characteristics, parametrized by reals, for each one of the following six types: uniformity and covering numbers of Yorioka ideals as well as both kinds of localization and anti-localization cardinals, respectively. This answers several open questions from Klausner and Mejía (*Arch. Math. Logic* 61 (2022), pp. 653–683).

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FRANCESCO PAOLO GALLINARO, ***Around Exponential-Algebraic Closedness***, University of Leeds, UK, 2022. Supervised by Vincenzo Mantova. MSC: Primary 03C65. Secondary 11F03, 11L99, 14K12. Keywords: abelian varieties, algebraic groups, Exponential-Algebraic Closedness, exponential function, modular *j*-function, quasiminimality.

Abstract

We present some results related to Zilber's Exponential-Algebraic Closedness Conjecture, showing that various systems of equations involving algebraic operations and certain analytic functions admit solutions in the complex numbers. These results are inspired by Zilber's theorems on raising to powers.

We show that algebraic varieties which split as a product of a linear subspace of an additive group and an algebraic subvariety of a multiplicative group intersect the graph of the exponential function, provided that they satisfy Zilber's freeness and rotundity conditions, using techniques from tropical geometry.

We then move on to prove a similar theorem, establishing that varieties which split as a product of a linear subspace and a subvariety of an abelian variety A intersect the graph of the exponential map of A (again under the analogues of the freeness and rotundity conditions). The proof uses homology and cohomology of manifolds.

Finally, we show that the graph of the modular j-function intersects varieties which satisfy freeness and broadness and split as a product of a Möbius subvariety of a power of the upper-half plane and a complex algebraic variety, using Ratner's orbit closure theorem to study the images under j of Möbius varieties.

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RAHMAN MOHAMMADPOUR, ***New methods in forcing iteration and applications***, Institut de Mathématiques de Jussieu-Paris Rive Gauche, Université de Paris, Paris, France, 2020. Supervised by Boban Veličković. MSC: 03E05, 03E35, 03E40, 03E55, 03E57. *Key words and phrases*. guessing model, approachability ideal, PFA, higher forcing axioms, Magidor models, side conditions.

Abstract

The Theme. Strong forcing axioms like Martin's Maximum give a reasonably satisfactory structural analysis of $H(\omega_2)$. A broad program in modern Set Theory is searching for strong forcing axioms beyond ω_1. In other words, one would like to figure out the structural properties of taller initial segments of the universe. However, the classical techniques of

forcing iterations seem unable to bypass the obstacles, as the resulting forcings axioms beyond ω_1 have not thus far been strong enough! However, with his celebrated work on generalised side conditions, I. Neeman introduced us to a novel paradigm to iterate forcings. In particular, he could, among other things, reprove the consistency of the Proper Forcing Axiom using an iterated forcing with finite supports. In 2015, using his technology of virtual models, Veličković built up an iteration of semi-proper forcings with finite supports, hence reproving the consistency of Martin's Maximum, an achievement leading to the notion of a virtual model.

In this thesis, we are interested in constructing forcing notions with finitely many virtual models as side conditions to preserve three uncountable cardinals. The thesis constitutes six chapters and three appendices that amount to 118 pages, where Section 1 is devoted to preliminaries, and Section 2 is a warm-up about the scaffolding poset of a proper forcing. In Section 3, we present the general theory of virtual models in the context of forcing with sets of models of two types, where we, e.g., define the "meet" between two virtual models and prove its properties.

The main results are joint with Boban Veličković, and partly appeared in *Guessing models and the approachability ideal, J. Math. Log. 21 (2021).*

Pure Side Conditions. In Section 4, we use two types of virtual models (countable and large non-transitive ones induced by a supercompact cardinal, which we call Magidor models) to construct our forcing with pure side conditions. The forcing covertly uses a third type of models that are transitive. We also add decorations to the conditions to add many clubs in the generic ω_2. In contrast to Neeman's method, we do not have a single chain, but α-chains, for an ordinal α with $V_\alpha \prec V_\lambda$. Thus, starting from suitable large cardinals $\kappa < \lambda$, we construct a forcing notion whose conditions are finite sets of virtual models described earlier. The forcing is strongly proper, preserves κ, and has the λ-Knaster property. The relevant quotients of the forcing are strongly proper, which helps us prove strong guessing model principles. The construction is generalisable to a $<\mu$-closed forcing, for any given cardinal μ with $\mu^{<\mu} = \mu < \kappa$.

The Iteration Theorem. In Section 5, we use the forcing with pure side conditions to iterate a subclass of proper and $\aleph_2$-c.c. forcings and obtain a forcing axiom at the level of $\aleph_2$. The iterable class is closely related to Asperó–Mota's forcing axiom for finitely proper forcings.

Guessing Model Principles. Section 6 encompasses the main parts of the thesis. We prove the consistency of the guessing principle $\mathrm{GMP}^+(\omega_3, \omega_1)$ that states for any cardinal $\theta \geq \omega_3$, the set of $\aleph_2$-sized elementary submodels M of $H(\theta)$, which are the union of an ω_1-continuous $\in$-chain of ω_1-guessing, I.C. models is stationary in $\mathcal{P}_{\omega_3}(H(\theta))$. The consistency and consequences of this principle are demonstrated in the following diagram. We also prove that one can obtain the above guessing models in a way that the ω_1-sized ω_1-guessing models remain ω_1-guessing model in any outer transitive model with the same ω_1, and we denote this principle by $\mathrm{SGMP}^+(\omega_3, \omega_1)$.

In the following diagram, TP stands for the tree property; wKH stands for the weak Kurepa Hypothesis; MP stands for Mitchell property, i.e., the approachability ideal is trivial modulo the nonstationary ideal; AP stands for the approachability property; $\mathrm{AMTP}(\kappa^+)$ states that if $2^\kappa < \aleph_{\kappa^+}$, then every forcing which adds a new subset of κ^+ whose initial segments are in the ground model, collapses some cardinal $\leq 2^\kappa$. The dotted arrow denotes the relative consistency, while others are logical implications.

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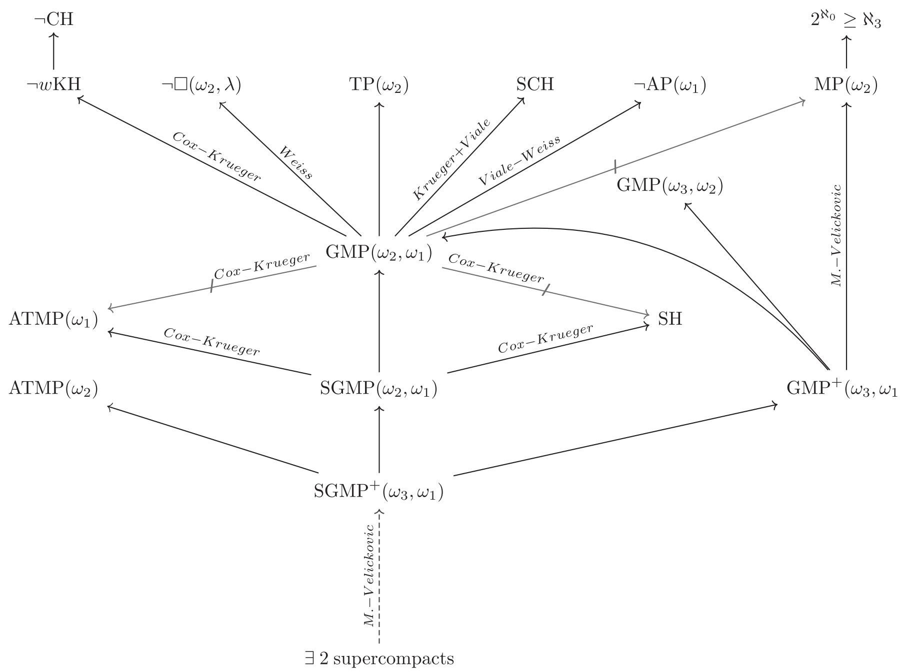

Appendices. Appendix A includes merely the above diagram. Appendix B presents a proof of the Mapping Reflection Principle with finite conditions under PFA. Appendix C contains open problems. Finally, the thesis's bibliography consists of 42 items.

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