AN A PRIORI MODEL FOR THE REDUCTION OF NUTATION OBSERVATIONS: $KSV_{1994.3}$ NUTATION SERIES

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Abstract. We discuss the formulation of a new nutation series to be used in the reduction of modern space geodetic data. The motivation for developing such a series is to develop a nutation series that has smaller short period errors than the IAU 1980 nutation series and to provide a series that can be used with techniques such as the Global Positioning System (GPS) that have sensitivity to nutations but can directly separate the effects of nutations from errors in the dynamical force models that effect the satellite orbits. A modern nutation series should allow the errors in the force models for GPS to be better understood. The series is constructed by convolving the Kinoshita and Souchay rigid Earth nutation series with an Earth response function whose parameters are partly based on geophysical models of the Earth and partly estimated from a long series (1979-1993) of very long baseline interferometry (VLBI) estimates of nutation angles. Secular rates of change of the nutation angles to represent corrections to the precession constant and a secular change of the obliquity of the ecliptic are included in the theory. Time dependent amplitudes of the Free Core Nutation (FCN) that is most likely excited by variations in atmospheric pressure are included when the geophysical parameters are estimated. The complex components of the prograde annual nutation are estimated simultaneously with the geophysical parameters because of the large contribution to the nutation from the S_1 atmospheric tide. The weighted root mean square (WRMS) scatter of the nutation angle estimates about this new model are 0.32 *mas* and the largest correction to the series when the amplitudes of the ten largest nutations are estimated is 0.17 ± 0.03 *mas* for the in phase component of the prograde 18.6 year nutation.

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1. Introduction

The formulation of modern nutation series such as the IAU 1980 nutations series (Wahr, 1981) are based on the (almost) linear response of the Earth to torques applied to the mantle of the Earth. These torques primarily arise from the luni-solar forces acting on the equatorial bulge of the Earth but there are significant contributions from the planets and from torques associated with the motions of the fluid parts of the Earth (atmosphere, oceans and fluid inner core). There have also been proposed contributions from the solid inner core as well (Mathews *et al.*, 1991a, de Vries and Wahr, 1991). The linear nature of the problem is used to separate the formation of the nutation series into two distinct parts: (a) the derivation of the nutations of a rigid Earth and (b) the derivation of the response function of the Earth which gives the ratio of the nutations of a "real" Earth to those of the rigid Earth. Much of the error in the IAU1980 nutations is due to imperfect knowledge of the geophysical parameters that effect the response function of the "real" Earth. The IAU 1980 nutations also suffers from the effects of truncation to 0.1 *mas* and the neglect of several second-order terms in the derivation of the rigid series.

In conventional formulations of the transformation between an Earth fixed frame and inertial frame, the only nearly diurnal motions are associated with nutations. However, recent analyses of space geodetic data have shown that other non-nutation related motions of the Earth rotation axis and the rate of rotation exist in the diurnal and semidiurnal bands. These motions arise from ocean tidal effects and tend to be dominated by tidally driven currents (Sovers *et al.*, 1993, Herring and Dong, 1994, Watkins and Eanes, 1994). For a complete representation of the transformation from Earth fixed coordinates to inertial space these additional motions whose amplitudes can reach 0.5 *mas* need also to be included. The data set used here include these other motions in the analysis of the space geodetic data.

A general representation of a nutation series is given by Mathews *et al.* (1991a) (see also Wahr (1981), de Vries and Wahr (1991)). The amplitude of the nutation at frequency σ is given by

$$\zeta_{Earth}(\sigma) = \eta_m(\sigma)\zeta_{rigid}(\sigma)$$

where $\zeta_{rigid}(\sigma)$ is the nutation amplitude for the rigid Earth with the same flattening as the real Earth; $\eta_m(\sigma)$ is the response of the Earth; and $\zeta_{Earth}(\sigma)$ is the nutation of the real Earth. The function, $\eta_m(\sigma)$, depends on many geophysical parameters of the Earth but is most sensitive to the elastic properties of the Earth, and the flattening of the Earth and its fluid core. At this time geophysical models of the Earth are not accurate enough to reliably determine $\eta_m(\sigma)$ mainly because of non-hydrostatic forces in the Earth, the effects of mantle anelasticity, and the effects of ocean tides. However, the form that $\eta_m(\sigma)$ should take is probably well represented by geophysical theory. The derivation of the nutation series discussed here is based on estimating the parameters in $\eta_m(\sigma)$ that are not well known or are effected by ocean tides and mantle anelasticity.

2. Series derivation

The series derived here is based on the Kinoshita and Souchay (1990) rigid Earth nutations series, with some slightly motivations discussed below, and the NASA Goddard Space Flight Center (GSFC) estimates of nutation angles from the analysis of VLBI data between 1979 and 1993. We refer to this series as KSV_{1994}. (The specific version we discuss here is the $KSV_{1994.3}$ series i.e., the third of a series of different types of analyses whose details are given below).

Before using the Kinoshita and Souchay (1990) rigid Earth nutation series we scaled the series by 0.9999404 to account for the rate of change of the nutation in longitude observed in VLBI and Lunar Laser Ranging (LLR) data (Herring *et al.*, 1991, Dickey *et al.*, 1994) of approximately −0.3 "/century that we interpret as arising from luni-solar precession and thus an error in the dynamical ellipticity of the Earth. We also combined duplicated terms in the series (each arising from different sources but with the same fundamental arguments) and we removed semidiurnal nutations due to the triaxiality of the Earth since these are implicitly included in the prograde diurnal polar motions terms in the VLBI analysis (see Herring and Dong, 1994) for discussion). The planetary nutation contributions to the nutation angles from Kinoshita and Souchay (1990) were removed from the observed nutation angles before they were used in the analysis.

Two other contributions were removed from observed nutations angles before they are analyzed: the annual modulation of the geodetic precession discussed in Fukushima (1991) (annual nutation in longitude 0.15 *mas*), and corrections to the 18.6 and 9.3 year nutations for planetary tilt effects (Williams, 1994) (largest correction 0.14 *mas* for nutation in longitude). In the analysis presented here no corrections were applied for ocean tides or mantle anelasticity. Instead, it was assumed that these corrections could be absorbed into the parameters of $\eta_m(\sigma)$.

The form of $\eta_m(\sigma)$ used here is based on that of Mathews *et al.* (1991a) with a small modification to better represent the effects of ocean tidal loading. $\eta_m(\sigma)$ is written as

$$\eta_m(\sigma) = R + R'(\Omega + \sigma) + \sum_{\alpha} \frac{R_a}{\sigma - \sigma_\alpha}$$

where Ω is the rotation rate of the Earth and we take σ to be the frequency of the nutation as seen on the rotating Earth; R and R' are constants derived from geophysical theory; and the sum over α represents the effects of resonances in the rotation of the Earth with strengths R_α and resonance frequencies σ_α. In the Mathews *et al.* (1991a) theory there were four resonances: the Chandler wobble, denoted CW; the retrograde free core nutation, RFCN; and two resonances associated with the solid inner core denoted by PFCN and ICW. The CW and ICW have resonance frequencies far from the diurnal band and have little effect on the nutations. The RFCN has a marked effect, and the PFCN which has a much smaller strength than the RFCN could have observable effects. After extensive searching for the PFCN, we could find no detectable evidence for this resonance. The fit to the VLBI data when the resonance was included were worse then when its strength was set to zero, thus indicating that resonance parameters are sufficiently far from the geophysically determined values, that we have been unsuccessful in isolating this resonance. In the analysis here we have set the strengths for the inner core resonances to zero.

The parameters, treated as complex values, estimated from the VLBI data were R, R_{RFCN}, and σ_{RFCN}. Trail analyses were performed with R' estimated, but we found slightly better fits to the VLBI data if more explicitly included the effects of loading phenomena. To include the load effects from ocean tides we modified the nutational response of the Earth to the following form:

$$\zeta_{Earth}(\sigma) = \eta_m(\sigma)(1 + \nu\frac{(\sigma + \Omega)}{\Omega})\, \zeta_{rigid}(\sigma)$$

where the complex quantity ν represents the effects of load (see for example Wahr and Sasao, 1981). Our analyses shows that the estimates of ν is highly correlated with estimates of R' and simultaneous estimation of these two complex parameters tends to be unstable and we have therefore estimated ν and keep R' fixed at its geophysically determined value. The values we used are from Mathews *et al.* (1991b).

The formulation above assumes that the forcing of the nutations is either proportional to the rigid Earth nutation or the torque applied to the Earth and that the response is modified by the elastic and anelastic parameters and the resonances of the Earth with the primary resonance being that due to the fluid core. There is however one nutation frequency for which these assumptions are probably not valid. The prograde annual nutation is likely to be affected by the thermal S_1 atmospheric tide which is much larger then the amplitude of the amplitude of the simple tidal forcing (see for example Chapman and Lindzen, 1970). We have therefore explicitly estimated the amplitude of this nutation while estimating the parameters of the resonance forcing.

The primary role of the RFCN resonance is to modify the amplitudes of the forced nutations but the resonance itself can also be excited by quasi-random excitations most likely from the atmospheric pressure variations (Sasao and Wahr, 1981). The amplitude and phase of this freely excited mode will vary with time (in much the same way that the Chandler wobble amplitude and phase does) and we therefore included estimates of the freely excited RFCN in two years intervals after 1984 with one estimate for the 1979-1984 interval because of the sparseness and lower quality of the VLBI data during this time.

The VLBI data set analyzed was the NASA/GFSC analysis of the data from 1979 to 1993 and consisted of 1840 pairs of estimates of nutation angles in obliquity and longitude. We root sum square (RSS) added 0.25 and 0.60 *mas* to the standard deviations given in the GSFC analysis to bring the χ^2 per degree of freedom of the postfit nutation angle residuals to approximately unity (see Herring *et al.*, 1991). From this data set we estimated 26 parameters comprised of 6 complex components for the time dependent RFCN amplitude, offsets and rates for the nutations in longitude and obliquity, the amplitude of the prograde annual, and the four complex parameters R, R_{RFCN}, σ_{RFCN}, and ν. The WRMS scatter of the postfit nutation angle residuals was 0.32 *mas*.

3. Results and Discussion

The fit to the VLBI nutation estimates was almost the same when only the 26 parameters listed above were estimated compared to a trial analysis in which corrections to the coefficients to the nutation series were estimated (0.321 *mas* versus 0.318 *mas*) indicating that the above formulation is adequate for representing the VLBI nutation angles and is likely to be a good predictor of future nutation angles.

The estimated rates of change of the nutations in longitude (interpreted as a change to the IAU 1976 precession constant) and in obliquity were -0.298 ± 0.01 "/cent and -0.024 ± 0.005 "/cent. The latter value is in very good agreement with that recently determined theoretically by Williams (1994).

The correction to the prograde annual nutation (relative to that value predicted by the new series) was $-0.01 - \iota 0.09 \pm 0.01$ *mas*. The amplitude of the RFCN varied from 0.34 ± 0.06 *mas* (1979-1984 interval) to 0.11 ± 0.02 *mas* (1992-1994 interval) and showed a steady decrease in amplitude over the 14 year interval of the VLBI data. When corrections to the coefficients of the series were estimated, all amplitude corrections were less than 0.016 *mas* except for the 18.6 year nutation for which the corrections were $-0.17 + \iota 0.11$ *mas* and $0.08 - \iota 0.13$ *mas* with uncertainties of 0.10 *mas* in both cases.

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