is based on the number of occurrences of a power of a without regard to sign.

Three valuable properties of this permanent are[2]

(a) that it may be expanded in terms of its minors

(b) that any minor, when expanded, is a "reciprocal" polynomial in a

(c) the effect of shifting a minor bodily across it is to multiply each term of its expansion by a constant power of a.

This determinant does not appear to have been noted previously.

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REFERENCES.

[1] Dodgson, Rev. C. L., "On condensation of determinants." *Proc. Roy. Soc., Lond.*, xv., 150.

[2] Kendall, M. G., Kendall, S. F. H. and Babington Smith, B., "The distribution of Spearmans coefficient of rank correlation ......" *Biometrica*, xxx., Pts. iii. and iv., p. 251.

Maxima and minima

By G. Lawson.

In my experience answers to questions about maxima and minima are as a rule hazy and unsatisfactory. I suggest that the principle "according as a function is increasing or decreasing, its derivative is positive or negative" might be applied more fully. All functions are here assumed continuous.

If y is a maximum at A, then

before A	at A	after A
(1) y is increasing		(2) y is decreasing
	(3) y is maximum	
(4) y_1 is $+$, by (1)		(5) y_1 is $-$, by (2)
	(6) $y_1 = 0$, by (4) and (5)	

(7) y_1 diminishing by (4) and (6)

(8) y_1 diminishing by (5) and (6)

(9) y_2 is $-$, by (7)

(10) y_2 is $-$, by (8)

(11) by (9) and (10)

either y_2 is $-$ *or* $y_2 = 0$ and is maximum.

Generalising, where y_n is a maximum, there $y_{n+1} = 0$ and either y_{n+2} is negative or $y_{n+2} = 0$ and is maximum. This leads directly to the usual conditions for a maximum

$$y_1 = 0,\ y_2 \text{ is negative}$$

$$\text{or}\quad y_1 = y_2 = y_3 = 0,\ y_4 \text{ is negative}$$

$$\text{or}\quad y_1 = y_2 = y_3 = y_4 = y_5 = 0,\ y_6 \text{ is negative, etc.}$$

Further the relation between the sign of d^2y/dx^2 and the concavity of an arc is often obscurely presented. Take an x or time axis horizontally and a y axis vertically and consider an arc AB everywhere concave down. Let C be a point on the arc such that AC and CB have equal horizontal projections, and let their vertical projections be ac and cb. Then algebraically we have from a figure $ac > cb$, that is, heights gained in equal successive times are diminishing and therefore there is a retardation and d^2y/dx^2 is negative. And we similarly associate concavity upwards with positive values of d^2y/dx^2.

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On certain modular determinants

By H. W. Turnbull.

An interesting determinant occurs in the fifth volume of Muir's *History*[1]. It is

$$\Delta = |a_{rs}|_n$$

[1] Sir Thomas Muir, *The History of Determinants*, 1900-1920 (Blackie, 1930), p. 340. Question 4269. *L'Intermédiaire des Math.*, **20** (1913), p. 218, proposed by E. Maillet: reply by E. Malo, **21**, pp. 173-176.