (CARA) ν; such utility functions play a prominent role in insurance mathematics and actuarial practice, since they are the only utility functions under which the principle of *zero utility* gives a fair premium that is independent of the level of reserves of an insurance company (see the book by Gerber [10, p. 68]).

Let $C^{(1,2)}$ be the space of $\phi(t,x)$ such that ϕ and its partial derivatives ϕ_t, ϕ_x, ϕ_{xx} are continuous on $[0,T] \times R$. To solve the above problem, we use the dynamic programming approach described in Fleming and Soner [9]. From standard arguments, we see that if the value function $V \in C^{(1,2)}$, then V satisfies the following HJB equation for $t < T$:

$$\sup_{\pi\in\Pi} \mathcal{A}^{\pi} V(t,x) = 0 \tag{2.4}$$

with the boundary condition

$$V(T,x) = u(x), \tag{2.5}$$

where $\mathcal{A}^{\pi}$ denotes the generator of the surplus process X_t^{π} controlled by the control policy $\pi_t = (A_t, q_t)$. Applying Itô's formula [19, Theorem 1.14, p. 6] for the jump-diffusion process to the value function $V(t,x)$,

$$\begin{aligned}\mathcal{A}^{\pi} V(t,x) = {} & V_t + [(a-r)A + rx + c - (1+\eta)(1-q)\lambda_1\mu_1]V_x \\ & + \frac{1}{2}(A^2\sigma^2 + \beta^2 + 2A\sigma\beta\rho)V_{xx} + \lambda_1 \int_0^{\infty} \{V(t,x-qy) - V(t,x)\} f(y)\,dy \\ & + \lambda_2 \int_R \{V(t,x+A\gamma(t,z)) - V(t,x) - A\gamma(t,z)V_x\} g(z)\,dz,\end{aligned}$$

where V_t and V_x are the partial derivatives with respect to t and x, respectively, and V_{xx} is the second partial derivative with respect to x. By the standard method used by Fleming and Soner [9], we now have the following theorem.

THEOREM 2.2. *Let $W \in C^{(1,2)}$ be a classical solution of (2.4) that satisfies (2.5). Then the value function V given by (2.3) coincides with W, that is, $W(t,x) = V(t,x)$. Furthermore, let $\pi^* = (A^*, q^*) \in R \times [\overline{q}, 1]$ be such that $\mathcal{A}^{\pi^*} V(t,x) = 0$ holds for all $(t,x) \in [0,T] \times R$. Then the strategy $\pi^* = (A^*, q^*)$ is the optimal strategy, that is, $W(t,x) = V(t,x) = V^{\pi^*}(t,x)$.*

3. The optimal results

To solve equation (2.4), we try to fit a solution of the form

$$V(t,x) = m - \frac{\kappa}{\nu} \exp[-\nu x e^{r(T-t)} + h(T-t)], \tag{3.1}$$

where $h(\cdot)$ is a suitable function such that (3.1) is a solution of equation (2.4). The boundary condition $V(T,x) = u(x)$ implies $h(0) = 0$. From (3.1),

$$\begin{cases} V_t = [V(t,x) - m][\nu x r e^{r(T-t)} - h'(T-t)], \\ V_x = [V(t,x) - m][-\nu e^{r(T-t)}], \\ V_{xx} = [V(t,x) - m][\nu^2 e^{2r(T-t)}], \\ \displaystyle\int_0^{\infty} [V(t,x-qy) - V(t,x)] f(y)\,dy = [V(t,x) - m] \int_0^{\infty} [\exp(\nu q y e^{r(T-t)}) - 1] f(y)\,dy, \\ \displaystyle\int_R [V(t,x+A\gamma(t,z)) - V(t,x)] g(z)\,dz \\ \qquad = [V(t,x) - m] \int_R [\exp(-\nu A\gamma(t,z) e^{r(T-t)}) - 1] g(z)\,dz. \end{cases} \tag{3.2}$$

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Substituting (3.2) back into equation (2.4) and cancelling like terms yields

$$\inf_{\pi\in\Pi} H(t,A,q) = 0, \tag{3.3}$$

where

$$\begin{aligned} H(t,A,q) = {} & -h'(T-t) - [(a-r)A + c - (1+\eta)(1-q)\lambda_1\mu_1]\nu e^{r(T-t)} \\ & + \frac{1}{2}[A^2\sigma^2 + \beta^2 + 2A\sigma\beta\rho]\nu^2 e^{2r(T-t)} \\ & + \lambda_1 \int_0^\infty [\exp(\nu q y e^{r(T-t)}) - 1] f(y)\,dy \\ & + \lambda_2 \int_R [\exp(-\nu A\gamma(t,z)e^{r(T-t)}) - 1 + \nu A\gamma(t,z)e^{r(T-t)}] g(z)\,dz. \end{aligned} \tag{3.4}$$

Then, for any $t \in [0,T]$,

$$\begin{cases} \dfrac{\partial H(t,A,q)}{\partial q} = -\nu e^{r(T-t)}(1+\eta)\lambda_1\mu_1 + \lambda_1 \nu e^{r(T-t)} \displaystyle\int_0^\infty y \exp(\nu q y e^{r(T-t)}) f(y)\,dy, \\ \dfrac{\partial^2 H(t,A,q)}{\partial q^2} = \lambda_1 \nu^2 e^{2r(T-t)} \displaystyle\int_0^\infty y^2 \exp(\nu q y e^{r(T-t)}) f(y)\,dy > 0, \\ \dfrac{\partial H(t,A,q)}{\partial A} = -\nu(a-r)e^{r(T-t)} + \beta\sigma\rho\nu^2 e^{2r(T-t)} + \nu^2 e^{2r(T-t)}\sigma^2 A \\ \qquad\qquad - \lambda_2 \nu e^{r(T-t)} \displaystyle\int_R \gamma(t,z)[\exp(-\nu A\gamma(t,z)e^{r(T-t)}) - 1] g(z)\,dz, \\ \dfrac{\partial^2 H(t,A,q)}{\partial A^2} = \lambda_2 \nu^2 e^{2r(T-t)} \displaystyle\int_R \gamma^2(t,z) \exp(-\nu A\gamma(t,z)e^{r(T-t)}) g(z)\,dz + \nu^2 e^{2r(T-t)}\sigma^2 > 0. \end{cases}$$

Therefore, $H(t,A,q)$ is a convex function with respect to q (or A). To get the minimizer $A_1(T-t)$, $q_1(T-t)$ of (3.4), we solve the following two equations:

$$(1+\eta)\mu_1 - \int_0^\infty y e^{yn} f(y)\,dy = 0, \tag{3.5}$$

$$a - r + \lambda_2 \int_R \gamma(t,z)(\exp[-\nu A\gamma(t,z)e^{r(T-t)}] - 1) g(z)\,dz = \nu e^{r(T-t)}(\sigma^2 A + \beta\sigma\rho), \tag{3.6}$$

with $n = q\nu e^{r(T-t)}$, which leads to the following lemma.

Lemma 3.1. *Equation (3.5) has a unique positive root ϱ with $0 < \varrho < \zeta$ and equation (3.6) has a finite root $\widehat{A}$.*

Proof. Let $f_1(n) = (1+\eta)\mu_1 - \int_0^\infty y e^{yn} f(y)\,dy$. Then

$$f_1'(n) = -\int_0^\infty y^2 e^{yn} f(y)\,dy < 0 \quad \text{and} \quad f_1''(n) = -\int_0^\infty y^3 e^{yn} f(y)\,dy < 0.$$

This means that $f_1(n)$ is a monotone decreasing concave function.

By the assumption that $E[Ye^{sY}]$ exists for $0 < s < \zeta$, $\lim_{s\to\zeta} E[Ye^{sY}] = \infty$ for some $0 < \zeta \le \infty$. Then we have $\lim_{n\to\zeta} f_1(n) < 0$. Since $f_1(0) = \eta\mu_1 > 0$, equation (3.5) has a unique positive root ϱ with $0 < \varrho < \zeta$.

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Next, let

$$f_2(A) = a - r + \lambda_2 \int_R \gamma(t,z)(\exp[-\nu A\gamma(t,z)e^{r(T-t)}] - 1)g(z)\,dz - \nu e^{r(T-t)}(\sigma^2 A + \beta\sigma\rho);$$

then

$$f_2'(A) = -\lambda_2 \nu e^{r(T-t)} \int_R \gamma^2(t,z)\exp(-\nu A\gamma(t,z)e^{r(T-t)})g(z)\,dz - \sigma^2 \nu e^{r(T-t)} < 0,$$

so $f_2(A)$ is a monotone decreasing function. Furthermore, $\lim_{A\to-\infty} f_2(A) > 0$ and $\lim_{A\to\infty} f_2(A) < 0$. Hence, equation (3.6) has a finite root $\widehat{A}$ and the proof is complete. □

Therefore, we get $\nu q_1(T-t)e^{r(T-t)} = \varrho$ and thus $q_1(T-t) = (\varrho/\nu)e^{-r(T-t)}$. Here ϱ is a constant, and it depends only on the safety loading η and the claim sizes distribution. Since the retention level $q_t \in [\overline{q}, 1]$, we discuss the optimal values in the following three cases.

Remark 3.2. If $(\varrho/\nu)e^{-rT} \geq 1$, then $\overline{q} < (\varrho/\nu)e^{-rT} < q_1(T-t)$ for any $t \in [0,T]$ and thus the net profit condition becomes trivial; if $(\varrho/\nu)e^{-rT} < 1$ and $\overline{q} > \varrho/\upsilon$, then $\overline{q} > q_1(T-t)$ for any $t \in [0,T]$ and thus $q_t^* = \overline{q}$; if $(\varrho/\nu)e^{-rT} < 1$ and $(\varrho/\nu)e^{-rT} \leq \overline{q} \leq \varrho/\nu$, then there exists a t_0 such that $q_1(T-t_0) = \overline{q}$. Therefore, in the following first two cases, we assume that $(\varrho/\nu)e^{-rT} \leq \overline{q} \leq \varrho/\nu$.

Case I: $\varrho \leq \nu$ or, equivalently, $0 < \eta \leq (1/\mu_1)M_Y'(\nu) - 1$. Then $q_1(T-t) \leq 1$ for any $t \in [0,T]$. Let $t_0 = T + (1/r)\ln[(1-\theta/\eta)\nu/\varrho]$; therefore, the optimal reinsurance strategy is

$$q_t^* = \begin{cases} 1 - \dfrac{\theta}{\eta} & \text{if } 0 \leq t < t_0, \\ \dfrac{\varrho}{\nu}e^{-r(T-t)} & \text{if } t_0 \leq t \leq T. \end{cases}$$

When $0 \leq t < t_0$, substituting $A_t^* = A_1(T-t) = \widehat{A}$, $q_t^* = \overline{q}$ into (3.3) yields

$$\begin{aligned} h_0'(T-t) = {} & -[(a-r)\widehat{A} + c - (1+\eta)(1-\overline{q})\lambda_1\mu_1]\nu e^{r(T-t)} \\ & + \frac{1}{2}[\widehat{A}^2\sigma^2 + \beta^2 + 2\widehat{A}\sigma\beta\rho]\nu^2 e^{2r(T-t)} \\ & + \lambda_1 \int_0^\infty [\exp(\nu\overline{q}ye^{r(T-t)}) - 1]f(y)\,dy \\ & + \lambda_2 \int_R [\exp(-\nu\widehat{A}\gamma(t,z)e^{r(T-t)}) - 1 + \nu\widehat{A}\gamma(t,z)e^{r(T-t)}]g(z)\,dz \end{aligned}$$

and, by integration,

$$h_0(T-t) = \widetilde{h}_0(T-t) + M_0, \tag{3.7}$$

where

$$\begin{aligned}\widetilde{h}_0(T-t) = &-\int_0^{T-t} [(a-r)\widehat{A} + c - (1+\eta)(1-\overline{q})\lambda_1\mu_1]\nu e^{r(T-s)}\,ds \\ &+ \frac{1}{2}\int_0^{T-t} [\widehat{A}^2\sigma^2 + \beta^2 + 2\widehat{A}\sigma\beta\rho]\nu^2 e^{2r(T-s)}\,ds \\ &+ \lambda_1 \int_0^{T-t}\int_0^{\infty} [\exp(\nu\overline{q}ye^{r(T-s)}) - 1]f(y)\,dy\,ds \\ &+ \lambda_2 \int_0^{T-t}\int_R [\exp(-\nu\widehat{A}\gamma(s,z)e^{r(T-s)}) - 1 + \nu\widehat{A}\gamma(s,z)e^{r(T-s)}]g(z)\,dz\,ds\end{aligned}$$

and M_0 is a constant that will be determined later.

When $t_0 < t \le T$, replacing $A_t^* = A_1(T-t) = \widehat{A}$, $q_t^* = q_1(T-t)$ back into equation (3.3) yields

$$\begin{aligned}h_1'(T-t) = &-[(a-r)\widehat{A} + c - (1+\eta)(1-q_1(T-t))\lambda_1\mu_1]\nu e^{r(T-t)} + \frac{1}{2}[\widehat{A}^2\sigma^2 + \beta^2 \\ &+ 2\widehat{A}\sigma\beta\rho]\nu^2 e^{2r(T-t)} + \lambda_1\int_0^{\infty} [\exp(\nu q_1(T-t)ye^{r(T-t)}) - 1]f(y)\,dy \\ &+ \lambda_2\int_R [\exp(-\nu\widehat{A}\gamma(t,z)e^{r(T-t)}) - 1 + \nu\widehat{A}\gamma(t,z)e^{r(T-t)}]g(z)\,dz, \end{aligned} \tag{3.8}$$

together with the initial condition $h_1(0) = 0$.

Let $M_0 = h_1(T-t_0) - \widetilde{h}_0(T-t_0)$; then $h_1(T-t_0) = \widetilde{h}_0(T-t_0) + M_0 = h_0(T-t_0)$.

Case II: $\nu < \varrho < \nu e^{rT}$ or, equivalently, $(1/\mu_1)M_Y'(\nu) - 1 < \eta < (1/\mu_1)M_Y'(\nu e^{rT}) - 1$. Let $t_1 = T - (1/r)\ln(\varrho/\nu)$; then $q_1(T-t) < 1$ for $t \in [0, t_1)$, $q_1(T-t) \ge 1$ for $t \in [t_1, T]$. Therefore, the optimal reinsurance strategy can be given as

$$q_t^* = \begin{cases} 1 - \dfrac{\theta}{\eta} & \text{if } 0 \le t < t_0, \\ \dfrac{\varrho}{\nu}e^{-r(T-t)} & \text{if } t_0 \le t < t_1, \\ 1 & \text{if } t_1 \le t \le T. \end{cases}$$

When $0 \le t < t_0$, substituting $A_t^* = A_1(T-t) = \widehat{A}$, $q_t^* = \overline{q}$ into (3.3),

$$h_2(T-t) = \widetilde{h}_0(T-t) + M_1, \tag{3.9}$$

where M_1 is a constant that will be determined later. When $t_0 \le t < t_1$, substituting $A_t^* = A_1(T-t) = \widehat{A}$, $q_t^* = q_1(T-t)$ into (3.3),

$$h_3(T-t) = h_1(T-t) + M_2, \tag{3.10}$$

where M_2 is a constant that will be determined later.

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When $t_1 \le t \le T$, replacing $A_t^* = A_1(T-t) = \widehat{A}$, $q_t^* = 1$ into (3.3) yields

$$\begin{aligned} h_4'(T-t) &= -[(a-r)\widehat{A} + c]\nu e^{r(T-t)} + \frac{1}{2}[\widehat{A}^2\sigma^2 + \beta^2 + 2\widehat{A}\sigma\beta\rho]\nu^2 e^{2r(T-t)} \\ &\quad + \lambda_1 \int_0^\infty [\exp(y\nu e^{r(T-t)}) - 1] f(y)\, dy \\ &\quad + \lambda_2 \int_R [\exp(-\nu\widehat{A}\gamma(t,z)e^{r(T-t)}) - 1 + \nu\widehat{A}\gamma(t,z)e^{r(T-t)}] g(z)\, dz \end{aligned} \tag{3.11}$$

together with the initial condition $h_4(0) = 0$.

Let $M_2 = h_4(T-t_1) - h_1(T-t_1)$ and $M_1 = h_3(T-t_0) - \widetilde{h}_0(T-t_0)$; then we have $h_4(T-t_1) = h_1(T-t_1) + M_2 = h_3(T-t_1)$ and $h_3(T-t_0) = \widetilde{h}_0(T-t_0) + M_1 = h_2(T-t_0)$.

Case III: $\varrho \ge \nu e^{rT}$ or, equivalently, $\eta \ge (1/\mu_1)M_Y'(\nu e^{rT}) - 1$. Then $q_1(T-t) \ge 1$ for any $t \in [0,T]$. Therefore, $q_t^* \equiv 1$ and the corresponding $h(T-t) = h_4(T-t)$. The function $h_4(T-t)$ can be calculated by using equation (3.11).

The proof of the following theorem follows from the above discussion.

THEOREM 3.3. *Let ϱ be the unique positive root of equation (3.5), $\widehat{A}$ be the finite root of equation (3.6) and $h_0(T-t)$, $h_1(T-t)$, $h_2(T-t)$, $h_3(T-t)$ and $h_4(T-t)$ be as in the equations (3.7)–(3.11), respectively. Then, for any $t \in [0,T]$, the optimal investment strategy is*

$$A_t^* = \widehat{A}.$$

Moreover:

(1) *if $0 < \eta \le (1/\mu_1)M_Y'(\nu) - 1$, then with $t_0 = T + (1/r)\ln[(1-\theta/\eta)\nu/\varrho]$, for any $t \in [0,T]$, the optimal reinsurance strategy of model (2.2) is*

$$q_t^* = \begin{cases} 1 - \dfrac{\theta}{\eta} & \text{if } 0 \le t < t_0, \\ \dfrac{\varrho}{\nu} e^{-r(T-t)} & \text{if } t_0 \le t \le T \end{cases}$$

and the value function is

$$V(t,x) = \begin{cases} m - \dfrac{\kappa}{\nu}\exp\{-\nu x e^{r(T-t)} + h_0(T-t)\} & \text{if } 0 \le t < t_0, \\ m - \dfrac{\kappa}{\nu}\exp\{-\nu x e^{r(T-t)} + h_1(T-t)\} & \text{if } t_0 \le t \le T; \end{cases}$$

(2) *if $(1/\mu_1)M_Y'(\nu) - 1 < \eta < (1/\mu_1)M_Y'(\nu e^{rT}) - 1$, then with $t_1 = T - (1/r)\ln(\varrho/\nu)$, the optimal reinsurance strategy of model (2.2) is*

$$q_t^* = \begin{cases} 1 - \dfrac{\theta}{\eta} & \text{if } 0 \le t < t_0, \\ \dfrac{\varrho}{\nu} e^{-r(T-t)} & \text{if } t_0 \le t < t_1, \\ 1 & \text{if } t_1 \le t \le T \end{cases}$$

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and the value function is

$$V(t,x)=\begin{cases} m-\dfrac{\kappa}{\nu}\exp\{-\nu x e^{r(T-t)}+h_2(T-t)\} & \text{if } 0\le t<t_0,\\ m-\dfrac{\kappa}{\nu}\exp\{-\nu x e^{r(T-t)}+h_3(T-t)\} & \text{if } t_0\le t\le t_1,\\ m-\dfrac{\kappa}{\nu}\exp\{-\nu x e^{r(T-t)}+h_4(T-t)\} & \text{if } t_1\le t\le T;\end{cases}$$

(3) *if* $\eta \ge (1/\mu_1)M_Y'(\nu e^{rT})-1$, *then for any* $t\in[0,T]$, *the optimal reinsurance strategy of model* (2.2) *is*

$$q_t^*\equiv 1$$

and the value function is

$$V(t,x)=m-\frac{\kappa}{\nu}\exp\{-\nu x e^{r(T-t)}+h_4(T-t)\}.$$

REMARK 3.4. If we know the distributions of the claim size Y and the jump size Z of the risky asset, and the expression of $\gamma(t,z)$, then we can obtain closed-form expressions for $h_0(T-t)$, $h_1(T-t)$, $h_2(T-t)$, $h_3(T-t)$ and $h_4(T-t)$, since $h_1(T-t_0)=h_0(T-t_0)$, $h_3(T-t_0)=h_2(T-t_0)$ and $h_4(T-t_1)=h_3(T-t_1)$. Then $V(t,x)$ is a continuous function for any $(t,x)\in[0,T]\times R$. By some simple calculations, we have $h_1'(T-t_0)=h_0'(T-t_0)$, $h_2'(T-t_0)=h_3'(T-t_0)=h_1'(T-t_0)$ and $h_3'(T-t_1)=h_4'(T-t_1)$. By (3.2), we have $V(t,x)\in C^{1,2}$, that is, $V(t,x)$ is a classical solution of the HJB equation (2.4).

4. Numerical analysis

In this section, we conduct a Monte Carlo simulation to illustrate the fact that the derived optimal strategies in Theorem 3.3 are indeed optimal strategies. Moreover, we present some illustrative numerical examples to study the relationships between the optimal proportional reinsurance and investment strategy and the parameters, and also investigate the effects of the jump on optimal investment.

Suppose that the claim sizes are exponentially distributed, that is, the density function $f(y)=(1/\mu_1)e^{-y/\mu_1}$, $y\ge 0$. Then equation (3.5) has a unique positive root

$$\varrho=\frac{1-\sqrt{1/(1+\eta)}}{\mu_1}. \tag{4.1}$$

Assume that the jump size Z of the risky asset with density

$$g(z)=\begin{cases} 2pe^{-2z} & \text{if } z\ge 0,\\ 3\overline{p}e^{3z} & \text{if } z<0,\end{cases}$$

where p, $\overline{p}\ge 0$ represent the probabilities of upward and downward jumps, respectively, with $p+\overline{p}=1$. In addition, we assume that $\gamma(t,z)\equiv e^z-1$ and $\rho=0$. Let $\alpha=\nu A e^{r(T-t)}$; then equation (3.6) becomes

$$\lambda_2\Big[p\alpha-p\alpha(2+\alpha)\int_0^\infty e^{-\alpha(e^z-1)}\,dz+\frac{\overline{p}}{4}-\frac{3\overline{p}}{\alpha^2}-\frac{12\overline{p}}{\alpha^3}-\frac{18\overline{p}}{\alpha^4}-\frac{6\overline{p}}{\alpha^3}e^\alpha+\frac{18\overline{p}}{\alpha^4}e^\alpha\Big] \\ =\sigma^2\alpha-a+r. \tag{4.2}$$

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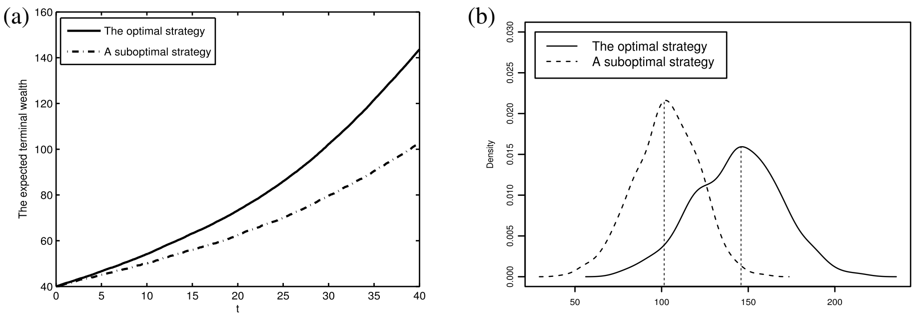

FIGURE 1. (a) Comparing the terminal wealth. (b) Comparing the density.

4.1. Monte Carlo simulation for the terminal wealth In Section 3, we have derived the closed-form expressions for the value function and optimal strategy by strict logical reasoning. In order to visualize our results, we conduct a simulation which contains two strategies, namely, the optimal strategy and a suboptimal strategy chosen at random.

To simplify our analysis, here we assume that $c = 3.5$, $r = 0.03$, $a = 0.05$, $\lambda_1 = 3$, $\lambda_2 = 2$, $\mu_1 = 1$, $\nu = 0.15$, $\eta = 0.6$, $p = 0.2$, $\sigma = 0.2$, $\beta = 0$, $x = 40$ and $T = 40$. Via Monte Carlo simulations, we perform the sample paths of the expected wealth process under the optimal strategy and a suboptimal strategy (A_t, q_t), respectively (see Figure 1(a)) via the MATLAB software package. Where $A_t \equiv 0$ is predefined, q_t is random and we assume that it has a uniform distribution on $[0, 1]$. Based on the data generated by Monte Carlo simulation in the last stage, we present the density functions of the terminal wealth by kernel density estimation for the two cases as in Figure 1(a) (see Figure 1(b)). Figure 1(a) shows that the expected wealth under the optimal strategy is bigger than that under the suboptimal strategy not only in the terminal moment but also throughout the period. In Figure 1(b), we find that the terminal wealth's density function is on the right. It means that terminal wealth under the optimal strategy will take larger values with higher probability. Since the exponential utility function is an increasing function of the terminal wealth, the simulation supports the fact that the derived closed-form solutions for optimal strategies in Section 3 are indeed optimal strategies.

4.2. The optimal proportional reinsurance strategy

EXAMPLE 4.1. Assume that $c = 3.5$, $\lambda_1 = 3$, $\mu_1 = 1$, $\nu = 0.1$, $\eta = 0.6$, $T = 40$, $r \in \{0.03, 0.04, 0.05\}$. We calculate the optimal proportional reinsurance strategy q_t^* by (4.1). The results are presented in Figure 2(a). We see that q_t^* is a decreasing function of r. As r is the risk-free interest rate, the larger r is, the greater the expected income of the risk-free asset, the larger the income the insurance company will obtain from investment and hence the less risk the insurance company will wish to share in each claim.

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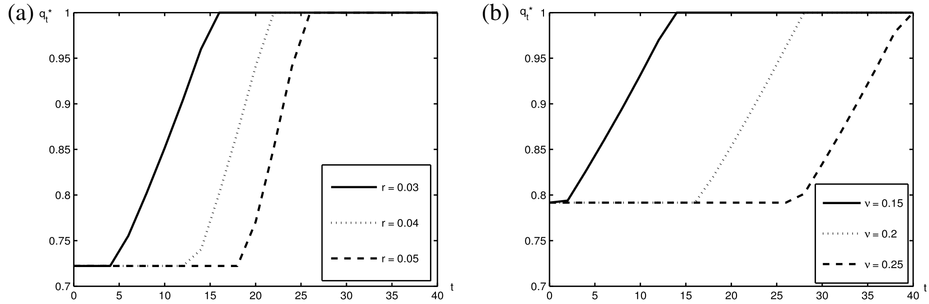

FIGURE 2. (a) Effect of r on q_t^*. (b) Effect of ν on q_t^*.

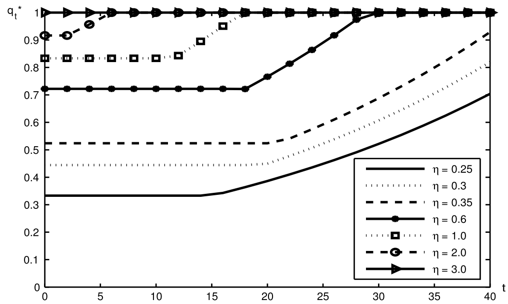

FIGURE 3. Effect of η on q_t^*.

EXAMPLE 4.2. Assume that $c = 3.5$, $\lambda_1 = 3$, $\mu_1 = 1$, $r = 0.02$, $\eta = 0.8$, $T = 40$, $\nu \in \{0.15, 0.2, 0.25\}$. We calculate the optimal proportional reinsurance strategy q_t^* by (4.1). The results are presented in Figure 2(b). Note that q_t^* is decreasing in ν. As ν is the absolute risk aversion parameter, the larger ν is, the less aggressive the insurer will be and hence the less retention level the insurer will hold.

EXAMPLE 4.3. Assume that $c = 3.5$, $\lambda_1 = 3$, $\mu_1 = 1$, $\nu = 0.15$, $r = 0.03$, $T = 40$, $\eta \in \{0.25, 0.3, 0.35, 0.6, 1.0, 2.0, 3.0\}$. We calculate the optimal proportional reinsurance strategy q_t^* by (4.1). The results are presented in Figure 3. Observe that q_t^* is increasing in η. A large η yields a high retention level of proportional reinsurance. As the premium of reinsurance increases, the insurer should retain a greater share of each claim.

4.3. The optimal investment strategy

EXAMPLE 4.4. Assume that $\sigma = 0.2$, $\lambda_2 = 2$, $p = 0.8$, $r = 0.03$, $\nu = 0.15$, $T = 40$, $a \in \{0.03, 0.04, 0.05\}$. We calculate the optimal investment strategy A_t^* by (4.2). The results are presented in Figure 4(a). Observe that A_t^* is an increasing function of a, which describes the rate of the income of the risky asset. The larger a is, the greater the

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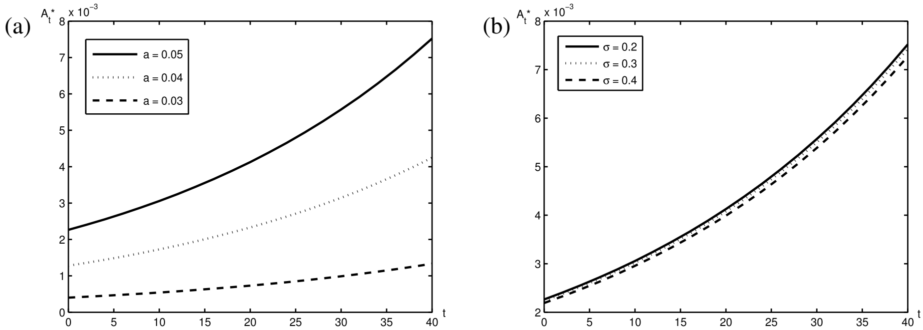

FIGURE 4. (a) Effect of a on A_t^*. (b) Effect of σ on A_t^*.

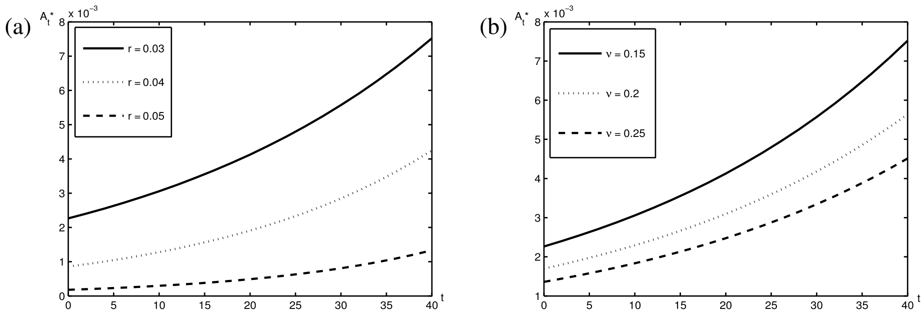

FIGURE 5. (a) Effect of r on A_t^*. (b) Effect of ν on A_t^*.

expected income of the risky asset will be and hence the more the insurance company will wish to invest in the risky asset.

EXAMPLE 4.5. Assume that $a = 0.05$, $\lambda_2 = 2$, $p = 0.8$, $r = 0.03$, $\nu = 0.15$, $T = 40$, $\sigma \in \{0.2, 0.3, 0.4\}$. We calculate the optimal investment strategy A_t^* by (4.2). The results are presented in Figure 4(b). We see that A_t^* is decreasing in σ, which is the volatility of the risky asset. The larger σ is, the riskier the risky asset will be and hence the less the insurance company will wish to invest in the risky asset.

EXAMPLE 4.6. Assume that $a = 0.05$, $\sigma = 0.2$, $\lambda_2 = 2$, $p = 0.8$, $\nu = 0.15$, $T = 40$, $r \in \{0.03, 0.04, 0.05\}$. We calculate the optimal investment strategy A_t^* by (4.2). The results are presented in Figure 5(a). We see that A_t^* is a decreasing function of r. Since r is the risk-free interest rate, the larger r is, the greater the expected income of the risk-free asset and hence the more the insurance company wishes to invest in the risk-free asset.

EXAMPLE 4.7. Assume that $a = 0.05$, $\sigma = 0.2$, $\lambda_2 = 2$, $p = 0.8$, $r = 0.03$, $T = 40$, $\nu \in \{0.15, 0.2, 0.25\}$. We calculate the optimal investment strategy A_t^* by (4.2). The results are presented in Figure 5(b). Note that A_t^* is decreasing in ν. As ν is the

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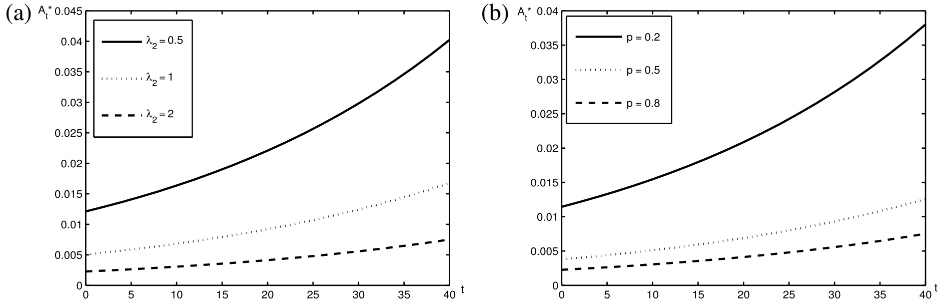

FIGURE 6. (a) Effect of λ_2 on A_t^*. (b) Effect of p on A_t^*.

absolute risk aversion parameter, the larger v is, the less aggressive the insurer will be and hence the less the insurance company will wish to invest in the risky asset.

EXAMPLE 4.8. Assume that $a = 0.05$, $\sigma = 0.2$, $v = 0.15$, $p = 0.8$, $r = 0.03$, $T = 40$, $\lambda_2 \in \{0.5, 1, 2\}$. We calculate the optimal investment strategy A_t^* by (4.2). The results are presented in Figure 6(a). We see that A_t^* is decreasing in λ_2, which is the number of jumps of the risky asset within unit time (see (2.1)). The larger λ_2 is, the greater the expected jump size of the risky asset will be and hence the less the insurance company will wish to invest in the risky asset.

EXAMPLE 4.9. Assume that $a = 0.05$, $\sigma = 0.2$, $v = 0.15$, $\lambda_2 = 2$, $r = 0.03$, $T = 40$, $p \in \{0.2, 0.5, 0.8\}$. We calculate the optimal investment strategy A_t^* by (4.2). The results are presented in Figure 6(b). We see that A_t^* is a decreasing function of p. As p represents the probability of an upward jump of the risky asset, the larger p is, the greater the expected jump size of the risky asset will be and hence the less the insurance company will wish to invest in the risky asset.

5. Conclusions

In this paper, we studied the optimal investment and proportional reinsurance problem for a jump-diffusion risk model in a general jump-diffusion financial market. Moreover, we added a reasonable constraint on the proportional reinsurance strategy which makes the model much more reasonable and realistic. We obtained explicit expressions for the optimal value function and optimal strategy. Finally, a Monte Carlo simulation was conducted to illustrate that the closed-form expressions we derived are indeed the optimal strategies, and some numerical examples were presented to illustrate the effects of parameters on the optimal strategies as well as the economic implications. There are some possible extensions of this paper. For example, one can try to add some other constraints in the model, such as transaction costs for investment and the dividend payment. Then the model may be more realistic and the problem more interesting. These are very challenging problems that constitute the research directions of our future work.

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