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Some varieties without the amalgam embedding property

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A variety of groups $\underline{\underline{V}}$ has the amalgam embedding property if every amalgam of two $\underline{\underline{V}}$-groups can be embedded in a $\underline{\underline{V}}$-group. In this note the author proves that if $\underline{\underline{V}}$ is a variety of exponent 0 which satisfies a law $W(x_1{}^n, x_2, \ldots, x_t)$ but not $W(x_1, x_2, \ldots, x_t)$ then $\underline{\underline{V}}$ does not have the amalgam embedding property.

A variety of groups $\underline{\underline{V}}$ has the amalgam embedding property (AEP) if every amalgam of two $\underline{\underline{V}}$-groups can be embedded in a $\underline{\underline{V}}$-group. No varieties other than the variety of all groups and abelian varieties are known to have AEP. (See [1] pp. 42-43 for references and comments on this problem).

In this note we prove the following:

THEOREM. *If* $\underline{\underline{V}}$ *is a variety of exponent* 0 *which satisfies a law* $W(x_1{}^n, x_2, \ldots, x_t)$ *but not* $W(x_1, x_2, \ldots, x_t)$ *then* $\underline{\underline{V}}$ *does not have* AEP.

Proof. Let F be the free $\underline{\underline{V}}$-group on t generators $y_1, y_2, \ldots, y_t$; then $gp(y_1)$ is infinite cyclic and $W(y_1, y_2, \ldots, y_t) \neq 1$, for every relator on the free generators of F is a law in $\underline{\underline{V}}$.

Let Z be the infinite cycle generated by z . Certainly Z is a $\underline{\underline{V}}$-group and so $A = (F, Z ; y_1 = z^n)$ is an amalgam of $\underline{\underline{V}}$-groups. If $\underline{\underline{V}}$

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had AEP then A would be embeddable in a $\underline{\underline{V}}$-group B, and $W(y_1, y_2, \ldots, y_t) = W(z^n, y_2, \ldots, y_t) = 1$ in B, which is a contradiction.

Reference

[1] Hanna Neumann, *Varieties of groups*, (Ergebnisse der Mathematik, und ihrer Grenzgebiete, Band 37, Springer-Verlag, Berlin, Heidelberg, New York, 1967).

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