ENERGY STORAGE AND INSTABILITY IN MAGNETIC FLUX TUBES

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1. INTRODUCTION

Now it is known that the solar corona consists of many loops which are believed to represent the structure of the magnetic field. Since the plasma is very tenuous in the corona, the equilibrium of the magnetic field is approximated by the force-free field:

$$\mathrm{rot}\ \underset{\sim}{B} \times \underset{\sim}{B} = 0 \quad . \tag{1}$$

In this paper we will propose a method of solution for equation (1), and will discuss on the energy build up and instability in the magnetic flux tubes.

2. METHOD OF SOLUTION FOR THE FORCE-FREE FIELD

Our method (Sakurai 1979) is based on a variational principle for the force-free field. We represent the magnetic field $\underset{\sim}{B}$ in terms of Euler potentials u and v as

$$\underset{\sim}{B} = \underset{\sim}{\nabla} u \times \underset{\sim}{\nabla} v \quad . \tag{2}$$

Then the solution of the variational problem

$$W \equiv \int \frac{B^2}{8\pi}\, dV = \text{extremum} \quad , \tag{3a}$$

$$u\ ,\ v = \text{given on the boundary} \tag{3b}$$

turns out to be the force-free field. Due to the boundary condition (3b), not only the distribution of magnetic flux but also the position of the footpoint of the field line is prescribed because every field line is labelled by a particular set of values of u and v .

This variational problem is solved directly by minimizing the

M. Dryer and E. Tandberg-Hanssen (eds.), Solar and Interplanetary Dynamics, 291-294.

https://doi.org/10.1017/S0074180900067735 Published online by Cambridge University Press

magnetic energy W. The finite element method (Strang and Fix 1973) is used to evaluate the integral (3a), and the minimization of the energy is performed by the variable metric method (Kowalik and Osborne 1968). Figure 1 shows the current free field of a pair of sunspots and the force-free field when the footpoints are rotated 180°. As the tube is twisted, the energy in the tube enhances as is shown in Figure 2.

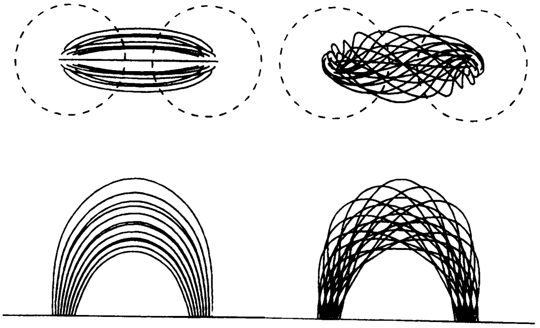

Figure 1 (left)
The current free field due to a pair of sunspots (left) and the force-free field when footpoints are rotated 180° (right): top view (above) and side view (below).

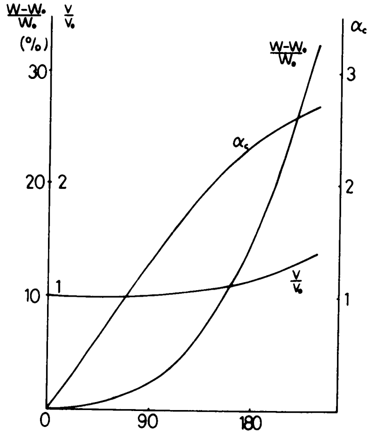

Figure 2 (right)
The change in energy W in the tube of volume V, versus the angle of rotation for the case shown in Figure 1. Subscripts 0 refer to the initial value. α_c is the value of α on the field line at the center of the tube. The unit of α_c is the inverse of the radius of the model sunspots.

A usual treatment for the force-free field is to write equation (1) as

$$\text{rot}\ \underset{\sim}{B} = \alpha \underset{\sim}{B} \quad , \quad \underset{\sim}{B} \cdot \underset{\sim}{\nabla}\alpha = 0 \tag{4}$$

and to assume that the quantity α does not vary in space (constant-α force-free field, e.g. Nakagawa and Raadu 1972). On the contrary, α is determined consistently in our method due to the boundary condition (3b), and generally the quantity α is not a constant.

3. INSTABILITY OF THE MAGNETIC FLUX TUBES

When the flux tube is twisted considerably, it may become unstable to magnetohydrodynamical instabilities. Resulting motion of the tube

https://doi.org/10.1017/S0074180900067735 Published online by Cambridge University Press

can be studied by a similar finite element technique applied to the Lagrangian (Sakurai 1976). The time development of a kink-type instability is calculated in the case of straight flux tube for various pitch of the helical field lines. Figure 3 shows the case when $P/R = 1/2$, where R is the radius of the tube and $2\pi P$ is the pitch of the field lines. It can be seen that the tube develops into a twisted loop just like an over-twisted rubber string. For smaller values of P/R, the initial deformation grows without much twisting, while for larger values of P/R the instability does not grow markedly and the tube evolves into a low arch. These kinds of motion are actually observed in eruptive prominences.

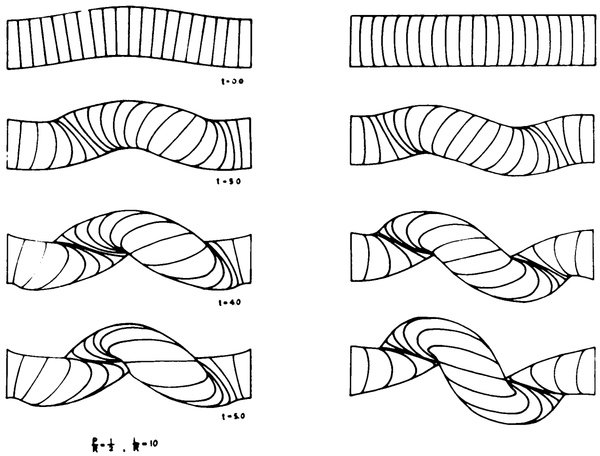

Figure 3
The motion of the twisted flux tube due to a kink instability: top-view (right) and side-view (left). The tube is divided into small cylinders in order to apply the finite element method. The unit of time is R/V_A, where R is the radius of the tube and V_A is the Alfvén speed in the tube, respectively.

4. SUMMARY

A new method to calculate the force-free field was proposed and it was shown that the motion of the footpoint of the field line feeds the energy into the flux tube. The instability of an over-twisted tube was also studied, and the calculation could explain the motion of the eruptive prominences. The energy build up and the instability in the magnetic field discussed here can be an important process in solar flares.

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DISCUSSION

Bratenahl: Were these calculations made in ideal MHD (i.e., infinite conductivity)?

Sakurai: Yes, and a variational formulation would be impossible for the system with dissipative processes.

Callebaut: (1) Does your variational analysis to determine a rotation correspond to an eigenvalue problem and does α correspond in some sense to an eigenvalue? (2) Is the theorem (well-known in quantum mechanics, etc.) that the eigenvalues are obtained with second order precision by using eigen functions which are only correct to first order, relevant to your work?

Sakurai: The quantity α does not appear explicitly in our formulation, but it can be said that α takes the role of Lagrangian multiplier in order to fix the connectivity of the field lines. The magnetic energy, a quantity to be minimized, has a second order accuracy but α will not have such property.

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