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One purpose for studying the gas flow in barred spiral galaxies is to use the observed distribution and kinematics of the gas as a tracer of the underlying gravitational field. By comparing model hydrodynamical calculations with observations of actual systems, one would like to define three basic properties of barred galaxies:

- 1) The bar strength. How significant is the deviation from axial symmetry in the region of the bar, measured by some parameter such as q_t , maximum aximuthal force in terms of the mean radial force (Sanders and Tubbs, 1980).
- 2) The mean radial distribution of matter. Clearly in a system with large deviations from circular motion, the "rotation curve" gives no direct information on the radial mass distribution.
- 3) The angular velocity of the bar. Where is the co-rotation radius (or Lagrange points) with respect to the bar axes? Are other principal resonances present?

Deriving these properties through the use of numerical hydro-dynamical calulations is not unambiguous because the detailed results do depend upon the kind of numcerical technique used - specifically upon the magnitude of the unphysical numerical viscosity (G.D. van Albada, this volume). We can, however, place some definite constraints upon these properties by looking both at numerical hydrodynamics and the character of periodic orbits in non-axisymmetric potentials.

It is obvious that the hydrodynamical equation of motion written in Lagrangian form without the pressure term is the equation of motion of a particle. This means that in the absence of pressure forces (thermal, turbulent, viscous or magnetic) a gas streamline is an orbit, and steady state flow in some frame would correspond to simple non-looping periodic orbits. But, in fact, it is possible to make a much stronger statement. Suppose that we consider an ensemble of particles moving on a variety of trajectories essentially filling the volume of phase space allowed by the energy or Jacobi constant. And now suppose that we allow particles to be "sticky" in the sense that over some characteristic interaction distance, random velocities are reduced; that is, we

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introduce a 'viscous' force which resists distortion of a fluid element. Then such viscous dissipation forces particle motion toward the simple periodic orbits, or, in the language of modern dynamics, the periodic orbits appear as attractors in the phase space of the Hamiltonian. An attractor, \overline{A} , is a region of the phase space surrounded by a neighbourhood \underline{U} such that any particle within \underline{U} approaches \underline{A} as $t \to \infty$ (Treve, 1978). Attractors can only arise in dissipative systems; in a conservative system a particle trajectory which is not periodic obviously cannot become periodic. In a galactic potential (axisymmetric or non-axisymmetric) there is one trivial attractor for a dissipative medium - the center of the galaxy. But I suggest here that for all practical purposes (t \to 1/H) periodic orbits also arise as attractors in the 4-dimensional phase space of the problem.

We do not provide here a general proof of this statement (see Melnikov, 1963 and the discussion by Lake and Norman, 1982), but do present two striking numerical examples.

The first involves an ensemble of particles distributed uniformly through the phase space of the Henon-Heiles potential (Henon and Heiles, 1964). If dissipation is added by an algorithm which reduces the velocity dispersion over some interaction distance, then it is found that within several orbit periods essentially all particle trajectories penetrate a surface of section within one of the small 'islands' about a periodic orbit. The periodic orbit "attracts" particle trajectories from a wide domain of the phase space (Sanders, 1982). The second example involves gas flow in the potential of a weak bar. Here there is only one simple family of periodic orbits present inside co-rotation, the parallel family, or \underline{X}_1 in the notation of Contopoulos and Papayannapoulos (1980). These orbits are shown in Fig. la. The gas flow in this potential calculated by a time-dependent numerical hydrodynamical code (G.D. van Albada, this volume) is shown in Fig. 1b. It is seen that gas streamlines inside co-rotation are practically identical to family X_1 . Moreover, because family X_1 is simple (no self-crossing or looping) there are no shocks or gas inflow toward the center.

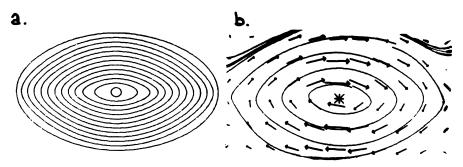
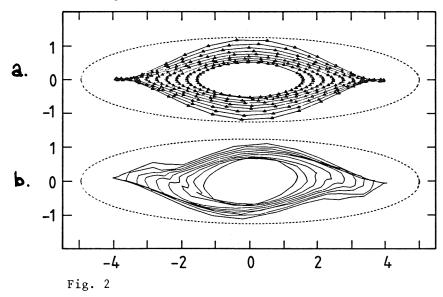


Fig. 1

But what if the lowest order periodic orbits are not so simple? What if all families of periodic orbits are self-crossing or looping. For example, if the bar becomes sufficiently strong, higher energy orbits of family \mathbf{X}_1 develop loops well inside co-rotation as is illustrated in Fig. 2a. Clearly in this case the fully developed gas flow cannot be along these orbits since streamlines cannot cross. The flow may be attracted to these orbits but something else must happen, and, of course, what happens is that shocks develop. In Fig. 2a we see a number of particles moving in these periodic orbits of family \mathbf{X}_1 , some of which loop. Pure particles, with no dissipation would stay on



such paths forever. But now if we add a bulk viscosity by means of an algorithm described by Lucy (1977) we find, after a short time that the rings of particles develop into the form shown in Fig. 2b (van Albada and Sanders, 1982). Particles on looping orbits attempt to cross through one another, lose energy and descend more abruptly toward the center where they collide with particles on lower energy orbits. Or in fluid dynamical terms a disturbance at the ends of the bar resulting from the attempted looping, propagates downstream as shocks lying roughly along characteristic curves; that is, linear shocks develop along the leading edge of the bar which may be identified with the linear off-set dust lanes seen in some SBb galaxies. This is effectively the same physical idea proposed by Prendergast in unpublished work more than 15 years ago.

Therefore the appearance of shocks in the gas flow in barred spirals may be identified with the development of loops in the basic parallel found of periodic orbits, X_1 , and this only occurs in strong bar potentials, $q_t \gtrsim 30\%$. This tells us that in at least some barred galaxies, the bars are not a small density wave but a major deviation from axial symmetry. Moreover, it is found that the shocks are stronger

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and more off-set if the perpendicular family of orbits is present deep within the bar; that is, if the two inner resonances are present within the bar. This tells us that the mean radial distribution of matter in some barred galaxies must be strongly centrally condensed with respect to the bar. However, the bar cannot lie within two inner resonances because then the gas seems to be preferentially attracted to the perpendicular family of orbits (X_2) ; i.e., the gas distribution is elongated perpendicular to the stellar bar. This means that, for a reasonable radial mass distribution, co-rotation cannot be far from the ends of the bar (within one bar semi-major axis).

In summary, the clear relationship between the numerically calculated gas response and the character of the simple periodic orbits tells us that there are some barred galaxies which deviate strongly from axial symmetry, which are centrally condensed, and which have a rapidly tumbling bar figure.

REFERENCES

Albada, T.S. van, Sanders, R.H.: 1982, M.N.R.A.S. 200 (in press).
Contopoulos, G., Papayannapoulos, T.: 1980, Astron. Astrophys. 92, 33.
Henon, M., Heiles, C.: 1964, Astron. J. 69, 73.
Lake, G., Norman, C.: 1982, preprint.
Lucy, L.B.: 1977, Astron. J. 82, 1013.
Mel'nikov, V.K.: 1963, Trans. Moscow Math. Soc. 12, 1.
Sanders, R.H.: 1982, in preparation.
Sanders, R.H.: 1982, in preparation.
Sanders, R.H., Tubbs, A.D.: 1980, Astrophys. J. 235, 803.
Treve, Y.M.: 1978, in Topics in Nonlinear Dynamics A Tribute to Sir Edward Bullared, ed. S. Jorna (New York: American Institute of Physics), p. 147.