9. THE ROTATION CURVE, MASS, LIGHT, AND VELOCITY DISTRIBUTION OF M 31

J. EINASTO and U. RÜMMEL

W. Struve Astrophysical Observatory, Tartu, Estonian S.S.R.

Abstract. A model for the Andromeda galaxy, M 31, has been derived from the available radio, photometric, and spectroscopic data. The model consists of four components – the nucleus, the bulge, the disc, and the flat component.

For all components the following functions have been found: the mass density; the mass-to-light ratio; the velocity dispersions in three perpendicular directions (for the plane of symmetry and the axis of the galaxy); the deviation angle of the major axis of the velocity ellipsoid from the plane of symmetry; the centroid velocity (for the plane of symmetry).

Our model differs in two points from the models obtained by other authors: the central concentration of mass is higher (in the nucleus the mass-to-light ratio is about 170), and the total mass of the galaxy (200×10^9 solar masses) is smaller. The differences can be explained by different rotation curves adopted, and by attributing more weight to photometric and spectroscopic data in the case of our model.

1. Introduction

The most convenient way to express in condensed and mutually consistent form the various observational data on galaxies is the construction of their models. In the case of bright galaxies there are available the following data: the photometric data for the galaxy as a whole, and for some subsystems (neutral and ionized hydrogen, young bright stars, novae, cepheids), spectrophotometric data (mean spectral type, stellar content) for the nucleus and the bulge, and kinematical data (the systematic radial motion and velocity dispersion) for the gaseous component, the nucleus, and the bulge.

On the basis of these data, using the necessary dynamical and geometric equations it is possible to construct a composite hydrodynamic model of the galaxy. The method has been described earlier (Einasto, 1968a, 1968b, 1969a, 1969c) and applied to the Andromeda galaxy (Einasto, 1969b; Einasto and Rümmel, 1969). In the present paper both the method and the model have been improved. For the first time it is possible to derive reasonable values for all hydrodynamical descriptive functions of the main components of a galaxy.

2. Theory

A. ASSUMPTIONS AND DESCRIPTIVE FUNCTIONS

We assume that the galaxy has an axis and a plane of symmetry, common for all subsystems, that the galaxy is in the steady state, and consists of a number of physically homogeneous subsystems. The equidensity surfaces of the subsystems are similar concentric ellipsoids.

The hydrodynamic descriptive functions, determining the space density of matter and the velocity dispersion tensor, are designated as follows:

Becker and Contopoulos (eds.), The Spiral Structure of Our Galaxy, 51–60.

$\varrho(a)$ – the space density of matter, a being the major semiaxis of the equidensity ellipsoid with the axial ratio $\varepsilon = b/a$;

$\sigma_R, \sigma_\theta, \sigma_z$ – the velocity dispersions in a galactocentric cylindrical coordinate system $(a^2 = R^2 + \varepsilon^{-2} z^2)$;

V_θ – the rotation velocity;

α – the inclination angle of the major axis of the velocity ellipsoid in respect to the plane of symmetry of the galaxy.

B. GEOMETRIC EQUATIONS

The space density of matter can be found from the observed projected luminosity density $L(A)$, where A is the major semiaxis of the projected equidensity ellipse with the apparent axial ratio E, $E^2 = \sin^2 i + \varepsilon^2 \cos^2 i$, where i is the angle between the axis of the system and the line of sight. From geometric considerations, neglecting the absorption of light, we have (Einasto, 1969a)

$$L(A) = \frac{2\varepsilon}{Ef} \int_A^{A^0} \frac{\varrho(a)\, a\, \mathrm{d}a}{\sqrt{a^2 - A^2}}, \tag{1}$$

where f is the mass-to-light ratio of the subsystem, considered as a constant, and A^0 the major semiaxis of the limiting ellipsoid of the subsystem.

C. HYDRODYNAMIC EQUATIONS

In a steady state galaxy the gravitational attraction of the galaxy is counterbalanced by the pressure (velocity dispersion) and the rotation. In cylindrical coordinates the hydrodynamic equilibrium equations are

$$\frac{1}{R}(\sigma_R^2 - \sigma_\theta^2) + \frac{1}{\varrho}\frac{\partial}{\partial R}(\varrho\sigma_R^2) + \frac{1}{\varrho}\frac{\partial}{\partial z}[\varrho\gamma(\sigma_R^2 - \sigma_z^2)] - \frac{V_\theta^2}{R} = -K_R, \tag{2}$$

$$\frac{1}{R}\gamma(\sigma_R^2 - \sigma_z^2) - \frac{1}{\varrho}\frac{\partial}{\partial R}[\varrho\gamma(\sigma_R^2 - \sigma_z^2)] + \frac{1}{\varrho}\frac{\partial}{\partial z}(\varrho\sigma_z^2) = -K_z, \tag{3}$$

where

$$\gamma = \tfrac{1}{2}\mathrm{tg}\,\alpha, \tag{4}$$

and K_R, K_z – the radial and vertical components of the gravitational acceleration of the whole galaxy. The latter quantities can be derived from the mass density distribution function (Einasto, 1969a). In the steady state galaxy the functions σ_R, σ_θ, σ_z, V_θ, γ fully determine the velocity ellipsoid as two axes of the ellipsoid lie in the meridional plane of the galaxy, and the radial and vertical components of the centroid motion are equal to zero.

D. ADDITIONAL EQUATIONS, CLOSING THE SYSTEM OF EQUATIONS

In order to obtain composite models of galaxies, the mass and light distribution of subsystems is first to be determined from photometric and spectroscopic data. Then the gravitational acceleration of the whole galaxy can be found. Finally the kinematical functions of the subsystems can be derived. Given the density and the acceleration the Equations (2) and (3) involve 5 unknown kinematical functions. As we have only two equations the problem is not closed: to solve the system of equations three additional equations are needed.

It is convenient to give the additional equations for the functions, which determine the orientation of the velocity ellipsoid, γ, and its shape

$$k_\theta(R, z) = \sigma_\theta^2/\sigma_R^2, \quad k_z(R, z) = \sigma_z^2/\sigma_R^2. \tag{5}$$

From the theory of the third integral of motion of stars follows (Kuzmin, 1953)

$$\gamma = Rz/(R^2 + z_0^2 - z^2), \tag{6}$$

where z_0 is a constant, depending on the gravitational potential of the whole galaxy.

The equations for k_θ and k_z are in general case complicated (Einasto, 1969c). In the present paper we have computed the kinematical functions for the plane and the axis of the galaxy only. The theory of the steady state galaxy gives (cf. Einasto, 1969a)

$$k_\theta(R, 0) = \tfrac{1}{2}\left[1 + \frac{\partial \ln V_\theta}{\partial \ln R}\right]. \tag{7}$$

We assume that in the first approximation the centroid velocity V_θ is proportional to the circular velocity V_c. In this case $k_\theta(R, 0)$ are identical for all subsystems. From the symmetry condition on the axis of the galaxy we have

$$k_\theta(0, z) = 1. \tag{8}$$

For flat subsystems the ratio $k_z(R, 0)$ can be found from the theory of irregular gravitational forces. Kuzmin (1961) has derived the following approximate relation

$$[k_z(R, 0)]^{-1} = 1 + [k_\theta(R, 0)]^{-1}. \tag{9}$$

On the other hand from the theory of the third integral we have for the axis $R=0$, supposing the ellipsoidal distribution of velocities (Einasto, 1969c)

$$k_z(0, z) = k_z(0, 0)/k_z(\sqrt{z^2 - z_0^2}, 0). \tag{10}$$

Formulae (9) and (10) can be used, if $R^2 \gg z_0^2$, and $z^2 \gg z_0^2$ correspondingly. For small R and z, $k_z(R, z)$ is to be interpolated, using the value $k_z(0, 0)$, derived from the virial theorem.

E. APPLICATION OF THE VIRIAL THEOREM

The nucleus of a galaxy can be considered in a good approximation to be an isolated dynamical system. In this case we may apply the tensor virial theorem (Kuzmin, 1964).

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Assuming a rigid body rotation and ellipsoidal shape for the nucleus we have

$$\overline{\sigma_R^2} + \tfrac{1}{3}\omega^2\overline{a^2} = \tfrac{1}{2}\beta_R G\mathcal{M}\bar{a}^{-1}, \tag{11}$$

$$\overline{\sigma_z^2} = \tfrac{1}{2}\beta_z G\mathcal{M}\bar{a}^{-1}. \tag{12}$$

In these formulae ω is the constant angular velocity, G the gravitational constant, $\mathcal{M}$, the mass of the nucleus, and

$$\overline{a^2} = \frac{1}{\mathcal{M}}\int_0^\infty \mu(a)\, a^2\, \mathrm{d}a, \tag{13}$$

$$\bar{a}^{-1} = \frac{2}{\mathcal{M}^2}\int_0^{\mathcal{M}} \frac{M(a)\,\mathrm{d}M(a)}{a}, \tag{14}$$

where

$$\mu(a) = 4\pi\varepsilon\varrho(a)\, a^2 \tag{15}$$

is the mass distribution function, and

$$M(a) = \int_0^a \mu(a)\,\mathrm{d}a \tag{16}$$

the integral mass distribution function.

The constants β_R and β_z depend on the shape of the system. Denoting $e^2 = 1 - \varepsilon^2$ we have

$$\beta_R = \frac{1}{2e^2}\left[\frac{\arcsin e}{e} - \varepsilon\right], \tag{17}$$

$$\beta_z = \frac{\varepsilon^2}{e^2}\left[\frac{1}{\varepsilon} - \frac{\arcsin e}{e}\right]. \tag{18}$$

From (11)–(12) we obtain

$$\bar{k}_z = \frac{\overline{\sigma_z^2}}{\overline{\sigma_R^2}} = \frac{\beta_z}{\beta_R}\left(1 + \frac{\omega^2\overline{a^2}}{3\overline{\sigma_R^2}}\right). \tag{19}$$

As in the nucleus of the Andromeda galaxy $\omega^2\bar{a}^2 \ll \bar{\sigma}_R^2$, the mean axial ratio of the velocity ellipsoid depends sufficiently only on the axial ratio of the system itself.

The value of $\bar{k}_z$, found for the nucleus of the galaxy, can be adopted for $k_z(0, 0)$.

3. The Model

The theory outlined has been applied to a model of the Andromeda galaxy, consisting of four components: the nucleus, the bulge, the disc, and the flat component.

The observational data used have been described in a published paper (Einasto, 1969b).

The distance 692 kpc of the galaxy is accepted, corresponding to the true distance modulus $(m-M)_0 = 24^{m}.2$ (Baade and Swope, 1963).

The inclination of the galaxy has been estimated by combining the data on the axial ratio of isophotes in the outer region of the galaxy, and the distribution of emission nebulae (Baade and Arp, 1964). The value $i = 12^\circ.8$ has been found. It is in good agreement with an earlier estimate by Baade $i = 12^\circ.7$, quoted by Schmidt (1957). Somewhat larger values found by Arp (1964), and by some other authors cannot be accepted, as in this case the true axial ratio of the equidensity surfaces of the disc population will be too small, of the order of 0.01. The disc component of a galaxy consists of the old population I stars. Their vertical dispersion of velocities at the distance $R = 10$ kpc from the centre is of the order of 20 km s^{-1}. From these data we can estimate the thickness and the axial ratio of equidensity surfaces; the latter quantity becomes of the order of 0.1.

The parameter z_0 was derived from the gravitational potential of the system. An effective value $z_0 = 0.5$ kpc has been found.

The principal descriptive function, the space density of matter, has been chosen in the form of a generalized exponential function

$$\varrho(a) = \varrho_0 \exp[-(a/a_0 k)^\nu], \tag{20}$$

where ϱ_0 is the central density of the component, a_0 – the effective (harmonic mean) radius of the component, ν – the structural parameter of the model, and k – a dimensionless parameter depending on ν. The central density depends on the mass, effective radius, and the axial ratio of the component:

$$\varrho_0 = \frac{h}{4\pi\varepsilon} \frac{\mathcal{M}}{a_0^3}, \tag{21}$$

where h is a dimensionless parameter depending on ν.

The derived parameters of the components are given in Table I.

To obtain better agreement with observations, the flat component of the galaxy has been represented by a sum of two functions (20), one of them being negative.

TABLE I

Parameters of the components of M 31

Quantity	Unit	Total	Nucleus	Bulge	Disc	Flat component	
						+	−
ν			1	1/4	1	1	1
k			0.5	1.263×10^{-4}	0.5	0.5	0.5
h			4	3112	4	4	4
ε			0.84	0.57	0.09	0.01	0.02
a_0	kpc		0.005	1	10	8	4
$\mathcal{M}$	$10^9 \mathcal{M}_\odot$	201.8	0.52	85.5	111.5	5.73	−1.43
L	$10^9 L_\odot$	13.13	0.003	4.95	6.46	2.29	−0.57
f		15.4	173	17.3	17.3	2.5	2.5
$\bar{\varrho} = \mathcal{M}/\frac{4}{3}\pi\varepsilon a^3{}_0$	$\mathcal{M}_\odot/pc^3$		1.2×10^6	35.8	0.296	0.267	−0.267

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The parameters of the model are subjected to the condition that a ring-like mass distribution and everywhere non-negative total density of the component could be provided.

The mass of the nucleus has been determined by means of the virial theorem. In an earlier paper (Einasto, 1969b) the mass has been found from the luminosity of the nucleus and its accepted mass-to-light ratio (Spinrad, 1966).

The calculated descriptive functions are presented graphically in Figures 1–6. In general, the results are similar with our previous ones (Einasto, 1969b; Einasto and Rümmel, 1969). Near the centre of the galaxy the functions are being improved to allow for the virial theorem, not used in our earlier papers.

4. Discussion

A. MASS DISTRIBUTION

Our model differs in two points from the models obtained by earlier authors (Schmidt, 1957; Brandt and Scheer, 1965; Roberts, 1966; Gottesman *et al.*, 1966): the central concentration of mass is much higher, and the total mass smaller (Einasto, 1969b,

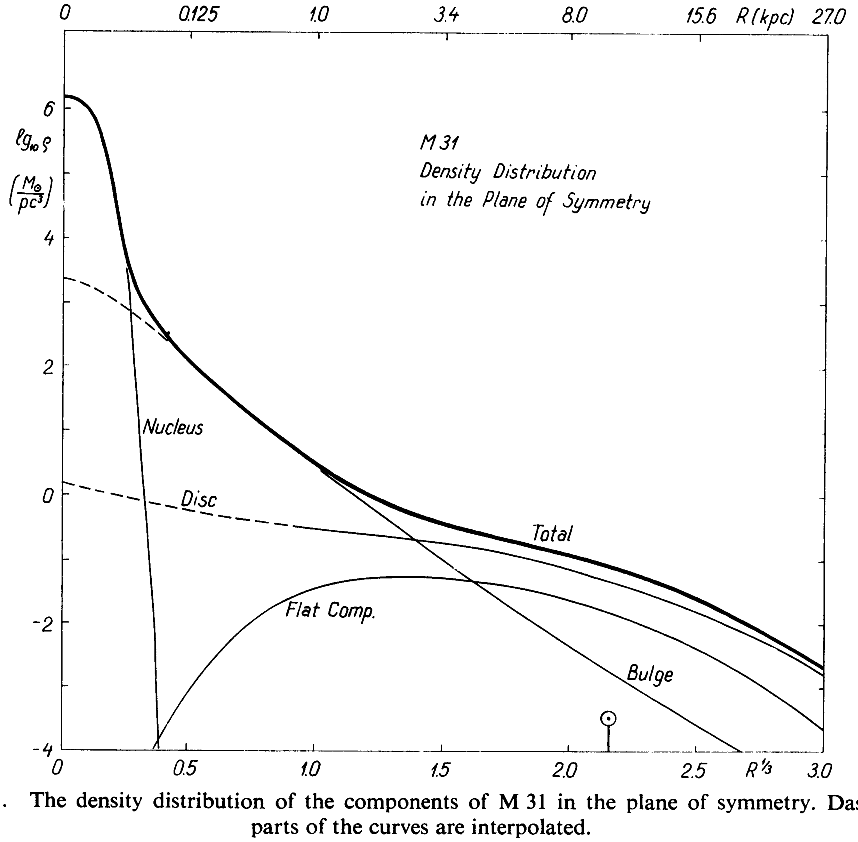

Fig. 1. The density distribution of the components of M 31 in the plane of symmetry. Dashed parts of the curves are interpolated.

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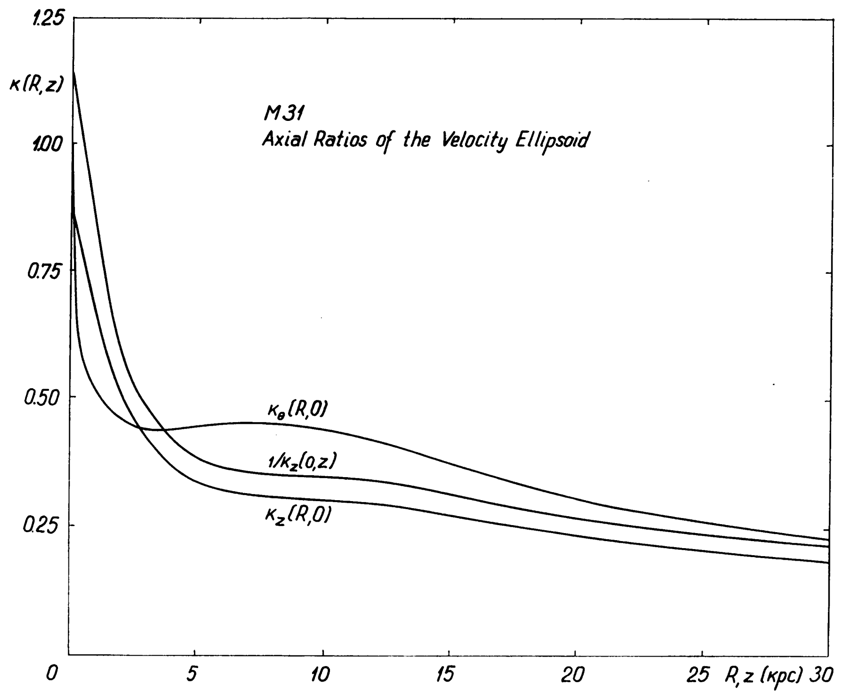

Fig. 2. The axial ratios of the velocity ellipsoid in the plane of symmetry and in the axis of M 31.

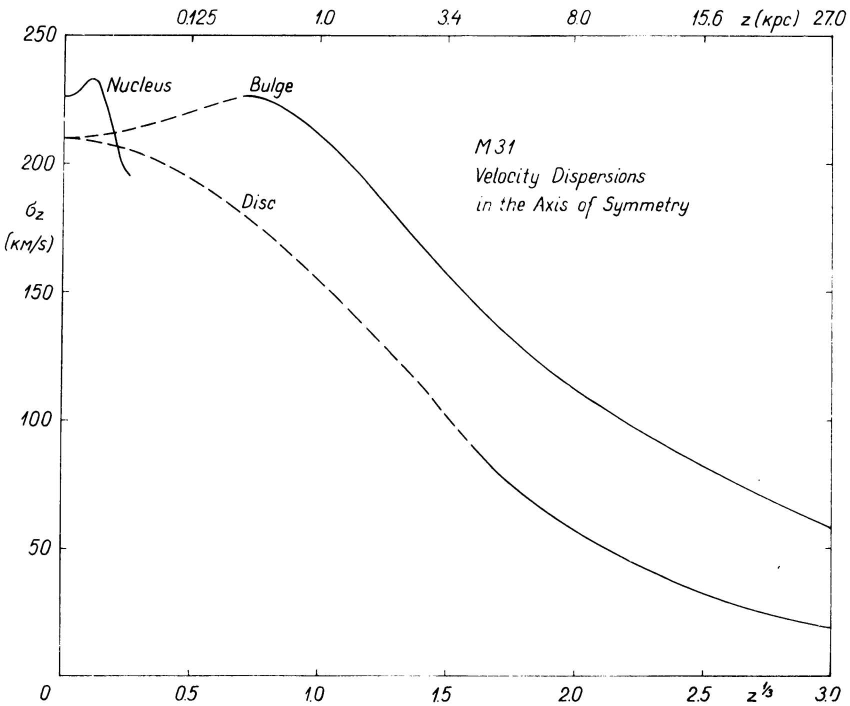

Fig. 3. The velocity dispersions of the components of M 31 in the axis of symmetry.

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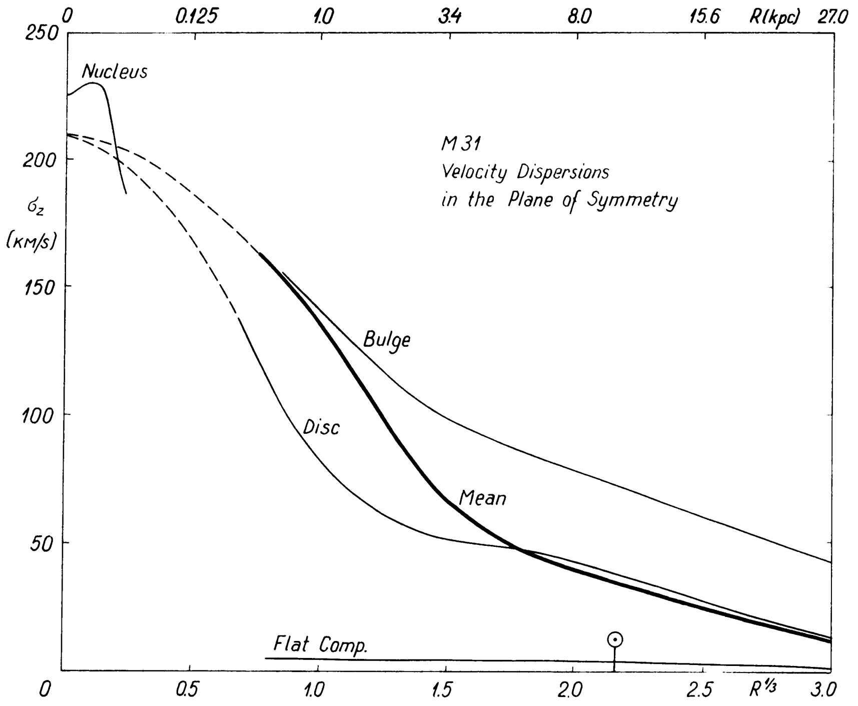

Fig. 4. The velocity dispersions of the components of M 31 in the plane of symmetry.

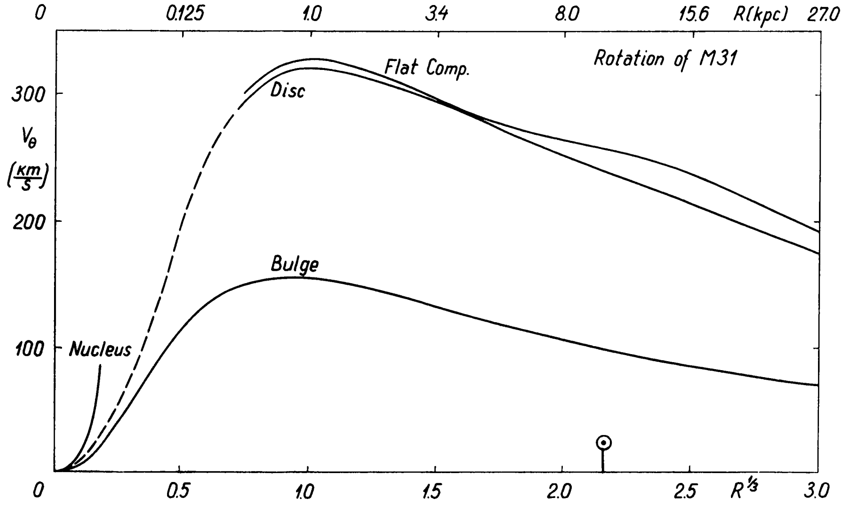

Fig. 5. The rotation velocities of the components of M 31 in the plane of symmetry.

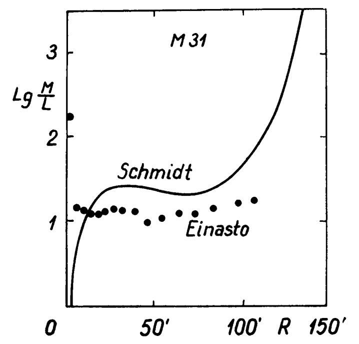

Fig. 6. The mass-to-luminosity ratio in M 31 according to Schmidt's (1957) model and to our present model.

Table III). In both cases the differences can be explained by various circular velocity curves adopted.

In the central region the velocities found earlier from the 21-cm radio-line measurements are underestimated due to the unsufficient correction for the antenna smearing effect. The rotation velocities, derived optically for the stellar component of the galaxy, cannot be identified with the circular velocities, as the pressure term (velocity dispersion) in hydrodynamical equations is predominating.

The great masses are found in most cases as a result of approximating the observed rotation velocities with a generalized Bottlinger law

$$V_\theta = \frac{V_0 R}{[1 + (R/R_0)^n]^{3/2n}}, \tag{22}$$

(V_0, R_0 – constants) and of identifying the rotation velocities with the circular velocity.

We have shown (Einasto, 1969a) that the generalized Bottlinger law cannot be applied to the circular velocity, as in this case great masses at very large distances from the centre of the galaxy occur. This is impossible due to the tidal effect of nearby galaxies (King, 1962).

Small radial gradient of the rotation velocity, observed in the periphery of some galaxies, in particular, in the Andromeda galaxy, is probably to be explained in another way, for instance as the appearance of systematic streaming motion in the galaxy.

B. MASS-TO-LIGHT RATIO

The mean mass-to-light ratio found, $f = 15.4$ is normal for a Sb galaxy. The flat population and the disc have also acceptable values, $f = 2.5$ and $f = 17.3$ respectively. The value for the bulge, $f = 17.3$ is a preliminary one, and a further study is needed.

The mass-to-light ratio for the nucleus, $f = 17.3$, seems at first glance to be too large. To explain this value we must suppose that the nucleus (a) consists of very old physically evolved stars, and (b) is dynamically not evolved.

The mean relaxation time of the nucleus is of the same order (10^{10} yr) as the age of the whole galaxy. Therefore the nucleus is dynamically indeed little evolved and has lost only a small fraction of his low-mass stars. As the nucleus has had too little time to form dynamically by star-star encounters, it must be formed in the proto-galaxy stage of the galaxy evolution.

The metal-content of stars in the nucleus is normal (Spinrad, 1966). Therefore, if the high mass-to-light ratio and the great age of the nucleus will be confirmed, we must conclude that in the nucleus the metal-enrichment has taken place in a very early stage of the galaxy evolution.

Note added in proof. The mass $\mathcal{M} = 5.2 \times 10^8\ \mathcal{M}_\odot$, obtained from the virial theorem for the nucleus, and the corresponding mass-to-light ratio $f = 170$ does not agree with the value $f = 17$ derived spectroscopically (Spinrad, 1966). This discrepancy may be removed, supposing that the nucleus contains besides stars an invisible central body – a dead quasar (Lynden-Bell, in press) or an object of unknown nature (Ambartsumian, 1958). In this case the virial theorem must be modified, and we get (Einasto, 1970, in preparation) for the point mass $\mathcal{M} = 1.4 \times 10^8 \mathcal{M}_\odot$, supposing $\mathcal{M} = 0.5 \times 10^8 \mathcal{M}_\odot$ for the mass of the stellar component of the nucleus (Einasto, 1969b).

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