Perseus in Sicily: From Black Hole to Cluster Outskirts
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Fully analytical solutions for Bondi accretion in galaxies with a central Black Hole

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Abstract. The fully analytical solution for isothermal Bondi accretion on a black hole (MBH) at the center of JJ two-component Jaffe (1983) galaxy models is presented. In JJ models the stellar and total mass density distributions are described by the Jaffe profile, with different scale-lengths and masses, and to which a central MBH is added; all the relevant stellar dynamical properties can also be derived analytically. In these new accretion solutions the hydrodynamical and stellar dynamical properties are linked by imposing that the gas temperature is proportional to the virial temperature of the stellar component. The formulae that are provided allow to evaluate all flow properties, and are then useful for estimates of the accretion radius and the mass flow rate when modeling accretion on MBHs at the center of galaxies.

Keywords. galaxies: elliptical and lenticular, cD, galaxies: ISM, galaxies: nuclei, hydrodynamics, stellar dynamics

1. Introduction

Observational and numerical investigations of accretion on massive black holes (hereafter MBH) at the center of galaxies often lack the resolution to follow gas transport down to the parsec scale. In these cases, the *classical* Bondi (1952) solution on to an isolated central point mass is then commonly adopted, for estimates of the accretion radius (i.e., the sonic radius), and the mass accretion rate (see Ciotti & Pellegrini 2017, hereafter CP17, and references therein). However, two major problems affect the direct application of the classical Bondi solution, namely the facts that 1) the boundary values of density and temperature of the accreting gas are assigned at infinity, and 2) in a galaxy, the gas experiences the gravitational effects of the galaxy itself (stars plus dark matter), and the MBH gravity becomes dominant only in the very central regions. The solution commonly adopted is to use values of the gas density and temperature "sufficiently near" the MBH. It is therefore important to quantify the systematic effects, on the estimates obtained from the classical Bondi solution for the accretion radius and the mass accretion rate, due to measurements taken at finite distance from the MBH, and under the effects of the galaxy potential well. A first step analysis of this problem was carried out in Korol *et al.* (2016, hereafter KCP16) where the Bondi problem was generalized to the case of mass accretion at the center of galaxies, including also the effect of electron scattering on the accreting gas. CP17 showed that the whole accretion solution can be given in an analytical way for the *isothermal* accretion in Jaffe (1983) galaxies with a central MBH. Ciotti & Pellegrini (2018, hereafter CP18, in preparation), extend the study to JJ two-component galaxy models (Ciotti & Ziaee Lorzad 2018, hereafter CZ18), where the *stellar* and *total* mass density distributions are both described by the Jaffe profile, with different scale-lengths and masses, and a MBH is added at the center. In particular, JJ models offer the *unique* opportunity to have a quite realistic family of galaxy models with a central MBH, allowing both for the fully analytical solution of the Bondi (isothermal) accretion problem, *and* the fully analytical solution of the Jeans equations.

2. The models

2.1. *The Bondi solution*

In the Bondi problem, the gas is perfect, has a spatially infinite distribution, and is accreting on to a MBH, of mass $M_{\rm BH}$. The gas density ρ and pressure p are linked by the polytropic relation

$$p = \frac{k_{\rm B}\rho T}{<\mu> m_{\rm p}} = p_\infty \tilde{\rho}^\gamma, \quad \tilde{\rho} \equiv \frac{\rho}{\rho_\infty}, \tag{2.1}$$

where $\gamma = 1$ in the isothermal case, $m_{\rm p}$ is the proton mass, and p_∞ and ρ_∞ are respectively the gas pressure and density at infinity. The sound speed is $c_{\rm s} = \sqrt{\gamma p/\rho}$, and in the isothermal case $T(r) = T_\infty$ and so $c_{\rm s}(r) = c_\infty$. With the introduction of the Bondi radius $r_{\rm B}$ and of the Mach number $\mathcal{M}$, given respectively by

$$r_{\rm B} \equiv \frac{GM_{\rm BH}}{c_\infty^2}, \quad \mathcal{M}(r) = \frac{v(r)}{c_{\rm s}(r)}, \tag{2.2}$$

the time-independent continuity equation becomes

$$x^2 \tilde{\rho}(x)\mathcal{M}(x) = \frac{\dot{M}_{\rm t}}{4\pi r_{\rm B}^2 \rho_\infty c_\infty} \equiv \lambda, \quad x \equiv \frac{r}{r_{\rm B}}, \tag{2.3}$$

where $\dot{M}_{\rm t}$ is the mass accretion rate on the MBH at the center of the galaxy, and λ the accretion parameter.

For a Jaffe galaxy of total mass $M_{\rm g}$ and scale-length $r_{\rm g}$, the gravitational potential and the two parameters determining the accretion solution are (CP17):

$$\Phi_{\rm g} = \frac{GM_{\rm g}}{r_{\rm g}} \ln\frac{r}{r + r_{\rm g}}, \quad \mathcal{R} \equiv \frac{M_{\rm g}}{M_{\rm BH}}, \quad \xi \equiv \frac{r_{\rm g}}{r_{\rm B}}. \tag{2.4}$$

In the Bondi problem for isothermal accretion reduces to the solution of

$$\frac{\mathcal{M}^2}{2} - \ln\mathcal{M} = f(x) - {\rm e}^\lambda, \quad f = \frac{\chi}{x} - \frac{\mathcal{R}}{\xi}\ln\frac{x}{x+\xi} + 2\ln x, \tag{2.5}$$

where $\chi \equiv 1 - L/L_{\rm Edd}$ measures the effect of electron scattering radiation pressure due to the accretion luminosity L. Solutions exist only for $\lambda \leqslant \lambda_{\rm t}$, the *critical* accretion parameter, a value determined by the position of the minimum $x_{\rm min}$ for the function f. For the Jaffe galaxy the position of the only minimum of f (corresponding to the sonic radius of the critical solution) can be calculated analytically, with $x_{\rm min} = x_{\rm min}(\chi, \mathcal{R}, \xi)$, so that quite surprisingly one can evaluate $\lambda_{\rm t}$ analytically. In the peculiar case of $\chi = 0$ (and/or $M_{\rm BH} = 0$), a solution of the accretion problem is possible only for $\mathcal{R} \geqslant 2\xi$. Moreover, the radial trend of the Mach number can also be calculated analytically in terms of the so-called W Lambert-Euler function.

CP18 apply the previous results to JJ models, that are characterized by a *total* Jaffe density distribution $\rho_{\rm g}$ (stars plus dark matter) of total mass $M_{\rm g}$ and scale-length $r_{\rm g}$, and a stellar Jaffe distribution of stellar mass $M_\star$, and scale radius r_*. Remarkably, almost all the stellar dynamical properties of JJ models with a central MBH can be expressed by analytical functions (CZ18), so that they are a family of two-component galaxy models with a central MBH allowing for a full analytical treatment of Bondi accretion and of stellar dynamics (CP18).

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2.2. *Structure and dynamics of the JJ models*

The stellar and total density distributions of JJ galaxies are given respectively by

$$\rho_*(r) = \frac{\rho_{\rm n}}{s^2(1+s)^2}, \quad \rho_{\rm g}(r) = \frac{\rho_{\rm n}\mathcal{R}_{\rm g}\xi_{\rm g}}{s^2(\xi_{\rm g}+s)^2}, \quad s \equiv \frac{r}{r_*}, \tag{2.6}$$

where we define

$$\rho_{\rm n} \equiv \frac{M_\star}{4\pi r_*^3}, \quad \Psi_{\rm n} \equiv \frac{GM_\star}{r_*}, \quad \mu \equiv \frac{M_{\rm BH}}{M_\star}, \quad \mathcal{R}_{\rm g} \equiv \frac{M_{\rm g}}{M_\star}, \quad \xi_{\rm g} \equiv \frac{r_{\rm g}}{r_*}. \tag{2.7}$$

Typically, $\mu \approx 10^{-3}$. When $\xi_{\rm g} \geqslant 1$, the density distribution of the dark halo $\rho_{\rm DM} = \rho_{\rm g} - \rho_*$ is nowhere negative, provided that $\mathcal{R}_{\rm g} = \alpha\xi_{\rm g}$ with $\alpha \geqslant 1$. A galaxy with $\alpha = 1$ is called a *minimum halo* model, and the associated halo is well approximated by the NFW (Navarro *et al.* 1997) profile over a very large radial range (CZ18). For JJ models the Osipkov-Merrit anisotropic Jeans equation, and the projected values of the velocity dispersion at the galaxy center, can be expressed analytically. In particular, the virial temperature of the stellar component can be written as $T_{\rm V} =< \mu > m_{\rm p}\sigma_{\rm V}^2/3$, where the virial velocity dispersion of stars is given by $\sigma_{\rm V}^2 = \Psi_{\rm n}\alpha\mathcal{F}_{\rm g}(\xi_{\rm g})$, and $\mathcal{F}_{\rm g}(\xi_{\rm g})$ is a simple analytical function, with $\mathcal{F}_{\rm g}(1) = 1/2$ and $\mathcal{F}_{\rm g}(\infty) = 1$.

2.3. *Linking stellar dynamics to fluidodynamics*

In CP18 the idea is to self-consistently "close" the accretion solution, determining a fiducial value for the gas temperature as a function of the galaxy model hosting accretion. For assigned values of $\xi_{\rm g} \geqslant 1$, $\mathcal{R}_{\rm g}$ (or α), and μ, we fix $T_\infty = \beta T_{\rm V}$, with $\beta > 0$, so that $c_\infty = \sigma_{\rm V}\sqrt{\beta/3}$. All the accretion parameters can therefore be obtained in terms of the galaxy properties as

$$\mathcal{R} = \frac{\alpha\xi_{\rm g}}{\mu}, \quad \xi = \frac{\mathcal{R}\beta\mathcal{F}_{\rm g}}{3}, \quad \frac{r_{\rm B}}{r_*} = \frac{3\mu}{\alpha\beta\mathcal{F}_{\rm g}}, \quad \frac{r_{\rm min}}{r_*} = x_{\rm min}(\chi, \mathcal{R}, \xi)\,\frac{r_{\rm B}}{r_*}, \tag{2.8}$$

where $r_{\rm min}$ is the position of the sonic radius. Then the critical accretion parameter $\lambda_{\rm t}$, and the Mach number profile, can be computed analytically using the results of CP17. It turns out that for JJ models with $\chi = 0$ (and/or $M_{\rm BH} = 0$) the Bondi solution exists only for $\beta \leqslant \beta_{\rm c} \equiv 3/(2\mathcal{F}_{\rm g})$ with $3/2 \leqslant \beta_{\rm c} \leqslant 3$, and that $\beta_{\rm c}$ determines the behavior of the solution also in presence of a MBH. The first three panels in Fig. 1 show some representative cases of the quantities in eq. (2.8). One of the most relevant features is the considerable jump of $r_{\rm min}$ from very external to very internal galactic regions, even for a slight increase of the gas temperature. Finally, CP18 evaluate the departure of the estimated mass accretion rate $\dot{M}_{\rm e}(r) \equiv 4\pi r_{\rm B}^2\lambda_{\rm cr}\rho(r)c_\infty$ obtained from the classical Bondi solution, from the true value $\dot{M}_{\rm t}$, as a function of the distance from the center:

$$\frac{\dot{M}_{\rm e}(r)}{\dot{M}_{\rm t}} = \frac{\lambda_{\rm cr}\tilde{\rho}(x)}{\lambda_{\rm t}} = \frac{\lambda_{\rm cr}}{x^2\mathcal{M}(x)}, \tag{2.9}$$

where $\mathcal{M}(x)$ is the solution of eq. (2.5). Here $\rho(r)$ is taken along the solution for accretion within the potential of the galaxy and used as "proxy" for the true value ρ_∞, and $\lambda_{\rm cr} = e^{3/2}/4$ is the isothermal critical accretion parameter of the Bondi solution on an isolated MBH. Note how the bias increases from values much lower than unity in the outer galactic regions to very large values near the center; this allows to directly estimate the so-called "boost factor" (see CP18 for a thorough discussion).

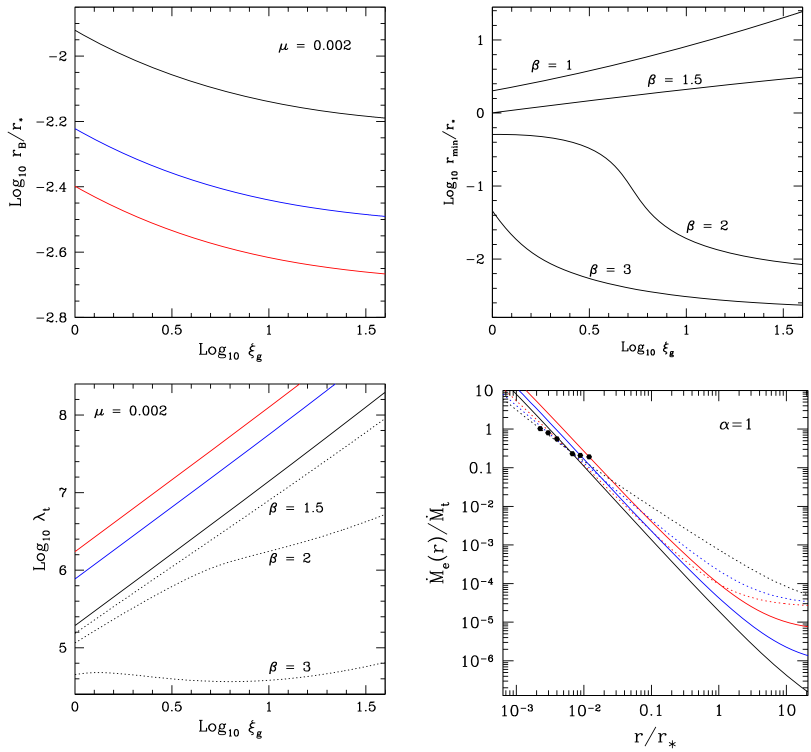

Figure 1. Relevant scale-lengths of the isothermal accretion solution in JJ models, as a function of $\xi_g = r_g/r_*$. A MBH-to-galaxy stellar mass ratio $\mu = 2 \times 10^{-3}$ is assumed, and $T_\infty = \beta T_V$. Top left: the ratio r_B/r_* with $\beta = 1$ and $\alpha = 1$ (black), 2 (blue), 3 (red). Top right: ratio r_{min}/r_* for $\alpha = 1$ and $\beta = 1, 3/2, 2, 3$: for large values of $\mathcal{R}$ and $\beta < \beta_c$ the ratio is almost independent of α but strongly dependent on gas temperature; for $\beta > \beta_c$, the sonic radius collapses near the center. Bottom left: the critical accretion parameter λ_t as a function of ξ_g, for $\alpha = 1, 2, 3$, $\chi = 1$, and $\beta = 1$ (solid lines). The dotted curves refer to $\alpha = 1$ and three different values of β. Bottom right: ratio between the estimate of the accretion rate $\dot{M}_e$ and the true accretion rate $\dot{M}_t$, as a function of r/r_*, in the minimun halo case ($\alpha = 1$), and $\xi_g = 1$ (red), $\xi_g = 3$ (blue), and $\xi_g = 20$ (black). The dotted lines correspond to $\beta = 3$, and the solid dots mark the position of the Bondi radius.

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