A note on the focal relations of a bicircular quartic.

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Let P be any point on a bicircular quartic having A, B, C for foci; so that $l\,.\,\mathrm{PA}+m\,.\,\mathrm{PB}+n\,.\,\mathrm{PC}=0$, where l, m, n are known. It will be shown how the fourth focus E (lying upon the circum-circle of ABC) may be found; and also the relations subsisting between any three focal distances.

If x, y, z be masses placed at A, B, C respectively and P any point in the plane of ABC,

$$x\,.\,\mathrm{PA}^2+y\,.\,\mathrm{PB}^2+z\,.\,\mathrm{PC}^2=(x+y+z)\,.\,\mathrm{PE}^2+\text{constant} \quad \text{(A)}$$

where E (the centroid of the masses) has its trilinear coordinates proportional to $x/a:y/b:z/c$.

When E lies on the circum-circle of ABC $\Sigma a^2/x=0$, and the constant vanishes. The relation (A) is then the result of eliminating l, m, n from the equations

$$\left.\begin{aligned} -(l/x)\,.\,\mathrm{PB}+(m/y)\,.\,\mathrm{PA}+\quad n\,.\,\mathrm{PE}&=0\\ (l/x)\,.\,\mathrm{PC}+\quad m\,.\,\mathrm{PE}-(n/z)\,.\,\mathrm{PA}&=0\\ l\,.\,\mathrm{PE}-(m/y)\,.\,\mathrm{PC}+(n/z)\,.\,\mathrm{PB}&=0 \end{aligned}\right\} \quad \text{(B)}$$

if $x+y+z+xyz=0$. The equations (B) are inconsistent unless $\Sigma l^2/x=0$, but if this relation holds [as well as (A)] they are derivable from the single equation $\Sigma l.\mathrm{PA}=0$, found by eliminating PE from any two of them.

From the equations $\Sigma l^2/x=\Sigma a^2/x=0$ and $\Sigma x+\Pi x=0$ we get

$$x(c^2m^2-b^2n^2)=\ldots\ldots=\ldots\ldots=\sqrt{\{\Sigma b^2c^2l^2-2abc\,\Sigma am^2n^2\}}.$$

Hence the coordinates of E are inversely proportional to

$$a(c^2m^2-b^2n^2),\ldots\ldots$$

and the relations between the focal distances are those given above (B).

If $l^2:m^2:n^2=a^2:b^2:c^2$ the equations for determining E are insufficient; this is *a priori* evident, for E may be any point on the circum-circle.

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