USING CLOSE ENCOUNTERS OF MINOR PLANETS FOR THE IMPROVEMENT OF THEIR MASSES

A. LÓPEZ GARCÍA
Astronomical Observatory, Valencia University
Valencia, Spain

YU. D. MEDVEDEV
Institute of Theoretical Astronomy, Russian Acad. of Sciences
St. Petersburg, Russia

AND

J. A. MORAÑO FERNÁNDEZ
Department of Astronomy and Astrophycics
Valencia, Spain

Abstract. The orbits of 6807 numbered minor planets on the time interval 1975–2010 have been integrated. The possibility of determining masses for asteroids with close encounters with each other has been investigated, including several kinds of situations. The importance of the observational time arc, as well as the distribution of observations, for determining the masses are shown. The mass of asteroid (24) Themis from its perturbations on asteroid (2296) Kugultinov, and the mass of (10) Hygiea from its perturbations on (1259) Ógyalla have been determined.

1. Introduction

Masses of large asteroids are needed for improving the modern planetary ephemerides. Williams (1984) showed that over 30 asteroids produce detectable perturbations in Mars' motion, but up to now the masses of only six asteroids have been determined. They are: (1) Ceres (Schubart, 1975; Standish and Hellings, 1989; Viateau and Rapaport, 1995); (2) Pallas (Schubart and Matson, 1979; Standish and Hellings, 1989); (4) Vesta (Schubart and Matson, 1979; Standish and Hellings, 1989); (10) Hygiea (Scholl *et al.*, 1987); (15) Eunomia (Hilton, 1996); and (704) Interamnia (Landgraf, 1992). Since the asteroids are so small, the best method to obtain their masses is through observations of individual encounters between pairs of

I. M. Wytrzyszczak, J. H. Lieske and R. A. Feldman (eds.),
Dynamics and Astrometry of Natural and Artificial Celestial Bodies, 199, 1997.

asteroids. In 1995 J. Hilton, J. Middour and P.K. Seidelmann (Hilton *et al.*, 1995) proposed a program for calculating asteroids masses. In their work they have published the list of encounters for 4538 numbered asteroids and investigated the possibility of determining asteroids masses from their encounters. In this work we extend this list up to 6807 asteroids. The influence of distribution of observations for the mass determination of larger asteroids is also investigated.

2. List of Close Encounters

The list of encounters which appears in Table 1 and Table 2 contains:

1) time of encounter;
2) number of larger and smaller asteroids;
3) expected deflection of velocity vector for smaller asteroids;
4) Earth-Sun-Asteroid position angle G at the moment of encounter.

The last value allows one to judge the possibility of observing the asteroids at the moment of their encounter. As will be shown below, if there are observations of the smaller asteroid at the time of closest approach, we can obtain the asteroid orbits and the mass for the larger asteroid with more accuracy than in the cases in which these observations are absent. To estimate the angle of deflection, θ, we use the following formula,

$$\tan\frac{\theta}{2} = 2.17 \cdot 10^{-21} \cdot \frac{r^3}{b \cdot v^2}$$

where r is the approximate diameter for the larger asteroid in km; v is the relative velocity of asteroids in AU/day; b is the minimum distance between asteroids.

Here, we assume that the density of asteroids is equal to 3.5 g/cm^3. To reduce the size of the tables we eliminate in Table 1 the close encounters for (1) Ceres and (4) Vesta and in Table 2 the encounters for which the angle G is larger than 100°.

3. Method for Determination of the Asteroid Mass

In paper (Scholl *et al.*, 1987) the method for determining the asteroid mass from its perturbations on the motion of the smaller asteroid has been explained in detail. Here, we turn our attention to the main steps for calculating the asteroid mass. Everhart's method (Everhart, 1985) is used for integrating the equations of motion of the asteroids. To increase the integration accuracy, the equations have been transformed by Encke's method (Dubyago, 1961). The perturbations from the Sun, nine major planets and three

TABLE 1. List of asteroid encounters for 1975–1997.

date	1st ast.	2nd ast.	θ	G	date	1st ast.	2nd ast.	θ	G
2445945	1	348	6197.6	54	2447871	10	2619	185.9	142
2450337	1	5303	2913.9	78	2449755	10	6006	184.3	127
2442805	1	5564	1383.2	46	2442771	24	2296	155.7	63
2443865	1	6655	906.4	131	2445304	145	6173	149.8	26
2444509	1	3344	883.2	82	2450076	10	465	120.0	122
2450250	4	17	726.0	24	2447694	10	4619	109.9	8
2450795	10	3946	347.8	2	2443521	10	4407	106.0	118
2444322	107	5570	221.2	104	2445687	10	6143	101.7	175

TABLE 2. List of asteroid encounters for 1997–2010.

date	1st ast.	2nd ast.	θ	G	date	1st ast.	2nd ast.	θ	G
2450902	10	3946	1087.1	94	2450817	3	5552	26.8	60
2451235	1	42	105.5	70	2452275	28	4056	26.4	75
2451165	4	2633	78.4	37	2451973	29	5533	22.8	84
2451102	1	631	74.3	35	2452608	29	6034	22.3	98
2452363	4	1652	50.6	68	2451274	9	548	22.2	10
2451404	48	4220	45.7	32	2451272	10	2061	21.2	38
2451689	4	4789	41.8	33	2451607	107	6254	17.3	61
2451297	4	4773	41.6	61	2451888	85	3487	16.1	34
2451214	704	1467	40.2	62	2451116	10	4094	15.5	91
2452021	4	3212	37.2	91	2450928	92	1569	13.5	48
2452275	28	4056	28.2	75	2451064	59	5163	13.4	39
2451160	4	5058	27.5	41	2452350	45	5809	12.4	96
2452795	39	6770	27.1	99	2451824	4	343	11.5	54

largest asteroids (Ceres, Pallas and Vesta) are taken into account, as well as the mutual perturbation between asteroids.

In a common case the 14 parameters – 6 components of initial vectors for the orbital position and velocity of the first asteroid and its mass, and the 7 similar parameters for the other one – must be determined when the asteroid masses are calculated from their mutual perturbations. But, if the mass of one of the asteroids is much larger than that of the other, then, as a rule, the perturbations of the smaller asteroid in the motion of the larger one are very small and only 7 parameters (6 orbital motion parameters for the smaller asteroid and the mass of the larger) should be determined. The

https://doi.org/10.1017/S0252921100046558 Published online by Cambridge University Press

coefficients of conditional equations for improving the orbital motion parameters and the mass are calculated by numerical integration. In particular, for the coefficients $\frac{\partial r}{\partial m}$ the following formula is used:

$$\frac{d}{dt^2}\frac{\partial \bar{r}}{\partial m} = \frac{\partial \sum W_i}{\partial m} + \frac{\partial \sum W_i}{\partial x}\frac{\partial x}{\partial m} + \frac{\partial \sum W_i}{\partial y}\frac{\partial y}{\partial m} + \frac{\partial \sum W_i}{\partial z}\frac{\partial z}{\partial m}$$

where $\bar{r}$ is the heliocentric position vector for the smaller asteroid, $\sum W_i$ is the sum of the acceleration acting on the motion of the smaller asteroid and m is the mass of the larger asteroid.

4. Determining Masses of Asteroids (24) Themis and (10) Hygiea

For determining the mass of (24) Themis, its encounter with (2296) Kugultinov has been chosen. Calculations have shown that asteroid (24) Themis approached (2296) Kugultinov with a minimum distance of 0.016 AU on 24 Dec. 1975. The distribution of observations for (2296) Kugultinov is sufficiently uniform and, in addition, the times of observation for 8 observations of this asteroid are close to the moment of encounter with (24) Themis. Using 62 observations of the smaller asteroid, corrections to the 6 components of the initial orbital vectors of position and velocity of (2296) Kugultinov and the mass of (24) Themis have been determined. The results of these calculations appear in Table 3.

TABLE 3. The components of the initial vectors of the orbital position (in AU), velocity (in AU/day) of (2296) Kugultinov and its errors at Epoch 26 December 1977 = JD 2443480.5. The mass of (24) Themis and its error (in Sun masses).

x	+2.95223266	$\pm 5.24 \cdot 10^{-06}$	$\dot{x}$	+0.00503387	$\pm 1.15 \cdot 10^{-08}$
y	−2.02106819	$\pm 5.13 \cdot 10^{-06}$	$\dot{y}$	+0.00589766	$\pm 7.57 \cdot 10^{-09}$
z	−0.96361196	$\pm 4.05 \cdot 10^{-06}$	$\dot{z}$	+0.00258960	$\pm 8.89 \cdot 10^{-09}$
	M_{24}	$2.89 \cdot 10^{-11}$		$\pm 1.26 \cdot 10^{-11}$	

The set of parameters obtained represents the observations of (2296) Kugultinov with mean square errors of $\sigma = 1''.63$.

Furthermore, we tried to determine the mass of asteroid (10) Hygiea. Seven sufficiently close encounters of Hygiea with several smaller asteroids have been found: 2 encounters with (1259) Ógyalla (on 21.02.1987 with a minimum distance equal to 0.097 AU and on 10.02.1984 with 0.036 AU); (1780) Kippes (on 14.05.1984 with 0.044 AU); (2619) Skalnaté Pleso (on 14.05.1989 with 0.023 AU); (4407) Taihaku (on 13.01.1978 with 0.053 AU);

(4619) Polyakhova (on 10.09.1989 with 0.073 AU); and (6143) 1993 JV (on 16.12.1983 with 0.019 AU). The results of the calculation of the mass of (10) Hygiea from its perturbations on these smaller asteroids are given in Table 4.

TABLE 4. The values of mass, its errors for (10) Hygiea (in Solar mass) and mean square error for the observations of smaller asteroids.

(1259) Ógyalla	$5.5 \cdot 10^{-11}$	$\pm 10.2 \cdot 10^{-11}$	1."47
(1780) Kippes	$-436 \cdot 10^{-11}$	$\pm 240 \cdot 10^{-11}$	1."89
(2619) Skalnaté Pleso	$190 \cdot 10^{-11}$	$\pm 84 \cdot 10^{-11}$	0."96
(4407) Taihaku	$122 \cdot 10^{-11}$	$\pm 47 \cdot 10^{-11}$	1."47
(4619) Polyakhova	$295 \cdot 10^{-11}$	$\pm 170 \cdot 10^{-11}$	1."22
(6143) 1993 JV	$21.8 \cdot 10^{-11}$	$\pm 6.1 \cdot 10^{-11}$	1."14

In the first column appears the number of the smaller asteroid, in the next, the mass of (10) Hygiea and its error, and in the last column the mean square errors for the observations of the smaller asteroid. As can be seen, a wide range of values for the mass of (10) Hygiea has been obtained. The mass obtained from the two encounters with (1259) Ógyalla is close to the value obtained previously by Scholl *et al.* (1987): $(4.7 \pm 2.3) \cdot 10^{-11} M_{\odot}$, but our result has a larger error. The probable explanation of this result is the nonuniform distribution for the observation of (1259) and the lack of observations near the moment of encounter. For the same reason the mass of Hygiea could not be obtained from its encounter with asteroid (1780), since for this asteroid the difference between the moments of observation and encounter is more than 3 years. Besides, asteroid (1780) had an encounter with another larger asteroid, (16) Psyche, 14 days after the moment of encounter with Hygiea. The large values of mass for Hygiea obtained using encounters with asteroids (2619) and (4619) can be explained by the short observation arc for these asteroids after the encounter. Therefore, new observations for these asteroids are needed. The small number of observations and the lack of them near encounter can explain the large mass value obtained from the encounters with asteroids (4407) and (6143). It should be noted also that (6143) 1993 JV had an encounter with the larger asteroid (24) Themis at a minimum distance 0.062 AU on 20.11.1989.

The following numerical simulation has been made in order to confirm the necessity of observations near the encounter. Artificial observations for (6143) with real distribution of epochs have been calculated. Observations with random errors of $\sigma = 1''$ and a mass for Hygiea equal to $5 \cdot 10^{-11}\ M_{\odot}$ were assumed. Using these observations the initial vectors for orbital position and velocity of (6143) 1993 JV and the mass of Hygiea have been

https://doi.org/10.1017/S0252921100046558 Published online by Cambridge University Press

determined. Furthermore, the variants when 3 observations near to the epoch of encounter are added have been considered. The following results have been obtained: $(-1.8 \pm 5.9) \cdot 10^{-11} M_{\odot}$ with the real distribution of observational epochs and $(5.0 \pm 5.1) \cdot 10^{-11} M_{\odot}$ for the second variant. The numerical simulation confirms also the importance of the existence of observations with epochs close to the moment of encounter.

5. Conclusion

A large quantity of asteroid encounters with each other has been discovered in the period 1975–2010. The mass of asteroid (24) Themis has been determined by assessing its perturbation effect on the smaller one (2296) Kugultinov. A series of values for the mass of asteroid (10) Hygiea has been determined. It is shown that very important for the determination of the mass of larger asteroids is not only the observational arc length, but also the uniform distribution of observations, and the presence of observations with epochs close to the encounter moment. In this connection, we consider that the distribution of regular communications with information about future close encounters should be very interesting, in order to obtain precise observations in such special epochs.

References

Dubyago, A.D.: 1961, *The Determination of Orbits*, The Macmillan Company, New York.

Everhart, E.: 1985, "An efficient integrator that used Gauss-Radau spacing", in: *Dynamics of Comets: Their Origin and Evolution* (A. Carusi, G.B. Valsecchi eds), Reidel, Dordrecht, 185–202.

Hilton, J.L., Middour, J., and Seidelmann, P.K.: 1995, "Prospects for determining masses for asteroids", in: *Dynamics, Ephemerides and Astrometry of the Solar System, IAU Symposium 172* (S. Ferraz-Mello, B. Morando, J.E. Arlot, eds), Kluwer, Dordrecht.

Hilton, J.L.: 1996, "The mass of 15 Eunomia", *Abstracts of the International Conference on Asteroids, Comets and Meteors*, held in Versailles.

Landgraf, W.: 1992, "A determination of the mass of (704) Interamnia from observations of (993) Moultona", in: *Chaos, Resonance and Collective Dynamical Phenomenon in the Solar System, IAU Symposium 152* (S. Ferraz-Mello ed.), Kluwer, Dordrecht, 179–182.

Scholl, H., Schmadel, L.D., and Röser, S.: 1987, "The mass of the asteroid (10) Hygiea derived from observations of (820) Academia", *Astron. Astrophys.* **170**, 311–316.

Schubart, J.: 1975, "The masses of the first two asteroids", *Astron. Astrophys.* **39**, 147–148.

Schubart, J. and Matson, D.L.: 1979, "Masses and densities of asteroids", in: *Asteroids* (T. Gehrels ed.), Tucson, 84–97.

Standish, E.M. and Hellings, R.W.: 1989, "A determination of the masses of Ceres, Pallas and Vesta from their perturbations upon the orbit of Mars", *Icarus* **80**, 326–333.

Viateau, B. and Rapaport, M.: 1995, "The orbit of (2) Pallas", *Astron. Astrophys., Suppl. Ser.* **111**, 305–310.

Williams, J.G.: 1984, "Determining asteroid masses from perturbation on Mars", *Icarus* **57**, 1–13.