Bull. Austral. Math. Soc.
Vol. 49 (1994) [171–172] 26E20, 49J52, 90C30

# Optimality conditions in nonsmooth optimization

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The purpose of this thesis is to establish, by a variety of techniques from convex and nonsmooth analysis, first order necessary and sufficient optimality conditions for general mathematical programming problems defined in abstract spaces and involving nonsmooth mappings. The conditions obtained are, in essence, generalizations of the classical Fritz John and Karush-Kuhn-Tucker theorems from finite dimensional differentiable programming.

In order to obtain verifiable optimality conditions involving subdifferentials for abstract cone-constrained programming problems it is necessary to develop new tools in nonsmooth analysis. In the first chapter this route is taken by establishing infinite dimensional chain rules for Lipschitz mappings (between Banach spaces) which possess a subdifferential multifunction satisfying a weak* closure condition. This condition is shown to be satisfied by the class of *compactly Lipschitzian mappings* which are then investigated and characterized by the upper semicontinuity and compactness properties of their generalized Jacobians. This generalized Jacobian is an extension, for vector-valued Lipschitz mappings, of the Clark subdifferential for real-valued locally Lipschitz functions. Both metric regularity and local solvability conditions are developed as applications of these results. In the second chapter a Fritz John-type theorem is obtained using Ekeland's variational principle. This result is then used to obtain Karush-Kuhn-Tucker conditions for Lipschitz programming problems without recourse to a constraint qualification. Applications of these results to concave minimization and certain stochastic programming problems are outlined and discussed.

It is well known that one approach to the derivation of necessary optimality conditions for differentiable programming problems is via a solvability theorem applied to a certain 'linearization' of the problem. This approach is extended using suitable generalizations of a fundamental solvability theorem known as Farkas Lemma. In the third chapter, various unifying generalizations of this result are presented with applications to global convex maximization, least squares problems and Lagrangian duality. In Chapter

Received 24th August, 1993
Thesis submitted to the University of Melbourne, March 1993. Degree approved July 1993. Supervisor: Dr B.D. Craven.

4 the class of *quasidifferentiable* programming problems is considered and asymptotic versions of the Karush-Kuhn-Tucker optimality conditions are developed via the solvability theorems from Chapter 3. In Chapter 6 this approach is revisited for a new class of mappings between locally convex topological vector spaces, here called *almost DC* (Difference Convex). Solvability theorems are obtained both with and without a Slater type regularity condition.

In Chapter 5 new approximation results for composite functions involving Lipschitz mappings are presented in the form of chain rules involving the Michel-Penot subdifferential. These results are used to derive, via the exact penalization approach, optimality conditions under the *calmness* regularity condition for a general composite programming model. Further consideration of quasidifferentiable programming problems is undertaken in this setting.

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