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Author for correspondence:
Bradley Paul Lipovsky,
E-mail: bpl7@uw.edu

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Density matters: ice compressibility and glacier mass estimation

Bradley Paul Lipovsky

Department of Earth and Space Sciences, University of Washington, Seattle, WA, USA

Ice flow models typically assume that ice is incompressible, a reasonable assumption because ice density changes are indeed small and have a correspondingly small effect on the overall mass balance of glaciers and ice sheets. Given the immense volume of the ice sheets, however, even relatively small changes may influence global mean sea level to a degree that severely impacts humanity (Hauer and others, 2020). Here, we quantify the role of gravitational compression and thermal contraction in estimating ice sheet mass.

We describe gravitational compression by considering an ice column in vertical hydrostatic equilibrium, $\partial\sigma_z/\partial z = -\rho g$ with vertical Cauchy stress σ_z, gravitational acceleration $g = 9.81$ m s^{-2}, ice density ρ and vertical coordinate z. Although the effect of compressibility is most commonly described through the equation of mass conservation (e.g., Lipovsky and Dunham, 2015), here we take a different analytical approach following Cathles (2015) that examines the long-term behavior of a linear compressible Maxwell viscoelastic material following, $\hat{\sigma}_z = [\lambda(\omega) + 2\mu(\omega)]\hat{\epsilon}_z$. Here, hats denote Fourier transforms and λ and μ are complex valued, frequency ω dependent constitutive parameters that correspond to volumetric and non-volumetric deformations, respectively. At times much greater than the Maxwell time (Reeh and others, 2003, ~1 d for ice) that are relevant to long-term glacier evolution, the model describes ice as a compressible fluid with $\mu(\omega) \to 0$ and $\lambda(\omega) \to K = 8.9$ GPa the bulk modulus (Schulson and Duval, 2009). These limits have the interpretation that long-term non-volumetric stresses relax but volumetric stresses do not. This explains why compression persists over glacial evolution timescales.

The total surface displacement due to gravitational compression w may then be calculated by combining the constitutive relation with the condition of hydrostatic equilibrium. The result is,

$$w = \frac{\rho_0 g H^2}{2K} \tag{1}$$

with ice thickness H and reference density $\rho_0 = 917$ kg m^{-3}. From Eqn 1 we may confirm that strains are small, e.g. $\rho_0 gH/K \sim 10^{-3}$ for 1 km thick ice.

Thermal contraction also changes the density of glacier ice because the vast majority of the interior of the ice sheets is at a temperature well below the freezing point. If the temperature of this ice were raised to the melting point it would undergo thermal expansion equal to $\alpha\Delta T(z)$ with thermal expansion coefficient $\alpha = 5.3 \times 10^{-5}$ °C^{-1} and temperature difference from the pressure melting point ΔT (Schulson and Duval, 2009). In order to put a lower bound on thermal strains, we assume a linear temperature profile with depth that reaches 0°C at the bed and is equal to the mean annual surface temperature at $z = H$. Although simplistic, this approach certainly results in a lower bound because many temperature profiles are steeper than linear (i.e. colder) due to the effect of advection and because much of the bed is colder than the melting point temperature (Cuffey and Paterson, 2010, Ch. 9). Using published ice temperature (Ettema and others, 2020; Le Brocq and others, 2010) and geometry datasets (Morlighem and others, 2017, 2020), we calculate gravitational compression and thermal contraction in the ice sheets.

We calculate that gravitational compression deforms the surface of the Greenland and Antarctic ice sheets by up to 5.8 and 11.3 m, respectively. For thermal contraction, the corresponding values are 2.9 and 6.2 m, respectively. The corresponding ice sheet-averaged values are 0.5 and 0.7 m for thermal contraction and 0.8 and 0.7 m for gravitational compression. Integrated over the entire ice sheet, the total mass bias from the gravitational effect is equal to 3000 Gt in Greenland and 30 200 Gt in Antarctica. Although these long-term biases are only ~ 0.2% of the total ice-sheet mass, they are significantly larger than recent annual ice mass losses (~318 Gt a^{-1}, Smith and others, 2020). Perhaps more important than this long-term bias are potential shorter-term biases.

Estimates of total glacier mass from satellite altimetry commonly account for compression of the solid Earth but not compression of the ice column (Smith and others, 2020). To gauge the importance of seasonal ice compression, we calculate the ice column compression caused by surface mass-balance anomalies in Antarctica as described by Medley and others (2020). The resulting ice-sheet surface vertical velocities are as large as 1–2 mm a^{-1} near the coasts and are therefore of comparable or slightly smaller magnitude as solid Earth uplift rates (e.g. Whitehouse and others, 2012; Ivins and others, 2013). Future ice-sheet mass-balance studies using satellite altimetry should therefore take into account the effect of ice-sheet compressibility.

Along with improved firn modeling, seasonal compression could plausibly explain some of the greater variability of altimetric versus gravimetric ice-sheet mass estimates (Shepherd and others, 2018). We note that seasonal compression could be directly measured with optical fiber strain sensing (Hartog, 2017).

Data availability statement. The codes used in this work are available at https://doi.org/10.5281/zenodo.5167989.

References

Cathles LM (2015) *Viscosity of the Earth's Mantle*. Princeton University Press.

Cuffey KM and Paterson WSB (2010) *The physics of glaciers*. Academic Press.

Ettema J and 6 others (2009) Higher surface mass balance of the Greenland ice sheet revealed by high-resolution climate modeling. *Geophysical Research Letters* **36**(12), 038110.

Hartog AH (2017) *An introduction to distributed optical fibre sensors*. CRC Press, Boca Raton, FL.

Hauer ME and 7 others (2020) Sea-level rise and human migration. *Nature Reviews Earth & Environment* **1**(1), 28–39.

Ivins ER and 5 others (2013) Antarctic contribution to sea level rise observed by GRACE with improved GIA correction. *Journal of Geophysical Research: Solid Earth* **118**(6), 3126–3141.

Le Brocq AM, Payne AJ and Vieli A (2010) An improved Antarctic dataset for high resolution numerical ice sheet models (ALBMAP v1). *Earth System Science Data* **2**(2), 247–260.

Lipovsky BP and Dunham EM (2015) Vibrational modes of hydraulic fractures: inference of fracture geometry from resonant frequencies and attenuation. *Journal of Geophysical Research: Solid Earth* **120**(2), 1080–1107.

Medley B, Neumann TA, Zwally HJ and Smith BE (2020) Forty-year simulations of firn processes over the Greenland and Antarctic ice sheets. *The Cryosphere* 1–35.

Morlighem M and 9 others (2017) BedMachine v3: complete bed topography and ocean bathymetry mapping of Greenland from multibeam echo sounding combined with mass conservation. *Geophysical Research Letters* **44**(21), 11–051.

Morlighem M and 9 others (2020) Deep glacial troughs and stabilizing ridges unveiled beneath the margins of the Antarctic ice sheet. *Nature Geoscience* **13**(2), 132–137.

Reeh N, Christensen EL, Mayer C and Olesen OB (2003) Tidal bending of glaciers: a linear viscoelastic approach. *Annals of Glaciology* **37**, 83–89.

Schulson EM and Duval P (2009) *Creep and fracture of ice*. Cambridge University Press.

Shepherd A and 9 others (2018) Mass balance of the Antarctic Ice Sheet from 1992 to 2017. *Nature* **558**, 219–222.

Smith B and 9 others (2020) Pervasive ice sheet mass loss reflects competing ocean and atmosphere processes. *Science* **368**(6496), 1239–1242.

Whitehouse PL, Bentley MJ, Milne GA, King MA and Thomas ID (2012) A new glacial isostatic adjustment model for antarctica: calibrated and tested using observations of relative sea-level change and present-day uplift rates. *Geophysical Journal International* **190**(3), 1464–1482.