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A non-linear boundary value problem arising in the theory of thermal explosions

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When a heat-producing chemical reaction takes place within a confined region, then under certain circumstances a thermal explosion will occur. In investigating from a theoretical viewpoint the conditions under which this happens, it is necessary to study the behaviour of the solution of a certain non-linear parabolic initial-boundary value problem.

A frequently used approach is to study the problem indirectly, by investigating whether positive steady-state solutions exist; the underlying assumption is that positive steady-state solutions exist if and only if a thermal explosion does not occur. The main theme of this thesis is the development and application of an alternative direct approach to the problem, involving the construction of upper and lower solutions for the parabolic problem and the application of appropriate comparison theorems. The assumption here is that a thermal explosion will not occur if and only if the solution of the parabolic problem remains bounded for all positive time.

Following three chapters of introductory material, Chapter 4 contains a survey of some of the important known results concerning the existence of positive steady-state solutions, especially those dealing with the effect on the theory of different assumptions as to the rate at which heat is produced in the reaction.

The comparison theorems that are used in the alternative approach, which are modified versions of known results, are proved in Chatper 5.

In Chapter 6, the equivalence of the two criteria mentioned above for

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the occurrence or non-occurrence of a thermal explosion is established under fairly general conditions. Also in this chapter, a critical value λ^* is defined for a parameter λ appearing in the problem, such that a thermal explosion will not occur if the value of λ is smaller than λ^*, but will occur if the value of λ is greater than λ^*.

In Chapter 7, upper and lower solutions are constructed for the time-dependent problem under a variety of assumptions as to the rate at which heat is produced in the reaction, and these are used to obtain a number of theorems concerning the behaviour of the solution of the problem, especially as the time variable tends to infinity. The information obtained from these theorems is related to and compared with that known from investigations of the existence of positive steady-state solutions. In conclusion, a theorem is proved concerning the effect of reactant consumption on the theory. This is examined in the light of some recent research, and an apparent defect which is thereby revealed in the usual criteria for the occurrence of a thermal explosion is discussed.

The theorems of Chapter 7 are employed in Chapter 8 to obtain rigorously derived bounds for the critical parameter λ^*, for a number of different shapes of the region in which the reaction takes place; these bounds are compared with known estimates for λ^* obtained using an empirically derived formula.

The thesis concludes, in Chapter 9, by using the methods of Chapters 7 and 8 to obtain some results concerning the case where the boundary condition is non-linear.