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Knots and links in low dimensions

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This thesis considers several problems in the theory of knots and links in low dimensions (locally flat embeddings of one or several copies of S^n into S^{n+2} for $n = 1$ or 2).

The first chapter summarizes the standard definitions and results to be used later. The next three chapters deal with classical links ($n = 1$) . In Chapter II the algebraic and geometric significance of the vanishing of certain of the Alexander ideals is considered. A new criterion for a ribbon link to be an homology boundary link is given, and a counter-example to Smythe's conjecture is constructed. A theorem of Murasugi relating the vanishing of $\Delta_1(L)$ for a 2-component link to the metabelian nilpotent completion of the link group and to the image of the longitudes in this completion is given a new, more conceptual proof, which applies to links with any number of components. As a corollary, the rank of the Alexander module $\alpha(L)$ is shown to be an invariant of concordance. The concept of $\mathbb{Z}/2\mathbb{Z}$-homology boundary link is introduced to show this invariant and the Murasugi nullity are essentially independent [*1*], [*3*], [*7*], [*8*], [*9*].

In Chapter III it is shown that the longitudes of a μ-component homology boundary link are in the second commutator subgroup of the link group if and only if the μth Alexander ideal is principal, thus generalizing a theorem announced without proof, for $\mu = 2$, by Crowell and Brown. In the proof of this result the technique of constructing covering spaces by splitting along Seifert surfaces is extended to homology boundary links, and this technique is discussed further [*4*].

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In Chapter IV Blanchfield duality is used to construct a primitive hermitian pairing on the torsion submodule of the Alexander module of a link, after a mild localization to kill the boundary components. This pairing is computable from Seifert surfaces for an homology boundary link, and the class of the pairing in a Witt group is an invariant of concordance for links concordant to boundary links. (This qualification is unnecessary; see [5].) (If $\mu = 1$ it is essentially the knot concordance invariant.) After a further localization, a Witt class also invariant under isotopy is obtained and it is shown that the set of such Witt classes for boundary links is an infinitely generated abelian group [5].

In the last chapter (Chapter V) three arguments are given to show that not every high knot group can be the group of a 2-knot (embedding of S^2 in a homotopy 4-sphere). Two of these arguments involve Milnor duality; the third uses orientability. The 2-knots with commutator subgroup finite or finitely generated nilpotent are considered in detail, and most of the groups not excluded by the three principal results are shown to be realizable by fibred knots in homotopy 4-spheres [2], [6].

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