1

Introduction

1.1 Initial Considerations

Some years ago at Cranfield, where we had set up a flow rig for testing the effect of upstream pipe fittings on certain flowmeters, a group of senior Frenchmen was being shown around and visited this rig. The leader of the French party recalled a similar occasion in France when visiting such a rig. The story goes something like this.

A bucket at the end of a pipe seemed particularly out of keeping with the remaining high-tech rig. When someone questioned the bucket's function, it was explained that the bucket was used to measure the flow rate. Not to give the wrong impression in the future, the bucket was exchanged for a shiny, new, high-tech flowmeter. In due course, another party visited the rig and observed the flowmeter with approval. "And how do you calibrate the flowmeter?" one visitor asked. The engineer responsible for the rig then produced the old bucket!

This book sets out to guide those who need to make decisions about whether to use a shiny flowmeter, an old bucket, nothing at all or a combination of these! It also provides information for those whose business is the design, manufacture or marketing of flowmeters. I hope it will, therefore, be of value to a wide variety of people, both in industry and in the science base, who range across the whole spectrum from research and development through manufacturing and marketing. In my earlier book on flow measurement (Baker 1988a/1989, 2002b, 2003), I provided a brief statement on each flowmeter to help the uninitiated. This book attempts to give a much more thorough review of published literature and industrial practice.

This first chapter covers various general points that do not fit comfortably elsewhere. In particular, it reviews guidance on the accuracy of flowmeters (or calibration facilities).

The second chapter reviews briefly some essentials of fluid mechanics necessary for reading this book. The reader will find a fuller treatment in Baker (1996), which also has a list of books for further reading.

A discussion of how to select a flowmeter is attempted in Chapter 3, and some indication of the variety of calibration methods is given in Chapter 4, before going in detail in Chapters 5–20 into the various high- (and low-) tech meters available.

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

In this edition, I have introduced three additional chapters to cover new commercial meters and to allow a brief and superficial review of multiphase hydrocarbon flowmeters. Chapter 21 deals with probes. Chapter 22 covers general issues relating to verification and clamp-on meters. Chapter 23 provides a brief introduction to remote data handling and Chapter 24 provides final personal reflections relating to manufacture and future developments.

In this book, I have tried to give a balance between the laboratory ideal, manufacturers' claims, the realities of field experience and the theory behind the practice. I am very conscious that the development and calibration laboratories are sometimes misleading places, which omit the problems encountered in the field (Stobie 1993), and particularly so when that field happens to be the North Sea. This may be more serious for flowmeters than for some other instruments, and may require careful consideration of the increase in uncertainty which results. In the same North Sea Flow Measurement Workshop, there was an example of the unexpected problems encountered in precise flow measurement (Kleppe and Danielsen 1993), resulting, in this case, from a new well being brought into operation. It had significant amounts of barium and strontium ions, which reacted with sulphate ions from injection water and caused a deposit of sulphates from the barium sulphate and strontium sulphate that were formed.

With that salutary reminder of the real world, we ask an important – and perhaps unexpected – question.

1.2 Do We Need a Flowmeter?

Starting with this question is useful. It may seem obvious that anyone who looks to this book for advice on selection is in need of a flowmeter, but for the process engineer it is an essential question to ask. Many flowmeters and other instruments have been installed without careful consideration being given to this question and without the necessary actions being taken to ensure proper documentation, maintenance and calibration scheduling. They are now useless to the plant operator and may even be dangerous components in the plant. Thus, before a flowmeter is installed, it is important to ask whether the meter is needed, whether proper maintenance schedules are in place, whether the flowmeter will be regularly calibrated, and whether the company has allocated to such an installation the funds needed to achieve this ongoing care. Such care will need proper documentation.

The water industry in the United Kingdom has provided examples of the problems associated with unmaintained instruments. Most of us involved in the metering business will have sad stories of the incorrect installation or misuse of meters. Reliability-centred maintenance recognises that the inherent reliability depends on the design and manufacture of an item, and if necessary this will need improving (Dixey 1993). It also recognises that reliability is preferable in critical situations to extremely sophisticated designs, and it uses failure patterns to select preventive maintenance.

In some research into water consumption and loss in urban areas, Hopkins, Savage and Fox (1995) found that obstacles to accurate measurements were

- buried control valves,
- malfunctioning valves,
- valve gland leakage,
- hidden meters that could not be read and
- locked premises denying access to meters.

They commented that "water supply systems are dynamic functions having to be constantly expanded or amended. Consequently continuous monitoring, revisions and amendments of networks records is imperative. Furthermore, a proper programme of inspection, maintenance and subsequent recording must be operative in respect of inter alia:

- networks,
- meters,
- control valves,
- air valves,
- pressure reducing valves,
- non-return valves."

They also commented on the poor upstream pipework at the installation of many domestic meters.

So I make no apology for emphasising the need to assess whether a flowmeter is actually needed in any specific application.

If the answer is yes, then there is a need to consider the type of flowmeter and whether the meter should be measuring volume or mass. In most cases, the most logical measure is mass. However, by tradition, availability and industrial usage, volume measurement may be the norm in some places, and as a result, the regulations have been written for volume measurement. This results in a Catch-22 situation. The industry and the regulations may, reasonably, resist change to mass flow measurement until there is sufficient industrial experience, but industrial experience is not possible until the industry and the regulations allow. The way forward is for one or more forward-looking companies to try out the new technology and obtain field experience, confidence in the technology and approval.

In this book, I have made no attempt to alert the reader to the industry-specific regulations and legal requirements, although some are mentioned. The various authors touch on some regulations, and Miller (1996) is a source of information on many documents. An objective of the Organisation Internationale de Métrologie Légale (OIML) is to prevent any technical barriers to international trade resulting from conflicting regulations for measuring instruments. With regard to flow measurement, it appears to have been particularly concerned with the measurement of domestic supplies and industrial supplies of water and gas (Athane 1994). This is because two parties, the supplier and the consumer, are involved, and the consumer is unlikely to be able to ascertain the correct operation of the meter. In addition, the supplier does not continually monitor these measurements, the meters may fail without anyone knowing, the usage is irregular and widely varying in rate, the measurements are not repeatable, and the commodities have increased in value considerably in recent years.

In order to reduce discussions and interpretation problems between manufacturers and authorised certifying institutes, the European Commission was mandating the European standardisation bodies (CEN and CENELEC) to develop harmonised standards that would give the technical details and implementation of the requirements based on OIML recommendations. These would be such that a measuring instrument complied with essential requirements, assuming that the manufacturer had complied with them (Nederlof 1994).

The manufacturer will also be fully aware of the electromagnetic compatibility (EMC), which relates to electromagnetic interference. In particular, the EMC characteristics of a product are that

- the level of electromagnetic disturbance the instrument generates will not interfere with other apparatuses, and
- the operation of the instrument will not be adversely affected by electromagnetic interference from its environment.

In order to facilitate free movement within the European area, the CE mark was designed to identify products that conformed to the European essential requirements. For further details relating to the European Community (EC), the reader is referred to the Measuring Instrument Directive (MID 2004, DTI 1993, Chambers 1994).

First, we consider the knotty problem of how accurate the meter should be.

1.3 How Accurate?

Inconsistency remains about the use of terms that relate to accuracy and precision. This stems from a slight mismatch between the commonly used terms and those that the purists and the standards use. Thus we commonly refer to an accurate measurement, when strictly we should refer to one with a small value of uncertainty. We should reserve the use of the word *accurate* to refer to the instrument. A high-quality flowmeter, carefully produced with a design and construction to tight tolerances and with high-quality materials as well as low wear and fatigue characteristics, is a precise meter with a quantifiable value of repeatability. Also, it will, with calibration on an accredited facility, be an accurate meter with a small and quantifiable value of measurement uncertainty. In the context of flowmeters, the word *repeatability* is preferred to *reproducibility*. The meanings are elaborated on later, and I regret the limited meaning now given to *precision*, which I have used more generally in the past and shall slip back into in this book from time to time! In the following chapters, I have attempted to be consistent in the use of these words. However, many claims for accuracy may not have been backed by an accredited facility, but I have tended to use the phrase "measurement uncertainty" for the claims made.

Hayward (1977a) used the story of William Tell to illustrate precision. William Tell had to use his crossbow to fire an arrow into an apple on his little son's head. This was a punishment for failing to pay symbolic homage to an oppressive Austrian ruler. Tell succeeded because he was an archer of great skill and high accuracy.

An archer's ability to shoot arrows into a target provides a useful illustration of some of the words related to precision. So Figure 1.1(a) shows a target with all the shots in the bull's-eye. Let us take the bull's-eye to represent ±1%, within the first ring ±3%, and within the second ring ±5%. Ten shots out of ten are on target, but how many will the archer fire before one goes outside the bull's-eye? If the archer, on average, achieves 19 out of 20 shots within the bull's-eye [Figure 1.1(b)], we say that the archer has an uncertainty of ±1% (the bull's-eye) with a 95% confidence level (19 out of 20 on the bull's-eye: 19 ÷ 20 = 0.95 = 95 ÷ 100 = 95%).

Suppose that another archer clusters all the arrows, but not in the bull's-eye, Figure 1.1(c). This second archer is very consistent (all the shots are within the same size circle as the bull's-eye), but this archer needs to adjust his aim to correct the offset. We could say that the second archer has achieved high repeatability of ±1%, but with a bias of 4%. We might even find that 19 out of 20 shots fell within the top left circle so that we could say that this archer achieved a repeatability within that circle of ±1% with a 95% confidence. Suppose this archer had fired one shot a day, and they had all fallen onto a small area [Figure 1.1(c)], despite slight changes in wind, sunshine and archer's mood; we term this good day-to-day repeatability. But how well can we depend on the archer's bias? Is there an uncertainty related to it?

Finally, a third archer shoots 20 shots and achieves the distribution in Figure 1.1(d). One has missed entirely, but 19 out of 20 have hit the target somewhere. The archer has poor accuracy, and the uncertainty in this archer's shots is about five times greater than for the first, even though the confidence level at which this archer performs is still about 95%.

If the third archer has some skill, then the bunching of the arrows will be greater in the bull's-eye than in the next circle out, and the distribution by ring will be as shown in Figure 1.1(e).

We shall find that the distribution of readings of a flowmeter results in a curve approximating a normal distribution with a shape similar to that for the shots. Figure 1.1(f) shows such a distribution where 95% of the results lie within the shaded area and the width of that area can be calculated to give the uncertainty, ±1% say, of the readings with a 95% confidence level. In other words, 19 of every 20 readings fall within the shaded area.

With this simplistic explanation, we turn to the words that relate to precision.

Accuracy

It is generally accepted that *accuracy* refers to the truthfulness of the instrument. An instrument of high accuracy more nearly gives a true reading than an instrument of low accuracy. Accuracy, then, is the quality of the instrument. It is common to refer to a measurement as accurate or not, and we understand what is meant. However,

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

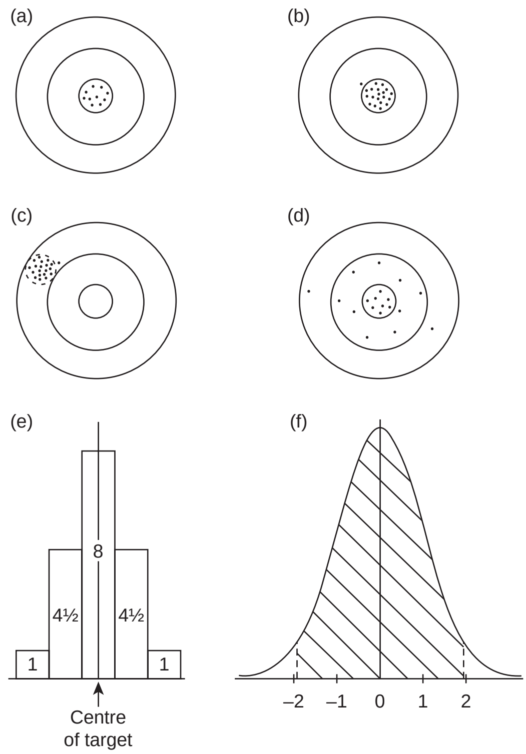

Figure 1.1. Precision related to the case of an archery target. **(a)** Good shooting – 10 out of 10 arrows have hit the bull's-eye. An accurate archer? **(b)** Good shooting? – 19 out of 20 arrows have hit the bull's-eye. An accurate archer and a low value of uncertainty (±1%) with a 95% confidence level. (**c**) Shots all fall in a small region but not the bull's-eye. Good repeatability (±1%) but a persistent bias of 4%. (**d**) Shots, all but one, fall on the target – 19 out of 20 have hit the target. A ±5% uncertainty with 95% confidence level. **(e)** Distribution of shots in (d) on a linear plot, assuming that we can collapse the shots in a ring semicircle onto the axis. **(f)** The normal distribution, which is a good approximation for the distribution of flowmeter readings.

the current position is that accuracy should be used as a qualitative term and that no numerical value should be attached to it. It is, therefore, incorrect to refer to a measurement's accuracy of, say, 1%, when, presumably, this is the instrument's measurement uncertainty, as is explained later.

Repeatability

In a process plant, or other control loop, we may not need to know the accuracy of a flowmeter as we would if we were buying and selling liquid or gas, but we may require repeatability within bounds defined by the process. *Repeatability* is the value below which the difference between any two test results, taken under constant conditions with the same observer and with a short elapsed time, are expected to lie with 95% confidence.

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

Precision

Precision is the qualitative expression for repeatability. It should not take a value and should not be used as a synonym for accuracy.

Uncertainty

Properly used, *uncertainty* refers to the quality of the measurement, and we can correctly refer to an instrument reading having an uncertainty of ±1%. By this we mean that the readings will lie within an envelope ±1% of the true value. Each reading will, of course, have an individual error that we cannot know in practice, but we are interested in the relationship of the readings to the true value. Because *uncertainty* is referred to the true value, by implication it must be obtained using a national standard document or facility. However, because it is a statistical quantity, we need also to define how frequently the reading does, in fact, lie within the envelope; hence the confidence level.

Confidence Level

The *confidence level*, which is a statement of probability, gives this frequency, and it is not satisfactory to state an uncertainty without it. Usually, for flow measurement, this is 95%. We shall assume this level in this book. A confidence level of 95% means that we should expect on average that 19 times out of 20 (19/20 = 95/100 = 95%) the reading of the meter will fall within the bracket specified (e.g. ±1% of actual calibrated value).

Linearity

Linearity may be used for instruments that give a reading approximately proportional to the true flow rate over their specified range. It is a special case of *conformity* to a curve. Note that both terms really imply the opposite. *Linearity* refers to the closeness within which the meter achieves a truly linear or proportional response. It is usually defined by stating the maximum deviation (or nonconformity e.g. ±1% of flow rate) within which the response lies over a stated range. With modern signal processing, linearity is probably less important than conformity to a general curve. *Linearity* is most commonly used with such meters as the turbine meter.

Range and Rangeability

An instrument should have a specified range over which its performance can be trusted. Therefore, there will be upper- and lower-range values. This reflects the fact that probably no instrument can be used to measure a variable when there are no limitations on the variable. Without such a statement, the values for uncertainty, linearity etc. are inadequate. The ratio of upper-range value and lower-range value may be called the *rangeability*, but it has also been known as the *turndown ratio.* The difference between upper- and lower- or negative-range values is known as *span.* It is important to note whether the values of uncertainty, linearity etc. are a percentage of the actual flow rate or of the full-scale flow [sometimes referred to as full-scale deflection (FSD), full-scale reading (FSR), maximum-scale value, or upper-range value (URV)].

1.4 A Brief Review of the Evaluation of Standard Uncertainty

Kinghorn (1982) points out the problem with terminology in matters concerning statistics and flow measurement. To the engineer and the statistician, words such as *error* and *tolerance* may have different meanings. The word *tolerance* was used for what is now known as uncertainty.

In providing an introduction to the terminology of uncertainty in measurement, I shall aim to follow the guidance in BIPM et al. (2008), which is usually known as the *Guide* or GUM, and also in a document consistent with the *Guide*, which provides the basis for uncertainty estimates in laboratories accredited in the United Kingdom (UKAS 2012). The reader should note that the *Guide* may also be available as ISO/IEC 98-3: 2008 and that a further valuable document is a guide on the vocabulary of metrology, ISO/IEC 99: 2007. The reader is strongly advised to consult this document, which is full of clear explanations and useful examples. Those wishing to pursue background arguments are referred to Van der Grinten's (1994, 1997) papers.

Random error, the random part of the experimental error, causes scatter, as the name suggests, and reflects the quality of the instrument design and construction. It is the part that cannot be calibrated out, and the smaller it is, the more precise the instrument is. It may be calculated by taking a series of repeat readings resulting in the value of the standard deviation of a limited sample n, and sometimes called the experimental standard deviation:

$$s\left(q_{\mathrm{j}}\right)=\left\{\frac{1}{n-1}\sum_{j=1}^{n}\left(q_{\mathrm{j}}-\bar{q}\right)^{2}\right\}^{1/2} \tag{1.1}$$

where $\bar{q}$ is the mean of n measurements q_{j}. The experimental standard deviation of the mean of this group of readings is given by

$$s\left(\bar{q}\right)=\frac{s\left(q_{\mathrm{j}}\right)}{\sqrt{n}} \tag{1.2}$$

Where too few readings have been taken to obtain a reliable value of $s(q_{\mathrm{j}})$, an earlier calculation of $s(q_{\mathrm{j}})$ from previous data may be substituted in Equation (1.2). In obtaining the overall uncertainty of a flowmeter or a calibration facility, there will be values of group mean experimental standard deviation for various quantities, and so UKAS (2012) defines a standard uncertainty for the *i*th quantity as

$$u(x_i) = s(\bar{q}) \tag{1.3}$$

where x_i is one of the input quantities. For those with access to UKAS (2012), this is, essentially, dealt with there as a Type A evaluation of standard uncertainty.

Systematic error, according to flowmeter usage, is that which is unchanging within the period of a short test with constant conditions. This is, essentially, dealt with in UKAS (2012) under the heading Type B evaluation of standard uncertainty. It is also called *bias.* However, in modern flowmeters and in calibration facilities, it is likely that this bias or systematic error will result in a meter adjustment, or a rig correction. The resulting uncertainty in that adjustment or correction will contribute to the overall uncertainty. The systematic uncertainty, therefore, may derive from various factors such as

a. uncertainty in the reference and any drift,
b. the equipment used to measure or calibrate,
c. the equipment being calibrated in terms of resolution and stability,
d. the operational procedure, and
e. environmental factors.

From these we deduce further values of $u(x_i)$.

There has been debate about the correct way to combine the random and systematic uncertainties. We can combine random and systematic uncertainties conservatively by arithmetic addition. This results in a conservative estimate. UKAS (2012) has followed the *Guide* in taking the square root of the sum of the squares of the standard uncertainties in consistent units. Thus the combined standard uncertainty is

$$u_c(y) = \sqrt{\sum [c_i\, u(x_i)]^2} \tag{1.4}$$

where y is the output quantity. To ensure consistent units, a sensitivity coefficient, c_i, will be required for each input x_i, although in practice this may be unity in most cases (as in Figure 4.3).

The final step (and we have glossed over many important details in UKAS 2012) is to deduce from u_c the bracket within which the reading of, say, the meter lies.

In the past, bearing in mind that u_c or its components have been derived from standard deviations, we have used Student's t value, which for a number of readings n is given by

n	t
10	2.26
20	2.09
>30	2.0

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

for a 95% confidence level. The *Guide* replaces this, in general, with a coverage factor, k, to obtain the expanded uncertainty

$$U = ku_c(y) \tag{1.5}$$

The recommended value is k = 2, which gives a confidence level of 95.45% taken as 95%, assuming a normal distribution. If this assumption is not adequate, then we need to revert to Student's t.

The net result is that the assumption of a factor of 2 has now been given a systematic basis. The reader who is interested in more details about the basis of normal and t distributions is referred to Appendix 1.A.

1.5 Note on Monte Carlo Methods

An alternative approach, which is an outcome of the speed and accessibility of personal computers, is based on the use of random number generators to model instrument errors, and on running many tests to obtain the overall uncertainty of the system, say, a flow calibration rig. This is known as the Monte Carlo method for assessing uncertainty.

Not being a statistician, my perception of these methods is that, essentially, a numerical model of the measurement system is set up on a computer, instrument and system errors are modelled using values obtained from a random number generator and the measurement procedure is, thereby, modelled. The program is then run very many times, to obtain the likely uncertainty by averaging all the results. The procedure may be less conservative in its assessment than the standard GUM (BIPM et al. 1993) approach.

Monte Carlo computer programs are available, some as freeware. One or more such programs may be specifically modelled on the latest GUM approach (e.g. GUM-Workbench may be available) (private communication from Peter Lau).

Some explanation of the procedure can be found in Coleman and Steele (1999).

1.6 Sensitivity Coefficients

Suppose that output quantity, a flow rate, has the relationship

$$y = x_1^p x_2^q x_3^r x_4^s \tag{1.6}$$

then if x_2, x_3 and x_4 are held constant, we can differentiate y with respect to x_1 and obtain the partial derivative. This is the slope of the curve of y against x_1 when the other variables are kept constant. It also allows us to find the effect of a small change in x_1 on y. This slope (or partial derivative) is the sensitivity coefficient c_1 for x_1 and may be found by calculation. It will have the value $c_1 = px_1^{(p-1)} x_2^q x_3^r x_4^s$, where the values of x_1, x_2, x_3 and x_4 will be at the calibration point and may be dimensional. In some cases, it may be a known coefficient (e.g. a temperature coefficient of

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

expansion). For cases where it is difficult to calculate, it may be possible to find the coefficient by changing x_1 by a small amount and observing the change in y. In some cases, the sensitivity coefficient may provide a conversion between different sets of units (e.g. where output quantity or velocity may be obtained from a dimension, a pressure, a movement or a voltage).

1.7 What Is a Flowmeter?

We take as a working definition of an ideal flowmeter:

> A group of linked components that will deliver a signal uniquely related to the flow rate or quantity of fluid flowing in a conduit, despite the influence of installation and operating environment.

The object of installing a flowmeter is to obtain a measure of the flow rate, usually in the form of an electrical signal, which is unambiguous and with a specified expanded uncertainty. This signal should be negligibly affected by the inlet and outlet pipework and the operating environment. Thus the uncertainty of measurement of a flowmeter should be reported as $y \pm U$, where U, the uncertainty band, might have a value of, say, 0.5%, and it should be made clear whether this is related to rate, full-scale deflection (FSD) or other value that might be a combination of these [e.g. in the form $\pm a$ (rate) $\pm b$ (FSD)]. The range should be given (e.g. 1 m^3/h to 20 m^3/h).

The statement of performance should include the coverage factor $k = 2$ and the level of confidence of approximately 95%, and, if appropriate, the authority that accredited the calibration facility (national or international).

In addition, the ranges of properties for which it can be used should be specified, such as fluid, flow range (beyond calibration), maximum working pressure, temperature range of fluid and ambient temperature range.

It is useful to introduce two factors that define the response of flowmeters, although they are most commonly used for linear flowmeters with pulse output. The K factor is the number of pulses per unit quantity. In this book, we shall take it as number of pulses per unit volume when dealing with turbine and vortex meters:

$$K = \frac{\text{Pulses}}{\text{True volume}}$$

whereas the meter factor is usually defined as

$$\text{Meter factor} = \frac{\text{True volume}}{\text{Indicated volume}}$$

The reader should keep a wary eye for other definitions of meter factor such as the reciprocal of the K factor.

Let us take a specific example of a fictitious, but reasonably realistic flowmeter. In Figure 1.2(a), a typical flowmeter envelope is shown. It defines an approximately linear flowmeter with a 10:1 turndown and an uncertainty of ±1% of rate with a confidence level of 95% against a traceable standard calibration. This is a reasonable performance for a flowmeter and probably satisfies most industry requirements. This, let us assume, is the performance specification the manufacturer carries in its sales literature. Actually the characteristic of the flowmeter may be the curve shown in Figure 1.2(a). If the company works to a high standard of manufacture, then the company may know that this characteristic lies within close tolerances in all cases. It may, therefore, only be necessary for the manufacturer to calibrate each flowmeter at, say, 90% of FSD, or 50% and 90% of FSD, in order to make the claim that the characteristic falls within the envelope specified in the sales literature.

If the meters are actually of this standard, it may well be feasible to calibrate them in much greater detail so that a 5-, 10- or even 20-point calibration may provide a characteristic that ensures that the reading is known to, say, ±0.2%. The values obtained from the calibration will then be programmed into a flow computer, which will interpret each reading of the flowmeter against this look-up table. Since we are comparing the flowmeter's signal to a linear one, if it were without error it would also be linear. Consequently, companies sometimes record linearity within their literature. In this case, it would also be ±1% with a 10:1 turndown.

The envelope just discussed gives the uncertainty at each flow rate in terms of the actual flow rate. Because of the physical basis of some flowmeters, this method is not appropriate, and the uncertainty may then be given in terms of the full scale. Figure 1.2(b) shows such an envelope where the performance of the flowmeter would be defined as ±1% FSD. It is apparent that the uncertainty in the flowmeter's reading at, say, half scale is ±2% of rate and at 20% of reading will be as much as ±5% of rate. The problem often arises wherein the user has a particular flow range that does not match that of the actual instrument. The user's full flow may be at only 60% of the instrument's range, and so for the user the instrument has an uncertainty, at best, of 1.7% of rate.

A third type of envelope is shown in Figure 1.2(c). This is particularly common in the specifications for water and gas meters. In the example shown, the meter has an uncertainty of ±2% of rate from full flow down to 20%. Below this value of flow rate, the uncertainty is ±5% of flow rate down to 2% of range. In practice, a meter might have more steps in its envelope.

In many cases, the manufacturer's specification of uncertainty may be a combination of these. As indicated earlier, it is common to have an uncertainty that combines a value based on rate and another on full-scale deflection. In addition, there may be allowances to be added for zero drift, temperature change and, possibly, even pressure change. In some flowmeters, viscosity is important but is probably accommodated by charts showing the variation in performance with viscosity.

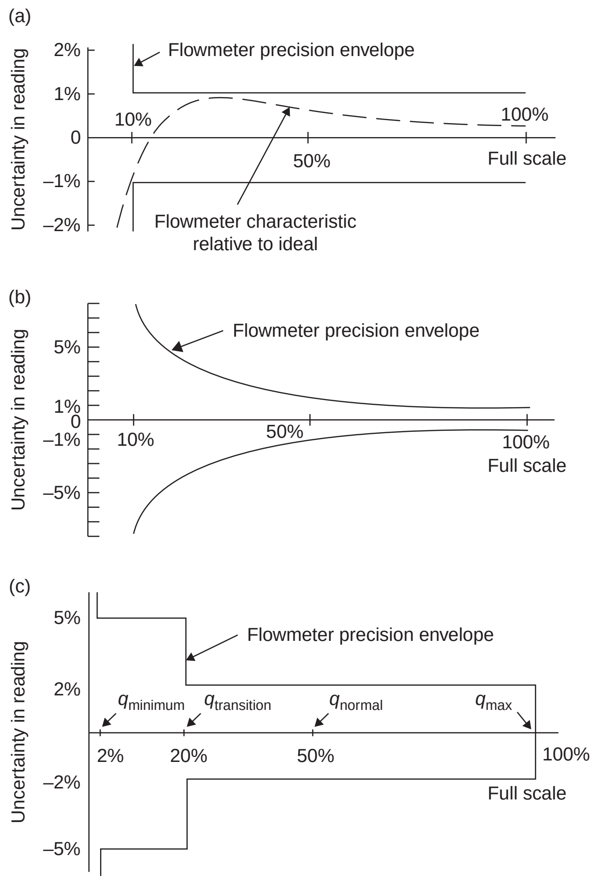

Figure 1.2. Required envelope for a flowmeter. **(a)** Envelope as a percentage of rate; (**b**) envelope as a percentage of FSD; (**c**) stepped envelope with increased uncertainty at low-range values. The Measuring Instruments Directive of the European Commission should be consulted for the latest definitions of Qmax etc. for the EC.

One note of caution! Clever electronics can take any signal, however nonlinear, and straighten the characteristic before the signal is output. Suppose such a procedure were used for the characteristic in Figure 1.2(a), below 10% of range. The characteristic is probably very sensitive to minor variations in this region, and any attempt to use the characteristic could lead to disguised, but serious, errors.

Variation of temperature and pressure can affect the performance of a flowmeter, as can humidity, vibration and other environmental parameters.

Often the units used in a manufacturer's catalogue are not those that you have calculated. For this reason, conversion factors that provide conversions to four significant figures of flow rate, velocity, temperature, pressure, length etc. have been included in Table 1.1. If not otherwise specified, the International System of units (SI) based on meter, kilogram, second is assumed.

Table 1.1. *Conversion for some essential measurements from Imperial, U.S. and other units to metric, to four significant figures (Note that the EC has time limits by which certain standard units must be used.)*

Length	1 in[a] = 25.4 mm
	1 ft = 0.3048 m
Volume	1 ft^3 = 0.0283 m^3
	1 ft^3 = 28.32 l (litre)
	1 bbl (barrel) = 0.1590 m^3
Mass	1 lb = 0.4536 kg
	1 long ton (2,240 lb) = 1,016 kg
	1 short ton (2,000 lb) = 907.2 kg
	1 metric tonne (2,205 lb) = 1,000 kg
Density	1 lb/ft^3 = 16.02 kg/m^3
Temperature	(Temperature in °F – 32)/1.8 = Temperature in °C
Pressure	1 psi = 6,895 N/m^2
Viscosity	Dynamic viscosity: SI (metric) unit is the Pascal second (Pas) to which the more common unit, the centipoise (cP), is related by 1 cP = 10^{-3} Pas.
	Kinematic viscosity: SI (metric) unit is m^2/s to which the more common unit, the centistoke (cSt), is related by 1 cSt = 10^{-6} m^2/s = 1 mm^2/s.
Velocity	1 ft/s = 0.3048 m/s
Volumetric flow rate	1 ft^3/s (1 cusec) = 0.02832 m^3/s
	1 Imp gal/s = 0.004546 m^3/s
	1 Imp gal/s = 4.546 1/s
	1 U.S. gal/s = 0.003785 m^3/s
	1 U.S. gal/s = 3.785 1/s
	1 Imp gal/h = 0.004546 m^3/h
	1 Imp gal/h = 4.546 1/h
	1 U.S. gal/h = 0.003785 m^3/h
	1 U.S. gal/h = 3.785 1/h
Mass flow rate	1 lb/h = 0.4536 kg/h
	1 lb/s = 0.4536 kg/s

[a] An approximate conversion is 4 in. to 100 mm.

1.8 Chapter Conclusions (for those who Plan to Skip the Mathematics!)

I have tried to bring together, within the compass of this book, essential information for all who may have dealings with flowmeters and flow measurement, although I have tried to avoid duplicating information available in other excellent books on the subject. For this reason, the chapters not only address the technical aspects but also the selection, maintenance, calibration and typical applications of the various meters. I hope that this book will provoke the prospective entrepreneur, the small and medium-sized enterprises (SME) or the major instrument company to assess the market needs and the relevant development and production needs of their companies for new devices.

The management of flowmeters, at all stages from selection through application in complex systems, to identifying malfunction, is clearly an area where modern information technology methods would be attractive. How does one select? How do we allow for the costs of ownership? How can we check performance and identify emerging problems? De Boom (1996) considered life-cycle analysis to help users select the most appropriate technology for their use. Menendez, Biscarri and Gomez (1998) used a model of a water supply net to deduce the errors in flow measurement from: analysis of the system, assignment to the meters of flow measurement uncertainty, estimate of flow distribution in the net and comparison of the estimated values with the measured values. Nilsson and Delsing (1998) and Nilsson (1998) considered malfunctions and inaccuracies in gas flowmeters. Scheers and Wolff (2002) saw the production measurement process from field data collection to final reporting as the entire chain.

As the reader moves into the following chapters, two sets of information may be useful. Table 1.1 lists the conversion factors for Imperial, U.S. and metric units, and Table 1.2 relates volumetric and mass flow rate to linear velocity in various sizes of tube (Baker 1988a/1989, 2002b, 2003). It is common in flow measurement to require the velocity of flow, and Table 1.2 provides an order of magnitude.

Finally, the whole matter of accuracy and the limits of accuracy, when related to all the parameters that influence a flowmeter's operation, remains an area with unanswered questions.

1.9 Mathematical Postscript

I have left this note to the end so that those who are not concerned with advanced mathematical concepts can ignore it.

I have included essential mathematics in the main text of this book. In certain flowmeters, the mathematical theory is more complex (e.g. the turbine meter), and the theory has, accordingly, been consigned to an appendix after the relevant chapter.

One important and interesting mathematical approach, which starts to develop a unified theory of flow measurement, was first suggested by Shercliff (1962) and significantly extended by Bevir (1970). Both applied it to electromagnetic flowmeters, where it has been highly successful. Hemp (1975) has also applied this theory to electromagnetic flowmeters, but he has developed the theory for other types of flowmeters: ultrasonic (1982), thermal mass (1994a) and Coriolis (1994b, and Hemp and Hendry 1995). In Chapter 12 on electromagnetic flowmeters, an appendix describes the essential mathematics. The weight function developed in this theory provides a measure of the importance of flow in each part of the meter with respect to the overall meter signal. The flow at each point of a cross-section is weighted with this function. Ideally, the weighting should result in a true summation of the flow in the meter to obtain a volume flow rate. It has been possible to approach this ideal for the electromagnetic flowmeter. For the other types of meters, the reader will be given only a brief explanation and will be referred to relevant papers.

A second mathematical physics theory, first (to my knowledge) applied by Hemp (1988) to flow measurement, is reciprocity. This, essentially, states that if you apply a voltage to one end of an electrical network and measure the current at the other end, you find that by reversing the ends and hence the direction you obtain the same relationship. Hemp has proposed this as a means of eliminating some errors in flowmeters to which the theory is applicable.

Table 1.2. *Velocity in pipes for various flow rates to two significant figures*

	m^3/h[a]	l/min	gal/min	gal/min	ft^3/min	Mean Velocity (m/s) in a Circular Pipe of Diameter							
						10 mm	25 mm	50 mm	100 mm	200 mm	500 mm	1000 mm	2000 mm
Very low	10^{-3}	0.017	3.7×10^{-3}	4.4×10^{-3}	5.9×10^{-4}	3.5×10^{-3}	5.7×10^{-4}	1.4×10^{-4}	3.5×10^{-5}				
	10^{-2}	0.17	3.7×10^{-2}	4.4×10^{-2}	5.9×10^{-3}	3.5×10^{-2}	5.7×10^{-3}	1.4×10^{-3}	3.5×10^{-4}	8.8×10^{-5}	1.4×10^{-5}		
	0.1	1.7	0.37	0.44	5.9×10^{-2}	0.35	5.7×10^{-2}	1.4×10^{-2}	3.5×10^{-3}	8.8×10^{-4}	1.4×10^{-4}	3.5×10^{-5}	
	1	17	3.7	4.4	0.59	3.5	0.57	0.14	3.5×10^{-2}	8.8×10^{-3}	1.4×10^{-3}	3.5×10^{-4}	8.8×10^{-5}
	10	170	37	44	5.9	35	5.7	1.4	0.35	8.8×10^{-2}	1.4×10^{-2}	3.5×10^{-3}	8.8×10^{-4}
	100	1,700	370	440	59	350	57	14	3.5	0.88	0.14	3.5×10^{-2}	8.8×10^{-3}
	1,000	1.7×10^{4}	3700	4,400	590	3,500	570	140	35	8.8	1.4	0.35	8.8×10^{-2}
	10^{4}	1.7×10^{5}	3.7×10^{4}	4.4×10^{4}	5,900	3.5×10^{4}			350	88	14	3.5	0.88
	10^{5}	1.7×10^{6}	3.7×10^{5}	4.4×10^{5}	5.9×10^{4}	3.5×10^{5}				880	140	35	8.8
Very high	10^{6}	1.7×10^{7}	3.7×10^{6}	4.4×10^{6}	5.9×10^{5}	3.5×10^{6}						350	88

Reproduced from Baker (1988a/1989) with permission of Professional Engineering Publishing.

[a] Since water has a density of 1,000 kg/m^3 (approximately), the mass flow rate in kilograms per hour of water may be obtained by multiplying this column by 1,000.

In preparing this second edition, I have added new material to the appendices. This is partly because much of the new material goes further into the mathematics or into experimental research, but also to accommodate many new references without disturbing the flow of the main text.

Appendix 1.A Statistics of Flow Measurement

1.A.1 Introduction

The engineer's main needs are to

- understand and be able to give a value to the uncertainty of a particular measurement;
- know how to design a test to provide data of a known uncertainty;
- be able to combine measurements, each with its own uncertainty, into an overall value; and
- determine the uncertainty of an instrument at the end of a traceable ladder of measurement.

The international and national documents set the recommended approach for flow metering. Most standard statistics books will provide the essentials (Rice 1988; cf. Campion, Burns and Williams 1973, which is often quoted but may not be easy to obtain), but good school texts may be more accessible (Crawshaw and Chambers 1984, Eccles, Green and Porkess 1993a, 1993b). Hayward (1977c) is an extremely well-written and elegant little book, which deserves to be updated and reprinted; Kinghorn (1982) provides a well-written and useful brief review of the main points; and Mattingly (1982) addresses some of the problems concerned with transfer standards. I would also draw the reader's attention to an excellent book on experimentation and uncertainty analysis by Coleman and Steele (1999).

1.A.2 The Normal Distribution

The normal distribution, Figure 1.A.1(a), is also known as the Gaussian distribution after Carl Friedrich Gauss, who proposed it as a model for measurement errors (Rice 1988). The notation used for the normal curve is $N(\mu, \sigma^2)$, which is the distribution under the curve

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[(x-\mu)/\sigma\right]^2} \tag{1.A.1}$$

where μ is the mean value of the data, and σ^2 is the variance. Alternatively, σ is the standard deviation for the whole population. We can simplify the curve (normalise it) by putting $z = (x - \mu)/\sigma$ and obtaining [Figure 1.A.1(b)]

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \tag{1.A.2}$$

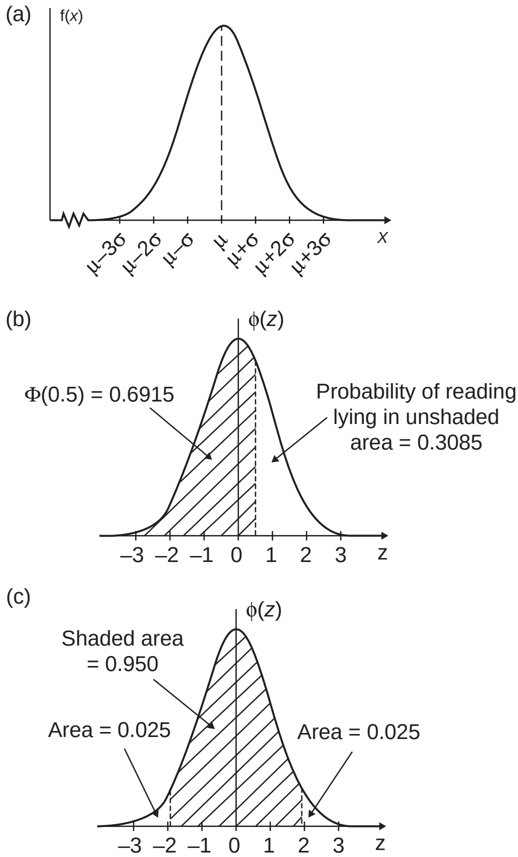

Figure 1.A.1. The normal distribution.

With the form of Equation (1.A.2), the curve does not vary with the size of the parameters μ and σ.

What the curve tells us (in relation to instrument measurements) is that the statistical chance of an instrument reading giving a value near to the mean μ is high, but the farther away the reading is from the mean, the less the chance is of its occurring (indicated by the curve decreasing in height the further one moves from the mean), and as values of the reading get farther still from μ, so the chance gets less and less.

The area under the curve of Equation (1.A.2) [Figure 1.A.1(b)], which reaches to infinity each way, is unity, and this is the probability of the reading lying within this curve (obviously). The area under the curve between $z = -\infty$ and some other value of z is given by

$$\Phi(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{1}{2}t^2}\,dt \tag{1.A.3}$$

and is the probability that a reading will lie within that range and is obtained numerically and given in Table 1.A.1 in normalised form. For instance, if $z = 0.5$,

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

Table 1.A.l. *A selection of values from the normal distribution function* $\Phi(z)$

z	$\Phi(z)$	Symmetrical central area under curve
0	0.5000	
0.5	0.6915	
1.28	0.8997	
1.282	0.9000	0.80
1.29	0.9015	
1.64	0.9495	
1.645	0.9500	0.90
1.65	0.9505	
1.96	0.9750	0.95
2.57	0.99492	
2.576	0.99500	0.99
2.58	0.99506	
3.29	0.99950	0.999
3.30	0.99952	

After D. V. Lindley and W. F. Scott, *New Cambridge Statistical Tables*, 2nd ed., Cambridge: Cambridge University Press. Table 4, pp. 34, 35.

$\Phi(z) = 0.6915$, where $\Phi(z)$ is the area under the curve from $z = -\infty$ to $z = 0.5$ in this case. So the chance of a reading lying beyond this point is 0.3085, or about 30%.

We shall be interested in the chance that a flowmeter reading will fall between certain limits each side of $z = 0$, the mean value. A chance of 95% is often used and is called a 95% confidence level. This means that 19 times out of 20 the reading will fall between the limits. This requires that the central area of the curve [Figure 1.A.1(c)] has a value of 0.95, or 0.475 each side of the mean. To obtain z from this value, we need to add $0.475 + 0.5 = 0.975$, and this gives a value (Table 1.A.l) of $z = 1.96$. If we put this in terms of x, we obtain

$$x - \mu = 1.96\,\sigma \tag{1.A.4}$$

or the band around the mean value of the reading within which 95% of the readings statistically should fall, is approximately $\pm 2\sigma$, or two standard deviations from the mean.

If we are interested, not in the spread of individual readings, but in the spread of the mean of small sets of readings, a statistical theorem called the Central Limit Theorem provides the answer. If a sample of n readings has a mean value of M, then the distribution of means like M is given by $N(\mu, \sigma^2/n)$. This is intuitively reasonable because one would expect that the scatter of means of groups of n readings would have a smaller variance, σ^2/n, than the readings themselves, as well as a smaller standard deviation, $\sigma/\sqrt{n}$. In this discussion, we have skated over the need to know the value of the standard deviation of the whole normal population. If we do not know σ, then we can approximate it with the value of the standard deviation s of the small set of n. So if $n \geq 30$, it is usually sufficiently precise to take $\sigma = s$. If $n < 30$, the standard deviation should be taken as $\sigma = s\sqrt{n}/\sqrt{n-1}$.

1.A.3 The Student *t* Distribution

We now need to look at one more subtlety of these estimates. The normal distribution assumed that we had obtained many readings and could with confidence know that they formed a normal distribution. We can agree that if the error is random, then it is a fair assumption that many readings would form a normal distribution. However, often we have only a few readings, and these may not be uniformly distributed within the curve of Figure 1.A.1. Too many may lie outside the 1.96σ limit. For this reason, we use the Student *t* distribution, which allows for small samples on the assumption that the distribution, as a whole, is normal. Figure 1.A.2 shows the effect of the small number of readings. Because, with a small number of readings, one has to be subtracted from all the others to obtain a mean, the number of independent values is one less than the number of readings, and so the statisticians say that there is one less degree of freedom than the number of readings. In Figure 1.A.2, ν is the symbol for the degree of freedom, and $\nu = n - 1$, where n is the number of readings. For $\nu \to \infty$, the *t* distribution tends to a normal curve with mean zero and variance unity.

Figure 1.A.2 shows clearly the larger area spreading beyond the normal curve in which the readings may lie and the reason for the greater uncertainty. The curves are used in a similar way to the normal curve, but, as an alternative, Table 1.A.2 provides the information we need. If we have 10 readings, say, and so select the value of $\nu = 9$ for the degree of freedom, and if we wish to find the limits for a confidence level of 95%, we shall need to use the 5% column. We obtain $t = 2.262$, which we can apply to obtain the limits for a 95% confidence of $\pm 2.262\,\sigma/\sqrt{n}$ on the mean values of groups of readings, where n is the number of readings in the group. We should note, however, that the 95% confidence level from Table 1.A.2 gives a *t* value that varies little from 2.0 if $\nu \geq 20$. The limits for 95% confidence will then be $\pm 2.0\,\sigma/\sqrt{n}$ on the mean values of groups of readings.

I have always been puzzled by the name Student, but Eccles et al. (1993b) explained that the originator of this technique was William S. Gosset, born in 1876, who used the pseudonym *Student.*

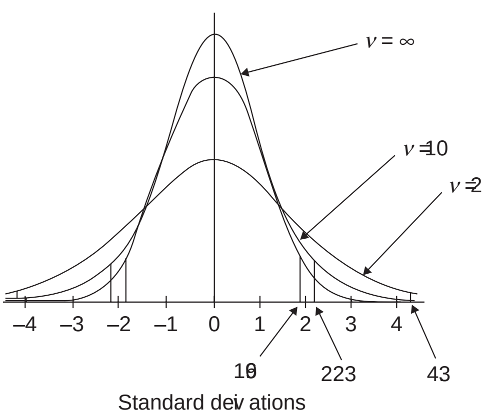

Figure 1.A.2. Student's *t* distribution curves compared with the normal curve. Note p = 5% as related to Table 1.A.2 for both tails.

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

Table 1.A.2. *A selection of values from the Student* t *function*

n	*v*	p(%) (Total)				
		20	10	5	1	0.1
		p/2 (%) (per tail)				
		10	5	2.5	0.5	0.05
2	1	3.078	6.314	12.71	63.66	636.6
3	2	1.886	2.920	4.303	9.925	31.60
5	4	1.533	2.132	2.776	4.604	8.610
10	9	1.383	1.833	2.262	3.250	4.781
20	19	1.328	1.729	2.093	2.861	3.883
30	29	1.311	1.699	2.045	2.756	3.659
61	60	1.296	1.671	2.000	2.660	3.460
121	120	1.289	1.658	1.980	2.617	3.373
∞	∞	1.282	1.645	1.960	2.576	3.291

After D. V. Lindley and W. F. Scott, *New Cambridge Statistical Tables*, 2nd ed., Cambridge: Cambridge University Press, Table 10, p. 45.

1.A.4 Practical Application of Confidence Level

The method described in Section 1.4 leads to the following steps (cf. Guide ISO/IEC 98-3: 2008, Hayward 1977b, 1977c):

i. Write down systematic uncertainties and derive the standard uncertainty for each component.
ii. Write down random uncertainties and derive the standard uncertainty for each component.
iii. Calculate the combined standard uncertainty for uncorrelated input quantities (and refer to UKAS 2012 if correlated).
iv. Obtain the expanded uncertainty using $k = 2$ for 95% confidence.

Taking a simple example, where we need to revert to t, suppose that we obtain a series of volumetric flow readings from a 50 mm ID flowmeter with the flow set at $10\text{m}^3/\text{h}$:

10.06, 10.01, 9.95, 9.99, 9.85, 10.02, 10.03, 10.12, 9.90, 9.98.

The results are plotted in Figure 1.A.3. The mean of these readings is 9.991, and the standard deviation is 0.07752. We could thus conclude that the true reading of this meter fell in the bracket $9.991 \pm 2.262 \times 0.07752/\sqrt{9} = 9.991 \pm 0.05845$, or between 9.93 and 10.05 with a 95% confidence. This fairly brackets the value of $10\ \text{m}^3/\text{h}$.

The actual readings should have all fallen within 9.991 ± 0.07752, or 9.91 and 10.07. In fact, three fell outside this bracket – rather higher than the 1 in 19 implied by the 95% confidence level. We might wish to look more closely at the procedure

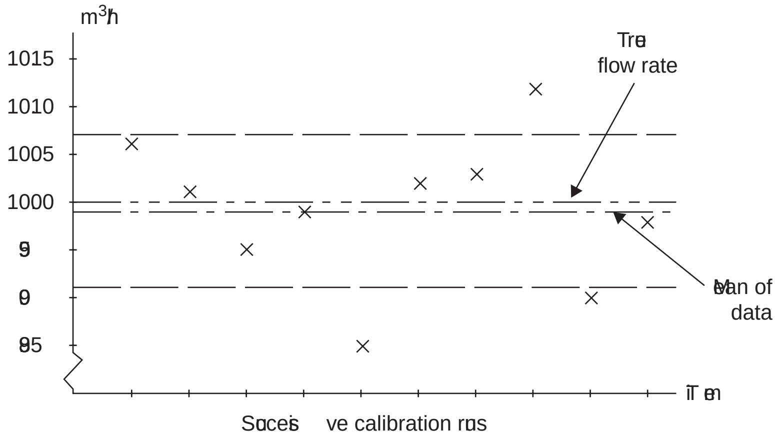

Figure 1.A.3. A set of flowmeter test readings (after Kinghorn 1982) for a fixed flow rate.

for obtaining these results since this suggests a possible problem with the means for obtaining the data.

1.A.5 Types of Error

There are essentially four types of error (Kinghorn 1982).

- *Spurious errors* result from obvious failures, obvious in the sense that they can be identified and documented. Readings with these should be eliminated.
- *Random errors* cause a variation in the output reading even when the input parameter has not changed.
- *Constant systematic error*, which is also called bias, may vary over the range but is constant in time, and could, in principle, be corrected out of the reading.
- *Variable systematic error* (bias) slowly varies with time, usually in a consistent direction, and may be caused by wear in bearings of a rotating meter, fatigue in components of a vibrating meter, erosion of geometry, etc.

Figure 1.A.4 illustrates these errors. Clearly one of the readings is so far out that there must be some explanation other than randomness. It is comforting to know that some of the most eminent experimentalists of the past have had cause to discard readings in critical experiments!

The scatter around the mean line will provide the basis of the calculation which we did in Section 1.A.4. The constant systematic error (bias) can be seen and could be built into a flow computer. The change in the mean value with time shows the changing systematic error, which is, in part, the reason for regular recalibration of meters.

The repeatability is related to the closeness of readings. If we expect a reading to lie within a band given by $\pm 2s$, the worst case difference between successive readings that fall within this band would be $4s$ $[(2 \times 2)s]$, but a less extreme working value is

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

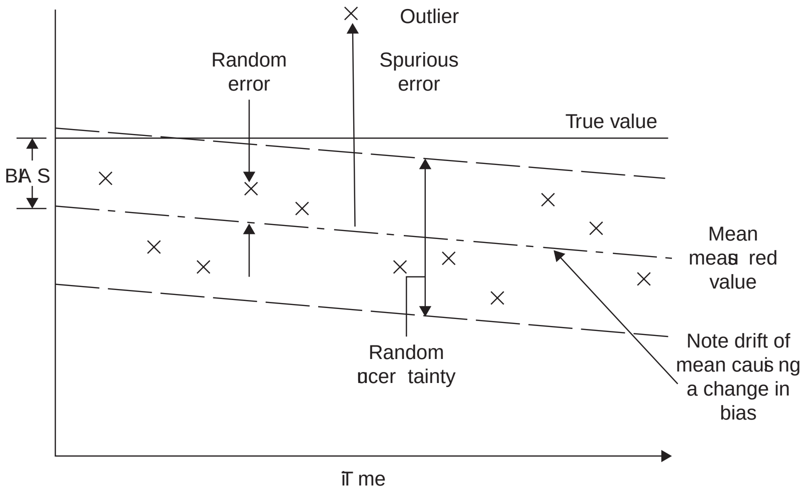

Figure 1.A.4. Diagram to show the various types of error (after Kinghorn 1982).

obtained from the root of the sum of the squares (or the quadrature; cf. Pythagoras and the length of the hypotenuse):

$$\sqrt{(2s)^2 + (2s)^2} \text{ or } 2\sqrt{2}s$$

1.A.6 Combination of Uncertainties

If we combine uncertainties due to the nature of a flowmeter's operating equation, then we take the following approach. Suppose that the flowmeter has the equation

$$q_v = x_1 x_2^n x_3 x_4^m \tag{1.A.5}$$

To obtain the uncertainty in q_v, we need the partial derivative of q_v with respect to x_1, x_2 etc. The required result can be achieved, either by differentiating the equation as it stands or by first taking logarithms of both sides. We shall skip this and go straight to the result:

$$\frac{u_c(q_v)}{q_v} = \pm\frac{u(x_1)}{x_1} \pm \frac{nu(x_2)}{x_2} \pm \frac{u(x_3)}{x_3} \pm \frac{mu(x_4)}{x_4} \tag{1.A.6}$$

The problem with this equation is that the arithmetic sum of the uncertainties is usually overly pessimistic. It is, therefore, recommended that they be combined in quadrature, or by the root-sum-square (rss) method. This leads to the following equation:

$$\frac{u_c(q_v)}{q_v} = \sqrt{\left(\frac{u(x_1)}{x_1}\right)^2 + \left(\frac{nu(x_2)}{x_2}\right)^2 + \left(\frac{u(x_3)}{x_3}\right)^2 + \left(\frac{mu(x_4)}{x_4}\right)^2} \tag{1.A.7}$$

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

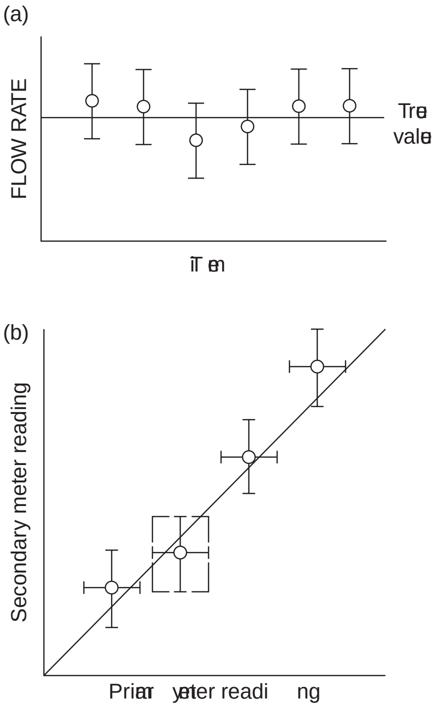

Figure 1.A.5. **(a)** Uncertainty limit bars on readings. **(b)** Uncertainty limit bars on both flow rate and readings.

There are complications beyond this equation. x_2 may appear in the equation as $x_5 + x_6$. In this case, $x_5 + x_6$ will need to be dealt with first and will require careful consideration as to whether the actual errors in these quantities are combining, cancelling or random.

1.A.7 Uncertainty Range Bars, Transfer Standards and Youden Analysis

It is sometimes useful to indicate the range of uncertainty estimated from the experimental method in each reading. This can be done by using bars that give uncertainty limits on each experimental point. This will then indicate, for a particular flow rate, the likely uncertainty in the reading. This is shown in Figure 1.A.5(a). In some cases where flow rate varies, there may be an uncertainty in both primary flow rate measurement and reading of second meters. In this case, uncertainty bars are needed in both directions, and the rectangle [Figure 1.A.5(b)] will define the limits of the possible uncertainty. In other words, the maximum uncertainty will have been obtained by quadrature.

Mattingly (1982) describes the procedure for checking the validity of different flow measurement laboratories using a Measurement Assurance Program (MAP) [or Proficiency Testing, as referred to by Mattingly in a draft report on the approach of the National Institute of Standards and Technology (NIST)] where a good-quality flowmeter acting as a transfer standard is exchanged between laboratories. Figure 1.A. 6(a) shows the results of such a cycle of checks; and the bars on

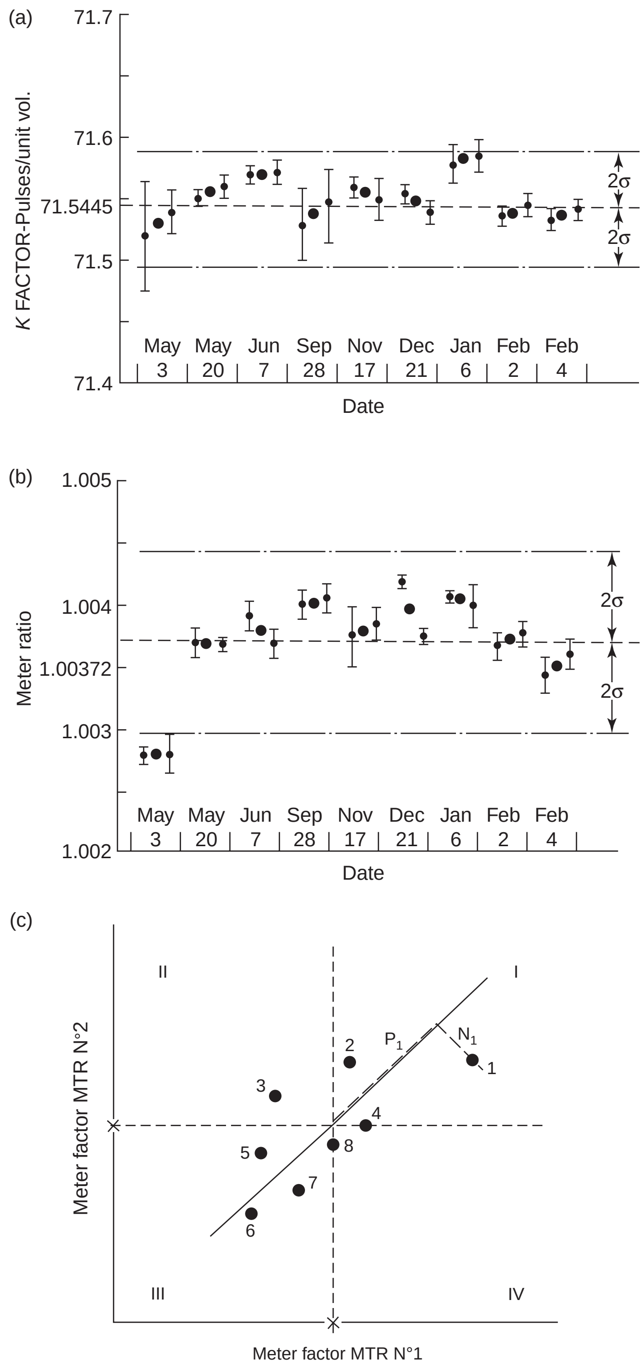

Figure 1.A.6. Turbine meter as a transfer standard (from Mattingly 1982; reproduced with the author's permission). **(a)** Typical turbine meter control chart for meter factor; **(b)** typical turbine meter control chart for ratio; **(c)** graphical representation of the Youden plot.

https://doi.org/10.1017/CBO9781107054141.002 Published online by Cambridge University Press

the experimental points presumably indicate the uncertainty of the turbine meter at a particular laboratory. Various people have suggested the use of two flowmeters usually in series, but as a possible alternative in parallel, to enhance the accuracy of a transfer standard. In this case, the ratio of the signals from the two meters will give an indication as to whether there has been a shift in either, and, if there has not been, the reading of the meters will provide, with greater confidence, the accuracy of the facility. In Figure 1.A.6(b), Mattingly (1982) gave a typical control chart for the ratio of a pair of National Bureau of Standards turbine meters. If the ratio falls outside an agreed tolerance band, the cause of the error will need to be found before confidence in the meter readings is restored.

For laboratory comparison, the transfer package with two meters should be run at one or two agreed flow rates. The position of the two flowmeters may be interchanged to obtain a second set of data. Using the Youden analysis in Figure 1.A.6(c), each laboratory is represented by a single point (1, 2, 3, etc.) resulting from plotting the meter factor of the two meters on the two axes. It is apparent that the position of the points relates to the type of error. Essentially, if the points lie in quadrants I or III, then the meters are reading the same, and the error can be attributed to the flow rig. If the readings are in quadrants II or IV, the flowmeters are not agreeing, and the error may be due to a malfunction in one of them. The reader is referred to Mattingly's (1982) article for further details of this procedure (cf. Youden 1959 and see Wu and Meng 1996 on the statistics of Youden circles).