1 Introduction

When I was a student taking courses in the mathematics of life insurance, or actuarial science, the textbook that we used, entitled *Life Contingencies*, made a brief historical reference to the origins of some mathematical models describing human mortality.[1] It was mentioned that Abraham De Moivre (1667–1754) proposed a model in 1724 in which the number of people living at any age decreased in a linear fashion until by age 86 no one was left alive. He used this model to simplify the calculation of the values of life annuities. Later I learned that 1725 was the correct date. When we were taught about the centrality of life tables to insurance calculations we learned in passing that the astronomer Edmond Halley (1656–1742) had constructed the first life table. My awareness of the connection between insurance and these two people was strengthened when I read some histories of statistics. One modern historian of statistics describes De Moivre's 1725 book on life annuities as "the first textbook on life insurance mathematics."[2] Delving into this a little further, I found that professional actuaries writing in the nineteenth century described all the mathematical progress that had been made in the techniques that they used to value life insurance contracts, including the work of Halley and De Moivre.[3] These early histories look like attempts to give legitimacy to the newly formed actuarial profession, as well as providing advertisements for the sound scientific basis on which the insurance industry was assumed to rest in the latter half of the nineteenth century.

It was a revelation to me to find that historians of science and social historians did not share this viewpoint. Through careful research these historians have shown a disconnection between mathematicians and the insurance industry as it developed in the eighteenth century. One of these historians, Lorraine Daston, has described the situation succinctly:[4]

> Despite the efforts of mathematicians to apply probability theory and mortality statistics to problems in insurance and annuities in the late seventeenth and early eighteenth centuries, the influence of this mathematical literature on the

https://doi.org/10.1017/9781316282229.002 Published online by Cambridge University Press

voluminous trade in annuities and insurance was negligible until the end of the eighteenth century.

Another historian, Eve Rosenhaft, has described it as a puzzle:[5]

> The historical literature on the development of life insurance has paid close attention to the relationship between risk-taking and risk avoidance in the transition to economic and social modernity. A central puzzle in the history of life insurance is this: the market in life insurance emerged in England early in the eighteenth century, *before* the calculus was developed that would make it possible to apply known mortality rates to the calculation of premiums and benefits. After mid-century, a second wave of foundings of life insurance and pension funds coincided with the existence of more or less reliable systems of calculation, but their founders ignored them, often defying good advice. So what ought to have been systems for the 'taming of chance' in everyday life resisted the influence of the scientific discourses – statistics and probability – that would have made them effective.

This certainly is a puzzle. If the mathematicians who applied probability theory to the valuation of standard life contingent contracts in today's life insurance industry were not actually involved in the insurance industry, what were they doing?

The historical evidence, on which historians such as Daston and Rosenhaft have relied, as well as the actuaries writing their own history, is for the most part the corpus of books that mathematicians in the eighteenth century wrote about the subject, mainly on life annuities. Were these book writers writing merely for one another? Or, were they writing in the hope that the insurance and annuity industry would take note of them? Any hopes about these budding industries were dashed as those in the industry apparently made little use of their work, as Daston and Rosenhaft have demonstrated.

This all became even more of a puzzle when I discovered that there was a perceived demand for these books as seen from the point of view of their printers or publishers. John Nourse was the publisher for two of the many writers on life annuities in the eighteenth century. In September 1741 Nourse paid Thomas Simpson, author of several mathematical books, 17 guineas for the copyright to his book *Doctrine of Annuities and Reversions*, published in 1742. Nourse also paid the mathematician and accountant James Dodson £25 and 4 shillings for the copyright to the third volume of his *Mathematical Repository*, which was devoted almost entirely to problems in life annuities.[6] Dodson was also to receive 25 free copies of the book. Nourse gave Dodson an advance of 10 guineas at least a year and a half prior to the *Mathematical Repository*'s appearing in print in 1755. The amounts Nourse paid for the copyrights seem to have been based on the number of printed sheets

of paper that went into the book. As laid out in the contract, he paid Dodson £1, 11 shillings and 6 pence per sheet. For Simpson, we can compare the 17 guineas he received for his *Doctrine of Annuities and Reversions* to the 73 guineas he received in 1750 for *Doctrine and Application of Fluxions*. At 576 pages, the book on fluxions is 4½ times as long as the 128-page book on annuities, while the copyright paid is just under the multiple of 4½. To put the amounts Dodson and Simpson received into perspective, a clerk in the late 1720s, working in a customs house tallying the collection of duties on imports and exports, made £40 to £50 per annum.[7] Based on that comparison, the copyright money that Nourse paid is not an insignificant amount as it might seem today. Clearly, Nourse was taking a financial risk in the publication of these books and thought he would make a profit from them. There must have been some market for these books – someone was buying them for some reason, or at least Nourse thought so.

For those unfamiliar with predecimal British currency here is a short account of it. Up until "Decimal Day," on February 15, 1971, British currency was expressed in pounds, shillings, and pence with the symbols l or £, s, and d, respectively. There were 20 shillings to the pound and 12 pence to the shilling. There were several other denominations in the currency (farthings and florins, for example), but the only other denomination that is relevant to annuities was the guinea, which was 21 shillings. In 1741 Nourse paid Simpson 17 guineas, or £17 17s. Dodson's contract with Nourse for the *Mathematical Repository* was £1 11s 6d per sheet. Typically when carrying out annuity valuations mathematicians decimalized their calculations. Sometimes they left their final result in decimal form and sometimes they converted the decimal value back to pounds, shillings, and pence.

A short solution to the enigma of the lack of interest shown by the insurance industry and yet considerable interest from book publishers is that mathematicians were doing something other than trying to set fair values to annuity and insurance contracts for the insurance and annuity industry. In fact, the mathematical activity up to about 1760 is the opposite side of the coin of the Daston–Rosenhaft insight. The mathematicians showed very little interest in the insurance and annuity industry. A hint at a little more detail of what they were doing is given in a 1752 paper in the *Philosophical Transactions* by the author of the 1755 *Mathematical Repository*, James Dodson.[8] Making no mention whatsoever of the insurance industry or government life annuity schemes, Dodson described the ubiquitous nature of life annuities in British society at the time. Many of the sorts of situations that he describes involving life annuities are treated in a general way by the mathematical writers on

life annuities in the eighteenth century. These situations are related to something that was central to the British economy of the eighteenth century – property. Many of the patrons of these mathematicians had interests in property.

Since the Norman Conquest of 1066 all land in England technically has been owned by the Crown but held by individuals through freeholds or leaseholds.[9] Someone who holds a freehold (also called an estate in fee simple) has complete use of the estate and the rights to dispose of it. The freeholder holds the land in perpetuity. By the eighteenth century a variety of types of leaseholds had evolved out of the feudal system.[10] As there are today, there were leases for a fixed period of time, or in the jargon of the eighteenth century, these were estates held for a term of years absolute. The normal term was 21 years, but it could be as long as 99 years or perhaps more. Unlike what exists today, there were many leases that were life-contingent, typically running for the lifetimes of the persons named in the lease. There were copyhold estates that evolved from the manorial system, as well as other forms of life leaseholds. Leases for fixed terms were typical in the east of England and leases that were life-contingent were typical in the west of England.[11] Lands held by dioceses of the Church of England and by colleges at Oxford and Cambridge, as well as some endowed charities, were let to others as life leaseholds.

Income from estates held for a term of years absolute is a form of fixed-term annuity or what is often called an annuity certain. Income from a freehold is a perpetuity. The value of the estate would involve, in part, finding the present value of an annuity certain. In terms of valuation, a long-term lease was treated as equivalent to a freehold. The present value of rents from a 99-year lease is nearly numerically equivalent to the present value of a perpetuity.

The common copyhold or life leasehold was let for the duration of the life of the last survivor of up to three persons named on the lease. For a lease on three lives, those named might be the farmer working the land, his wife, and his oldest son. If one of the persons named on the lease died – the term used in the eighteenth century is that one of the lives dropped – then that person could be replaced by another on payment of a sum of money to the landlord, commonly called a fine. This leasing arrangement provided a means by which land could remain within a family for many years. It was also a form of life annuity for the lessor, but based on the lives of his tenants, not on his own life. Copyhold leases usually involved a large fine payable to enter the lease and similarly large fines on renewal, with a nominal annual rent.

During the eighteenth century there was a move among estate holders to convert their life leases to leases for a fixed period of time. Conversion allowed the landlord more flexibility in trying to maximize his rents and more flexibility with the development of the land. Life leases on church lands remained in place until well into the nineteenth century. The risks associated with this kind of life annuity are mainly tied up in the ability of the lessee to pay the rent. On non-payment of rent, the tenant could be evicted and the land rented to someone else to continue the annuity for the lessor. The land is always there, unlike a monetary fund set up by those operating a life annuity company that is not backed by land.

Other kinds of arrangement with respect to land might have life-contingent provisions. For a gentleman who holds a life tenancy on lands which he sublets to others for his source of income there might be dower rights attached. Should this tenant for life predecease his wife, she would have the right to half the estate during her lifetime. A similar situation could apply to a landowner. In this case it comes in the form of a marriage settlement which conveys a portion of the land to the widow for her use on the landowner's demise. On her death, the land would revert to the heirs of the estate. Income from dower rights or a marriage settlement is a form of life annuity for the widow.

If the mathematicians did not devise methods for the valuation of life contingent contracts until the 1690s or later, how were they valued before then? There was a long-established tradition for the valuation of leases for lives, coming out of a statute of Henry VIII related to leases which lumped together leases of 21 years and leases for three lives.[12] The tradition was to equate the length of a life, which is uncertain, to a fixed period of time, specifically 21 years to the last survivor among three lives, or 7 years to one life. Consequently, any valuation method that can be applied to fixed-term leases can also be applied to the valuation of leases for lives. Should a life drop in a lease for three lives, the fine to renew the lease was based on tacking an extra 7 years on to the end of the lease. When Halley carried out the proper valuation of life annuities in 1693, using the life table he constructed, the values turned out to be quite different from what had been established by custom. Custom was hard to break. Many were content with the "old ways." Despite advances made by mathematicians in the valuation of life annuities by the 1750s, in his 1752 paper Dodson hinted at a continuing tension between calculation and custom. There needed to be compelling reasons to discard the "old ways" and to adopt the new technology.

Closely linked to the valuation of life annuities, as well as life insurance, is the life table. Construction of a life table requires proper data, and these data were not always available during the eighteenth century.[13] And when the data were available there was the question of their soundness, reliability, and applicability. Near the end of the century, Jeremy Bentham summed up the problem succinctly and put forward the idea in some notes on annuities that it should be the government that should take responsibility for the collection of such data.[14]

> Mathematicians though themselves unerring may lead men into error, and will do so whenever the stock of data they have to work upon is imperfect or erroneous: but to furnish them with proper data, and proper data is not their own province but is a government concern.

The move to central registration of births and deaths and to a regular census to determine population size did not occur until the middle of the nineteenth century.[15] By the beginning of the nineteenth century insurance companies dealt with this problem by constructing life tables based on their own mortality experiences or on a mortality study carried out in a locality where sufficient information was thought to have been collected. Throughout the eighteenth century, mathematicians working as consultants for life-contingent contracts relied on published tables that were possibly "imperfect or erroneous," in the words of Bentham.

Once the life-insurance blinkers are removed when examining mathematicians and their work in life annuities during the seventeenth and eighteenth centuries, a whole new world of activity opens up. I have found several manuscripts and printed sources that support mathematical activity in life-contingent contracts related to property throughout the entire eighteenth century. It has been a fruitful paper chase. The result of this chase has provided new insights beyond the mathematics in standard works on life annuities such as De Moivre's *Annuities upon Lives* and Simpson's *Doctrine of Annuities and Reversions*. Throughout the book I will try to put this new world of mathematical activity, including the eventual adoption of it by the insurance and annuity industries, into its historical context.

One of the stumbling blocks in the valuation of life-contingent contracts is that it requires an enormous amount of calculation without the aid of a computer or even a mechanical calculator. This inspired mathematical ways to get around the burden of calculation, as well as the construction of tables to ease the burden. Since I have a computer at my disposal, I was inspired to look more closely at many of these

calculations. When I let the numbers speak I got further insights into what these mathematicians were doing, the type of problems they solved, and the issues they faced.

To understand the mathematical work on life-contingent contracts, it is necessary to start at the beginning – the valuation of annuities as they relate to land in the seventeenth century.