

## OBITUARY

PHILIP HOLGATE (1934–1993)



Philip Holgate was born in Chesterfield on 8 December 1934. His family moved from Derbyshire to Devon in 1945, and he was educated at Newton Abbot Grammar School from 1945 to 1952, and the University College of the South-West (now Exeter University, but which then awarded London degrees) from 1952 to 1955. He qualified as both a teacher (King's College, London, 1955–56) and a statistician (University College, London, 1956–57).

After teaching mathematics and physics at Burgess Hill School, in Hampstead and then in Borehamwood, Philip joined the Statistics Section at Rothamsted Experimental Station (1961–62). He then spent five years in the Biometrics Section of the Nature Conservancy. His first publications date from this period, and the interests he acquired then were to develop into what became his most enduring, and distinctive, scientific interests.

Philip joined the Department of Statistics, Birkbeck College, University of London, in 1967, and remained at Birkbeck for the rest of his career, until his death from a heart attack on 13 April 1993. He was promoted to Reader in Statistics in 1968,

then to Professor of Statistics in 1970, and he served as Head of the Department of Statistics until 1987. Following the merger of the Departments of Statistics and Mathematics in 1989, he served as Chairman of the Department of Mathematics and Statistics until his death.

His interests were unusually broad, ranging over probability and statistics, pure and applied, stochastic geometry, stochastic processes—particularly in biological settings—non-associative algebra, and its applications to algebras arising in genetics, and history of mathematics in general and of probability and statistics in particular. But the unifying themes throughout most of his work were biological and ecological, and here the influence of his early years with Rothamsted and the Nature Conservancy shows clearly.

Philip's own unusual career development stood him in good stead in his teaching duties at Birkbeck, which specialises in providing part-time education to students who work by day, most of whom missed the chance to study full-time when young. He taught—successfully and with characteristic conscientiousness—across a broad range, in probability and statistics, stochastic processes, and mathematical areas such as numerical analysis. He supervised fourteen PhD theses to completion; his former students still active in academic life include Bob Gilchrist (1981), John Gates and Gillian Iossif (1983). It is a mark of Holgate's versatility that on his death, no fewer than three supervisors were needed to take over his pupils—in infinite Bernoulli convolutions, time series and non-associative algebras.

Philip Holgate was the complete professional. In addition to his research and teaching duties, he gave exemplary service to the academic community generally. He served many universities as external examiner, of statistics courses or of doctoral theses, the Royal Statistical Society (as Editor of the *Journal, Series B*, 1980–84), the London Mathematical Society (on Council, and as Book Reviews Editor) and the Biometric Society (on Council, and as Regional President). He made many invited visits to overseas universities, and spoke at numerous international conferences. He served the University of London, for example as Chairman of the Board of Studies in Statistics. In his administrative work at Birkbeck, he served, in addition to his terms as Head or Chairman of Department, twice as Dean of Science and three times as Vice-Dean.

Both his dedication and his wisdom and objectivity were widely appreciated, as witnessed by the following tribute from the former Master of Birkbeck, Baroness Blackstone. 'Philip worked selflessly for Birkbeck, not just for his own Department. He could always be relied upon to take a completely objective view of the interests being debated. He always argued thoughtfully, taking account of the common interest, and had more integrity than almost anybody I have ever worked with.' For further tributes, we refer to the earlier obituary by John Haigh (7).

As a person, Philip Holgate had an evident scholarship, wisdom and integrity that inspired respect in all who knew him. He had a great deal of 'hinterland'—broad cultural interests and reading, hobbies and relaxations, and a fulfilling personal life with his partner and collaborator, Susannah Brown. He had a good sense of humour, and his unassuming and gentle manner served, if anything, to emphasise the impact of his views, which were unfailingly considered and objective. His one failing was his suspect health, of which I never heard him complain. Characteristically, he chose to carry on with the work he loved, and die with his boots on. He is much missed by his former colleagues, in Birkbeck, London and in the wider world, and by all who knew him.

*Mathematical work*

Holgate's life as a researcher spanned the years from 1961 to 1993. In the working environment of his pre-academic years of 1961–67, he was steeped in biological and ecological problems arising in the real world, and devoted himself to attacking them with the armoury of mathematics, statistics, probability, modelling and numerical analysis. These early influences were decisive for his long-term interests: the great majority of his many papers were devoted to the application, direct or indirect, of mathematical and statistical methods to the life sciences.

The interaction between biology and the mathematics of chance, which so influenced Holgate's work and to which he contributed so much, is largely a twentieth-century development. Following the re-discovery of Mendelism in 1900, the early years of the century saw first a conflict between Darwinism and Mendelism, and then a synthesis between them (for background see, for example, Ewens (6, Section 1.1)). Prominent here were the three founding fathers of mathematical genetics, R. A. Fisher (1890–1962) and J. B. S. Haldane (1892–1964) in the UK, and Sewall Wright in the USA. By the time Holgate began his work, the field was maturing: good books existed for a mathematically-oriented readership—for example, Bartlett (2) and Bailey (1) on the population modelling side, and Moran (13) on the genetics side—and journals such as *Biometrika* and *Biometrics* (which published much of Holgate's work) had long been established.

A typical Holgate paper contains several ingredients, not uncommon separately but unusual together. It is inspired by a problem from the real world—and here his early years in Rothamsted and the Nature Conservancy were invaluable, as the unfathomable complexity of the natural world is an inexhaustible source of good problems. It contains some real mathematics—and here Holgate's mathematical ability, wide reading and breadth of interests found full scope. And it contains real data, subjected to whatever by way of statistical or numerical analysis might be necessary. It is worth noting that Holgate's early work was in the era of desk machines and punched cards. He acknowledged both assistance with hand computation, and access to the Rothamsted Orion Computer, in early papers, and ended his career in the modern era of statistical and mathematical packages and a computer on every desk.

There are striking parallels between the careers of Philip Holgate and one of the founding fathers of British statistics from the previous generation, Maurice Bartlett (1910–). In both, one sees the profound impact of the formative years being spent in the practical setting of a technical job outside academia. ('From 1934 to 1938 Bartlett was statistician at the I.C.I. Agricultural Research Station at Jealott's Hill, Berks., this being his first full-time job': Cochran (4, Section 1).) In both, one sees a spectrum of academic interests clearly showing the influence of this early experience, and extending through statistics, stochastic processes and probability, with emphasis on connections with the life sciences. The careers of both men illustrate beautifully the symbiotic relationship in the life sciences between the natural world as an unending source of questions, and mathematics as an unending source of tools with which to address them.

In his own list of his work, Holgate divided his publications into seven categories. We shall follow the same format here, and consider these in Holgate's ordering (see Holgate's bibliography at the end of this obituary).

1. *Statistical distributions and inference* [1, 2, 5, 9, 10, 11, 16, 25, 28, 32, 33, 34, 39, 53, 60, 70, 71]. Of the seventeen papers here, most are directly motivated by Holgate's interests in biology and ecology. Sometimes this is clear from the title of the paper, as in [2] on animal trapping, [11] on wildfowl counts, [39] on animal population size and [53] on 'abundance diversity', approached by information-theoretic methods. Sometimes an ecological question leads on to theoretical questions in statistics: thus [5] starts from questions on weakness in the tooth crowns of red deer, [9] on animal trapping, [10] on numbers of plants of two species in secondary rain forest in Trinidad, and [16] on estimating the basal proportion area in forestry.

2. *Stochastic geometry* [3, 6, 7, 8, 36, 40, 77]. The seven papers here are mainly motivated by questions on distances between plants, such as trees in a forest, arising out of Holgate's early work with the Nature Conservancy. One seeks information on the mechanism generating the data—whether it is a Poisson point process, for instance, and if so with what intensity—from observations of plant positions and inter-plant distances. In [40], for instance, written with Susannah Brown, one seeks to distinguish between a regularly planted forest that has been subject to random thinning and re-growth, and a natural or primaeval one.

3. *Stochastic processes in biology* [4, 12, 13, 17, 18, 19, 20, 31, 35, 43, 44, 46, 47, 49, 63, 72]. Of these sixteen papers, five [4, 12, 13, 44, 49] are on genetics. For example, in [12] the Sewall Wright effect—the tendency to homozygosity in a finite population—is studied, under self-fertilisation ('selfing') and random mating, using branching processes. In [13], the limiting proportion of a given allele in a growing population is studied, again under random mating and selfing. The Nature Conservancy and Rothamsted influences are clearly visible.

The remaining papers study population problems by the use of applied probability methods in general, and of branching process methods in particular. In [17], branching processes are used to study extinction probability, for annual, biennial and other reproductive patterns; in [35] they are used to study the spread or survival of family names, sometimes used in genetic studies as informative about the degree of inbreeding in the population. The interesting study [20] focusses on birth-and-death processes which explode—grow to infinity in finite time, motivated by the phenomenon of occasional explosive growth—for example, of rodent populations such as lemmings (Elton (5)); these surges are regarded as 'outbreaks', for example, from the point of view of pest control. Prey–predator interaction is studied in [43] and [63]; in the latter, predators can choose between two species of prey, and switch to the more abundant species. In [72], the Gompertz model of population growth is studied; here, one models the inhibiting effect of the finite carrying capacity of the environment on growth. The deterministic Gompertz model is made stochastic, and the effect of randomness on existence of equilibrium, and other characteristics, is studied.

4. *Non-associative algebras, including theory of genetic algebras* [21, 37, 51, 58, 64, 68, 73, 74, 78, 82, 83, 84, 87, 88]. Holgate's most enduring interest, and the subject of his deepest work, was in genetic algebras and their applications. Although Mendel's work on genetics dates back to 1865, it came to widespread attention only in 1900 (see Fisher's introduction to the Mendel centenary volume (12) for background to this 'rediscovery'). One of the earliest mathematical results on Mendelian genetics was the Hardy–Weinberg law of 1908, showing that under broad conditions, the

stationary distribution of genotypes is achieved in one generation of random mating (see, for example, (13, II)). This work was taken further by the great Russian probabilist S. N. Bernstein (1880–1968) in 1922–24 (3), leading to the Bernstein *stationarity principle*.

Algebraic methods are immediately relevant. If one gene has two alleles,  $A$  and  $a$  say, then the possible genotypes are  $AA$ ,  $aa$  and  $Aa$ . Heterozygotes  $Aa$  producing gametes (sex cells: eggs and sperm) do so in equal proportions, so the frequency distribution may be written  $\frac{1}{2}A + \frac{1}{2}a$ . A mating between two  $Aa$  types, symbolically written  $Aa \times Aa$ , thus becomes  $(\frac{1}{2}A + \frac{1}{2}a) \times (\frac{1}{2}A + \frac{1}{2}a) = \frac{1}{4}AA + \frac{1}{2}Aa + \frac{1}{4}aa$ , etc. Now our genetic inheritance comes to us symmetrically from our two parents (apart from sex-linked characteristics, considered below, and aspects such as mitochondrial DNA, which Holgate did not work on), so the operation  $\times$  is *symmetric*. It is not, however, *associative*: complete information about who has mated with whom in a family tree is crucially relevant, and thus all brackets need to be retained.

Non-associative algebras in general lack enough structure to be mathematically tractable, and one needs additional structural properties to be able to make progress. The two best-known types of non-associative algebras are *Lie algebras* (dating from work of Sophus Lie in the 1870s—the term is due to Weyl in the 1930s) and *Jordan algebras* (commutative algebras satisfying  $(x^2y)x = x^2(yx)$ , which date from work of P. Jordan on quantum mechanics in 1933—the term is due to Artin (14)).

After Bernstein, and J. B. S. Haldane in 1930, the next serious attempt to study mathematical genetics via non-associative algebras was made by I. M. H. Etherington in 1939, and later work up to 1951. Subsequent workers included R. D. Shafer in 1949, and H. Gonshor in 1960. There resulted a number of classes of non-associative algebras with genetic significance, including (in increasing order of generality) ‘special train algebras’ (Etherington), genetic algebras (Shafer, Gonshor), ‘train algebras’, baric algebras, and algebras with genetic realisation. To one side of this chain of inclusions, and motivated by Bernstein’s stationarity principle, are Holgate’s Bernstein algebras [42]. The study of these classes of algebras, and their genetic significance, has developed into an important area, to which Holgate was one of the major contributors. One of his key contributions [37] was to give a characterisation of Schafer’s genetic algebras alternative to the original one. Schafer’s approach was algebraically motivated, and its genetic significance was not transparent; Holgate gave an approach which made the genetic meaning clear. His work used the Borel–Serre theorem from Lie algebra theory. Another, earlier contribution was [21], on linearisation. The operations of genetics, involving the mating of two parents, are naturally quadratic. However, Haldane showed in 1930 that the methods of linear algebra can sometimes still be used—linearisation—on passing to a higher-dimensional space. Holgate [21] showed that ‘Haldane linearisation’ works for all genetic algebras. For background here, see Reed (15, Section 5.1), and Etherington’s comments in Subsection 5 below.

In a rather different line of work, published posthumously [87], Holgate used algebraic methods to study kinship relationships of the type occurring in anthropology.

5. *Applications of genetic algebras* [14, 22, 23, 24, 29, 42, 45, 48, 50, 52, 54, 55, 61, 65, 66, 67, 75, 79, 80, 81, 85, 86]. When Philip Holgate read his paper ‘Population algebras’ [54] to the Royal Statistical Society, the founding father of genetic algebras, Etherington, commented in the discussion (pp. 16–17) that ‘After

R. D. Shafer's seminal paper of 1949, all the best new ideas on the subject have been introduced either by Professor Holgate himself, or by his disciples'. Etherington particularly praised his method of linearisation (see above). Other highlights of Holgate's work on genetic algebras include the following.

[14] Genetic algebras associated with polyploidy (populations with more than two sets of chromosomes: algebraicisation of earlier work of Haldane, Moran and others).

[22] Jordan algebras in population genetics—again casting light on the genetic significance of algebraic work by Shafer.

[29] Study of sex linkage.

[44] Algebraic study of Bernstein algebras.

The introduction of algebraic methods into an area where there is scope for them always results in an increase in power, and the area of genetics was no exception. On the other hand, continual contact with problems from the real world is needed to keep such an area well motivated and focussed. Holgate's mastery of both the algebraic and the genetic sides was one of the keys to his success in the area; his sheer mathematical ability, and his enduring enthusiasm for the subject over nearly thirty years, were others. It is perhaps surprising that the textbooks current when Holgate began his work made so little use of genetic algebras (Moran (13), for instance, makes no use of them). However, as the importance of the subject became increasingly appreciated, good monographs and surveys were written, in which Holgate's work is fully recognised. See, for instance, Lyubich (10, 11), Wörz-Busekros (16) and Reed (15).

6. *Probability theory* [15, 26, 27, 30, 38, 41, 57, 76]. Of these eight papers, four [30, 41, 57, 76] are on random power series. Holgate's interest here was inspired by work of Ludwig Arnold in 1966 and 1967; the subject has roots in Polish work, by Steinhaus in 1930 and Zygmund in 1933. For background and references, see Kahane (9, Chapter 4). The other papers include an extension of Pólya's theorem on the transience–recurrence dichotomy for random walks to walks with dependence between steps [15], an early contribution to the subject of random graphs [26], and work on probabilistic number theory [38], using results from Halberstam and Roth (8, III).

7. *History of mathematics; philosophy of science* [56, 59, 62, 69, 89, 90]. Holgate was very well read and had wide interests, characteristics which fit well with an interest in the history of the subject. He made three contributions to the well-known 'Studies in the history of probability and statistics' series in the journal *Biometrika*: on Buffon [56], on glimpses in Waring and Sylvester of the theory of random polynomials [62], and finally on the work of the Polish school—Steinhaus, Zygmund, Marcinkiewicz, Kac and others—on independent functions, for them the bridge between analysis and probability, between the World Wars [90].

#### *Postscript*

This obituary has been regrettably delayed; this is due to oversight by the London Mathematical Society in the first instance, and overwork by myself in the second. However, this provides the opportunity to tie up some loose ends left by Philip Holgate's untimely death. It is sad to record that the College's reaction to his death was to attempt to close his Department, and then to change it to a



Department of Statistics only (1995); it is pleasant to record the re-establishment of the Department of Mathematics and Statistics (1998). I succeeded Philip Holgate as fourth Professor of Statistics at Birkbeck in 1995 (the previous holders of the chair were D. R. (now Sir David) Cox and Harold Ruben), and the Department was augmented by Dr Rüdiger Kiesel's appointment in 1996. The research students who lost their supervisor on Philip Holgate's death have now completed their studies, and his bibliography is now complete. There are few things as durable as a scientific reputation, and Philip Holgate's reputation thrives as much as, and as part of, the subject to which he gave his life's work.

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