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Probability of Paternity in Rh Blood Groups

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Essen-Möller (1938) derived the so-called Essen-Möller's formula from Bayes' theorem for the probability of paternity. Since this formula was developed a number of articles on the same problem have been published. These are concerned with the following three aspects: The first deals with analyses of the mode of inheritance of individual characters and derivation therefrom of the X and Y values of the Essen-Möller's formula. The second is concerned with application of this method for practical cases and statistical analyses of data obtained for estimation of the grade of fitness of the theoretical values to practical cases (Wichmann & Tuppa, 1954). The third is concerned with discussions on the methodological problems related to Essen-Möller's formula, which require very careful use of the method for practical purposes (Ludwig & Wartmann, 1952; Ludwig, 1954/55).

Essen-Möller explained the principle of the method as applied to two genetically distinct characters, one that follows Mendelian laws like the blood groups and the other which does not. The example used by him for the former is the probability of paternity of ABO blood groups.

A number of studies have been made on the probability of paternity concerning blood groups. For instance, Komatu (1939) obtained the probabilities for ABO, MN, Qq $\pm$ blood groups and S system and their values in the Japanese, and advanced further a general theory on the probability of paternity (1952). Elbel & Sellier (1955) presented the tables for the calculation of Essen-Möller's formula for A_1A_2BO, MN, P and Rh blood groups. For Rh blood groups probabilities of paternity of three systems, Cc, Dd and Ee, were obtained separately. Similar tables were procured for Rh blood groups in the Japanese population by Uetake & Yokoyama (1954). It is necessary to keep in mind that the hereditary traits should be genetically independent from each other for application of the Essen-Möller's formula. Linked characters cannot be treated as independent and must be treated as a unit. According to Fisher (Race & Sanger, 1954) the three allelomorphs of Rh blood groups, Cc, Dd and Ee, are located on the same chromosome. Their linkage is very close and the crossing over is assumed to be very rare. Therefore, these three allelomorphs may be taken as one hereditary unit for the present purpose. Consequently, the units used for cal-

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culation of Essen-Möller's formula are chromosomes on which these allelomorphs are located and not the individual genes, C, c, D, d, E and e.

If we are to apply the table derived by Elbel & Sellier or by Uetake & Yokoyama to calculate the probability of paternity with Essen-Möller's formula (1), we must use only one of the following three formulas (2), (2') and (2'').

$$\text{Probability of paternity} = \frac{1}{1 + \dfrac{Y}{X}}; \tag{1}$$

$$\frac{Y}{X} = \frac{Y_1}{X_1} \cdot \frac{Y_2}{X_2} \cdots \frac{Y_{Cc}}{X_{Cc}} \cdots \frac{Y_n}{X_n}, \tag{2}$$

$$\text{or} = \frac{Y_1}{X_1} \cdot \frac{Y_2}{X_2} \cdots \frac{Y_{Dd}}{X_{Dd}} \cdots \frac{Y_n}{X_n}, \tag{2'}$$

$$\text{or} = \frac{Y_1}{X_1} \cdot \frac{Y_2}{X_2} \cdots \frac{Y_{Ee}}{X_{Ee}} \cdots \frac{Y_n}{X_n}, \tag{2''}$$

where Y/X stands for the combined critical value and Y_j/X_j ($j = 1, 2, \ldots,$ Cc or Dd or Ee, ..., n) for the critical value of individual characters. If we make use of only one of these three systems, the other two will be excluded from calculation, even though they are all immunologically tested, because the latter systems always link with the former and are not genetically independent from each other. Therefore, it is not correct to use the following formula,

$$\frac{Y}{X} = \frac{Y_1}{X_1} \cdot \frac{Y_2}{X_2} \cdots \frac{Y_{Cc}}{X_{Cc}} \frac{Y_{Dd}}{X_{Dd}} \cdot \frac{Y_{Ee}}{X_{Ee}} \cdots \frac{Y_n}{X_n} \tag{3}$$

Selection of one among three formulae should be made before results of testing of Rh blood groups become clear so that selection is made free from influence of the results. Therefore applications of the tables prepared by Elbel & Sellier and by Uetake & Yokoyama are limited due to the fact that the three systems are to be handled separately. If, on the other hand, chromosomes are treated as hereditary units, three systems can be used for the calculation at the same time and the grade of exactness or fitness is increased. Calculations in this method are more complicated than when the tables procured by those authors are used. Probabilities of paternity for all theoretically possible mother-child-father combinations are so complicated that it is desirable to divide the table into probabilities one each for H, K and Y. H (or H' or H'') stands for the probability of mother-child-father combination, K (or K' or K'') for that of mother-child combination and Y or (Ph-n) for the frequency of Ph-n (phenotype represented by class number n).

A number of different chromosomes are known to be concerned with Rh blood groups and some of them are found frequently while others only rarely in the population. Fort he sake of simplification six common genes C, c, D, d, E and e and eight chromosomes containing them are used in this paper. Frequencies of these chromosomes are denoted as s, t, u, v, w, x, y and z. The eight chromosomes and their frequencies are given in Table I. Although very rare chromosomes such as CdE and CDE are considered to be almost absent, these are not omitted from calculation. Since the tables presented herein include calculations for chromosomes which appear to occur but rarely, as far as known to date, these tables can be applied to all populations with different chromosome frequencies. From these eight chromosomes thirty eight different kinds of genotypes are theoretically expected, of which eight are homozygous while other twenty eight are heterozygous individuals.

For calculation of probabilities of paternity the following assumptions are made:

1. Chromosome distributions are in the state of equilibrium.
2. Marriage of parents is at random.
3. Mutations of Rh genes and crossing over of chromosomes are negligible.
4. Selections due to the incompatibility of Rh blood groups between mothers and children are also negligible.

Method

Principle for calculation of the probability of paternity in Rh blood groups advanced by the author was already communicated briefly elsewhere (1955). The principle will be reiterated in details below.

1. Cases where five anti-sera (anti-C, -D, -E, -c, -e) are used for testing:

When these five anti-sera are used, thirty six genotypes may be classified into eighteen phenotypes. Table 2 shows the class numbers of these phenotypes in the first column, and the relation between phenotypes and corresponding genotypes in the second and third columns. The frequency of each phenotype can be obtained by chromosome counting as shown in the fourth column of the table.

Now, a table for H, the mother-child-father combination, is constructed as follows: Since a series of thirty six different genotypes are made of combinations of eight different chromosomes, 1296 (36×36) different matings are expected theoretically. Among them 36 matings are identical in genotype. Thus squaring the frequencies of all genotypes, probabilities of all possible mother-father combinations are obtained.

Taking into account the genetics of Rh blood groups types and proportions of the offsprings from each mating can be figured out. Furthermore from the probabilities of mother-father combinations the probabilities of mother-father-child combinations, which is equal to that of mother-child-father combinations represented by H can be calculated. For instance, if the genotypes of mother and father are cDE/cdE (Ph-5) and CDe/cde (Ph-8), respectively, the product of the frequencies of the two genotypes indicates the probability for this mating.

$$uw \times st = stuw.$$

From this mating four different genotypes, cDE/CDe (Ph-11), cDE/cde (Ph-6), cdE/CDe (Ph-11) and cdE/cde (Ph-3), are expected in the offspring with the same probability, 1/4 stuw. Thus the probabilities are obtained for all mother-child-father genotype combinations.

As Table 2 shows Ph-5 consists of two different genotypes, cDE/cDE and cDE/cdE; Ph-8, of three, CDe/cDe, CDe/cde and cDe/Cde; and Ph-6, of three, cDE/cDe, cDE/cde and cDe/cdE, in the phenotype combination, where mother, child and father belong respectively to Ph-5, Ph-6 and Ph-8, some different kinds of mother-child-father genotype combinations are contained. Let the probability of these phenotypes combination be represented by $H_{5 \cdot 6 \cdot 8}$, then it will be given by the sum of probabilities for all genotype combinations and the following series are obtained:

mother Ph-5		child Ph-6		father Ph-8	probability of genotype combination
cDE/cDE	—	cDE/cDe	—	CDe/cDe	su^2v
cDE/cDE	—	cDE/cde	—	CDe/cde	stu^2
cDE/cDE	—	cDE/cDe	—	cDe/Cde	u^2vx
cDE/cdE	—	cDE/cDe cdE/cDe	—	CDe/cDe	suvw suvw
cDE/cdE	—	cDE/cde	—	CDe/cde	stuw
cDE/cdE	—	cDE/cDe cdE/cDe	—	cDe/Cde	uvwx uvwx

$$\Sigma = H_{5 \cdot 6 \cdot 8} = uvw(s + x) + u(u + w)(st + sv + vx)$$

In the same way all probabilities for $H_{l \cdot m \cdot n}$, where mother, child and father belong respectively to Ph-l, Ph-m and Ph-n, are obtained (see Table 3). It is to be noted here that these quantities are equal if the class numbers l and n of $H_{l \cdot m \cdot n}$, namely phenotypes of the mother and father, are mutually replaceable without reference to the phenotype of the child, m. We have, therefore, abbreviated the table for $H_{l \cdot m \cdot n}$, with the indications for the parts to be referred to instead of the description of the quantities in detail, if n is smaller than l.

The next step is to obtain the probability of mother-child combination, that is $K_{l \cdot m}$, where phenotypes of mother and child are respectively Ph-l and Ph-m. This probability can be obtained by adding the probabilities of all mother-child-father combinations represented by $H_{l \cdot m \cdot n}$, of which l and m correspond tot hose of $K_{l \cdot m \cdot 1}$ without reference to n, phenotype of the father. Then, we shall illustrate the procedure to obtain $K_{l \cdot m}$, using $K_{5 \cdot 6}$ as example. As $K_{5 \cdot 6}$ is built of the probabilities of all mother-child-father combinations of which mother and child belong respectively to Ph-5 and Ph-6, it will be calculated by adding the series of $H_{l \cdot m \cdot n}$ in Table 3, as seen in the table of the next page.

Similarly probabilities for all mother-child combinations in $K_{l \cdot m \cdot}$, can be obtained and these are shown in Table 4. The probability of any pair of mother and child is equal to that of the pair obtained by interchanging the mother and the child in the

mother	child	father	$H_{l\cdot m\cdot n}$	probability of combination
Ph-5	Ph-6	Ph-1	$H_{5\cdot6\cdot1}$	$t^2u\alpha_3$
		Ph-2	$H_{5\cdot6\cdot2}$	$tuv\alpha_3 + uv\alpha_2(\alpha_3 + w)$
		Ph-3	$H_{5\cdot6\cdot3}$	$tuw\alpha_3$
		Ph-6	$H_{5\cdot6\cdot6}$	$uvw\alpha_3 + u\alpha_3\beta_4$
		Ph-7	$H_{5\cdot6\cdot7}$	$tux\alpha_3$
		Ph-8	$H_{5\cdot6\cdot8}$	$uvw\alpha_1 + u\alpha_3\beta_1$
		Ph-9	$H_{5\cdot6\cdot9}$	$tuz\alpha_3$
		Ph-11	$H_{5\cdot6\cdot11}$	$uvw\alpha_4 + u\alpha_3\beta_5$

$\Sigma\, H_{l\cdot m\cdot n} = K_{5\cdot6} = uvw + u(t + v)(u + w)$

original pair, that is $K_{l\cdot m} = K_{m\cdot l}$ It is noteworthy that this symmetry is not accidental, but essential (Komatu, 1951).

Quantity for X in Essen-Möller's formula is given by dividing $H_{l\cdot m\cdot n}$ by $K_{l\cdot m}$. On the other hand, as quantity for Y in the same formula, the frequency of Ph-n observed in the population and denoted by (Ph-n), is used. Consequently, we get the probability of paternity in Rh blood groups from Essen-Möller's formula (1) when mother, child and putative father belong to Ph-l, Ph-m and Ph-n respectively, as follows:

$$\frac{1}{1+\frac{Y}{X}} = \frac{1}{1+\frac{(\text{Ph-n})\,.\,K_{l\cdot m}}{H_{l\cdot m\cdot n}}} \qquad (4)$$

where critical value is

$$\frac{Y}{X} = \frac{(\text{Ph-n}).K_{l\cdot m}}{H_{l\cdot m\cdot n}\cdot} \qquad (4')$$

2. Cases where four anti-sera (anti-C, -D, -E, -c) are used for testing:

By using these four anti-sera, twelve different kinds of phenotypes may be recognized in Rh blood groups, and class numbers of these phenotypes are indicated by Ph-1′, Ph-2′, ..., Ph-12′. Table 5 shows the relation between these phenotypes and their genetical constitutions and their frequencies represented by chromosome frequencies. In this table we can recognize that Ph-1′, Ph-2′, Ph-5′, Ph-6′, Ph-9′, Ph-10′ are quite identical with Ph-1, Ph-2, Ph-7, Ph-8, Ph-13, Ph-14, respectively, in their genetical constitutions; but that each of other phenotypes primed is constructed of two phenotypes not primed, that is, Ph-3′, Ph-4′, Ph-7′, Ph-8′, Ph-11′ and Ph-12′ correspond respectively to Ph-3 + Ph-4, Ph-5 + Ph-6, Ph-9 + Ph-10, Ph-11 + Ph-12, Ph-15 + Ph-16 and Ph-17 + Ph-18.

In this case also, to obtain the probability of paternity, the tables for $H_{l'\cdot m'\cdot n'}$ and $K_{l'\cdot m'}$ corresponding to those for $H_{l\cdot m\cdot n}$ and $K_{l\cdot m}$ as stated in the last case must be prepared. It is, however, convenient for the present calculation to prepare the table for $K_{l'\cdot m'}$ first instead of $H_{l'\cdot m'\cdot n'}$. $K_{l'\cdot m'}$, which reveals the probability of mother and child combination who belong to Ph-l′ and Ph-m′, respectively, can be obtained by adding the probabilities of all mother-child combinations represented by $K_{l\cdot m}$, of which l and m may be included in l′ and m′ by referring the collation given in Table 5. For example, as Ph-4′ is composed of Ph-5 + Ph-6, and Ph-8′ of Ph-11 + Ph-12, that, if we consider the combination of mother belonging to Ph-4′ and child belonging to Ph-8′ the probability of such combination, namely $K_{4'\cdot 8'}$, will be calculated from Table 5 as the sum of four $K_{l\cdot m}$ as follows:

$$K_{4'\cdot 8'} = K_{5\cdot 11} + K_{5\cdot 12} + K_{6\cdot 11} + K_{6\cdot 12}. \tag{5}$$

If each primed phenotype of mother, child and father, l′, m′ and n′, consists of one non-primed phenotype only, $H_{l'\cdot m'\cdot n'}$ standing for the probability of combination of these three members will be equal to the value for $H_{l\cdot m\cdot n}$, of which l, m and n correspond to l′, m′ and n′. On the other hand, if at least one among mother, child and father consists of two non-primed phenotypes, $H_{l'\cdot m'\cdot n'}$ would be a sum of more than two probabilities represented by $H_{l\cdot m\cdot n}$. $H_{l'\cdot m'\cdot n'}$ is then the sum of all $H_{l\cdot m\cdot n}$ selected from Table 3 and to be included in it, as in the case of $K_{l'\cdot m'}$ composed of several $K_{l\cdot m}$.

For example, let us take the mother-child-father combination, whose phenotypes belong respectively to Ph-4′, Ph-8′ and Ph-8′. In this case, as $K_{4'\cdot 8'}$ illustrated above as an example (5) gives all possible combinations of mother and child possessing non-primed characters, it is possible now to refer to Table 3 for the parts to be picked out, where mother and child belong respectively to Ph-5 and Ph-11, Ph-5 and Ph-12, Ph-6 and Ph-11, and Ph-6 and Ph-12. Taking from each of these four parts in this table, the combinations in which fathers' phenotypes belong to Ph-8′, that is, Ph-11 and Ph-12, we can get all mother-child-father combinations, probabilities of which are represented by $H_{l\cdot m\cdot n}$ to be included in $H_{4'\cdot 8'\cdot 8'}$. Details of calculation are as follows:

mother	child	father	$H_{l\cdot m\cdot n}$	probability of combination
Ph-5	Ph-11	Ph-11	$H_{5\cdot 11\cdot 11}$	$suw\alpha_3 + u\alpha_3\beta_2$
Ph-5	Ph-11	Ph-12	$H_{5\cdot 11\cdot 12}$	zero
Ph-5	Ph-12	Ph-11	$H_{5\cdot 12\cdot 11}$	$uwy\alpha_2 + u\alpha_3\beta_5$
Ph-5	Ph-12	Ph-12	$H_{5\cdot 12\cdot 12}$	$uwy\alpha_3 + u\alpha_3\beta_6$
Ph-6	Ph-11	Ph-11	$H_{6\cdot 11\cdot 11}$	$u^2x\alpha_2 + v^2z\alpha_3 + \beta_4(s\alpha_3 + y\alpha_2)$
Ph-6	Ph-11	Ph-12	$H_{6\cdot 11\cdot 12}$	$uvz\alpha_3 + y\alpha_3\beta_4$
Ph-6	Ph-12	Ph-11	$H_{6\cdot 12\cdot 11}$	$uvz\alpha_2 + y\alpha_2\beta_4$
Ph-6	Ph-12	Ph-12	$H_{6\cdot 12\cdot 12}$	$u^2z\alpha_2 + y\alpha_1\beta_4$

$$\Sigma\, H_{l\cdot m\cdot n} = H_{4'\cdot 8'\cdot 8'} = suw\alpha_3 + u\alpha_3\beta_2 + uwy\alpha_2 + \ldots + u^2z\alpha_2 + y\alpha_1\beta_4.$$

Thus the probability of $H_{4'\cdot 8'\cdot 8'}$ is the sum of partial probabilities. The phenotype combination which may be represented by $H_{5\cdot 11\cdot 12}$ is not listed in Table 3, because a combination between mother, child and father would not appear unless a doubt exists about the paternity of both parents. For this reason probability revealed by $H_{5\cdot 11\cdot 12}$ is equal to zero.

Then the probability of paternity in the present case where the putative father belongs to Ph-n′ is

$$\frac{1}{1 + \dfrac{(\text{Ph-n}') \cdot K_{l'\cdot m'}}{H_{l'\cdot m'\cdot n'}}} \qquad (6)$$

Therefore, that of the case, in which mother, child and putative father belong respectively to Ph-4′, Ph-8′, Ph-8′, will be:

$$\frac{1}{1 + \dfrac{2\left\{s(u+w) + y(t+u+v+w)\right\}\left\{s(t+v) + 2s(u+w) + y(t+u+v+w)\right\}}{(t+u+v+w)\left\{2s(u+w) + y(t+u+v+w)\right\}}}$$

(where $x = z =$ zero) (6′)

As shown in this calculation without using anti-e serum, we shall be able to recognize the fact, that if one anti-serum is reduced from five testing anti-sera, it will take more complicated work to calculate the probability of paternity, and for this reason it can be said that decrease of the number of anti-sera used, increases generally complexity in calculation of the probability. This is true also in the next level of testing with three anti-sera.

3. Cases where three anti-sera (anti-C, -D, -E) are used for testing:
If three antisera are used, Rh blood groups might be classified into eight phenotypes (Ph-1″, Ph-2″, ..., Ph-8″). Table 7 shows the relation of these phenotypes and their genetic constitutions. These double primed phenotypes consist of one, two or four non-primed phenotypes respectively. Also the probability of paternity in this level of testing may be calculated in the same way as stated in the last level of testing with four anti-sera. Now, let us denote the combined probability of mother and child by $K_{l''\cdot m''}$ and that of mother, child and father by $H_{l''\cdot m''\cdot n''}$. $K_{l''\cdot m''}$ is, like $K_{l'\cdot m'}$, composed of one or more probabilities represented by $K_{l\cdot m}$ and the relations between them are listed in Table 8.

The process to obtain the probability of paternity in this level of testing is illustrated with an example, where mother, child and father belong respectively to Ph-6″, Ph-8″ and Ph-8″. From Table 8 $K_{6''\cdot 8''}$ is found to be:

$$K_{6''\cdot 8''} = K_{8\cdot 11} + K_{8\cdot 15} + K_{14\cdot 11} + K_{14\cdot 15}. \qquad (7)$$

Therefore, considering such construction of $K_{l''\cdot m''}$ we can obtain $H_{l''\cdot m''\cdot n''}$ from Table 3. If father's phenotype is Ph-8″ in this example, $H_{6''\cdot 8''\cdot 8''}$ would be the sum of individual probabilities represented by $H_{l\cdot m\cdot n}$ and derived from Table 3.

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mother Ph-6″	child Ph-8″	father Ph-8″	$H_{l \cdot m \cdot n}$	probability of combination
Ph-8	Ph-11	Ph-11	$H_{8 \cdot 11 \cdot 11}$	$s^2 w\alpha_2 + v^2 z\alpha_1 + \beta_1 (u\alpha_1 + y\alpha_2)$
		Ph-12	$H_{8 \cdot 11 \cdot 12}$	$swy\alpha_2 + uvz\alpha_1 + \beta_1 (\beta_6 + uy)$
		Ph-15	$H_{8 \cdot 11 \cdot 15}$	$svz\alpha_1 + y\alpha_1\beta_1$
		Ph-16	$H_{8 \cdot 11 \cdot 16}$	$vyz\alpha_1 + y\alpha_4\beta_1$
	Ph-15	Ph-11	$H_{8 \cdot 15 \cdot 11}$	$svz\alpha_2 + y\alpha_2\beta_1$
		Ph-12	$H_{8 \cdot 15 \cdot 12}$	$suz\alpha_2 + y\alpha_3\beta_1$
		Ph-15	$H_{8 \cdot 15 \cdot 15}$	$s^2 z\alpha_2 + y\alpha_1\beta_1$
		Ph-16	$H_{8 \cdot 15 \cdot 16}$	$syz\alpha_2 + y\alpha_4\beta_1$
Ph-14	Ph-11	Ph-11	$H_{14 \cdot 11 \cdot 11} = H_{11 \cdot 11 \cdot 14}$	$sux\alpha_1 + s\alpha_1\beta_2$
		Ph-12	$H_{14 \cdot 11 \cdot 12} = H_{12 \cdot 11 \cdot 14}$	$sux\alpha_4 + s\alpha_1\beta_6$
		Ph-15	$H_{14 \cdot 11 \cdot 15}$	zero
		Ph-16	$H_{14 \cdot 11 \cdot 16}$	zero
	Ph-15	Ph-11	$H_{14 \cdot 15 \cdot 11} = H_{11 \cdot 15 \cdot 14}$	$sxy\alpha_2 + s\alpha_1\beta_5$
		Ph-12	$H_{14 \cdot 15 \cdot 12} = H_{12 \cdot 15 \cdot 14}$	$sxy\alpha_3 + s\alpha_1\beta_6$
		Ph-15	$H_{14 \cdot 15 \cdot 15}$	$sxy\alpha_1 + s\alpha_1\beta_3$
		Ph-16	$H_{14 \cdot 15 \cdot 16}$	$sxy\alpha_4 + sy\alpha_1 (\alpha_4 + z)$

$$\Sigma H_{l \cdot m \cdot n} = H_{6'' \cdot 8'' \cdot 8''} = s^2 w\alpha_2 + v^2 z\alpha_1 + \beta_1 (u\alpha_1 + y\alpha_2) + swy\alpha_2 + \dots + sxy\alpha_4 + sy\alpha_1 (\alpha_4 + z).$$

If the disputed father belongs to Ph-n″ and the frequency of Ph-n″ in the population represented by (Ph-n″), the probability of paternity in this case is derived from the following formula:

$$\frac{1}{1 + \dfrac{(\text{Ph-n}'') \cdot K_{1'' \cdot m''}}{H_{1'' \cdot m'' \cdot n''}}} \qquad (8)$$

Thus if the phenotypes of mother, child and putative father belong to Ph-6″, Ph-8″ and Ph-8″ respectively, the probability of paternity is obtained from the following formula

$$\frac{1}{1 + \dfrac{(\text{Ph-8}'') \cdot K_{6'' \cdot 8''}}{H_{6'' \cdot 8'' \cdot 8''}}} \qquad (8')$$

4. Cases where other levels of testing are used:

There may be many other cases where the kinds of anti-sera used in Rh blood grouping tests are not identical to those of the cases stated above, as in cases tested with anti-C, -D, -E and -e, or with anti-C, -D and -c, or with anti-C and -D, etc. In these cases, we can calculate the probability of paternity by the way similar to those already explained. To prepare tables for testing these levels, it is necessary to rearrange the tables presented in this paper for the purpose of obtaining the quantities corresponding to K and H.

In cases in which only one anti-serum or two anti-sera, which react with the antigens allelomorphic to each other, as anti-C and anti-c or anti-E and anti-e, are used for the testing, there is no need to calculate the probability of paternity by the method presented here starting from Tables 1, 2, 3 and 4. In such cases the tables which were derived by Elbel & Sellier, and Uetake & Yokoyama are available and simple formulas, (2) or (2′) or (2″) are available instead of formula (4).

Summary

A method of calculating the probability of paternity, based upon Bayes' theorem, for Rh blood groups has been developed. Tables have been prepared for calculation of the probability of paternity for the most common levels of testing, i.e., with five anti-sera (anti-C, -D, -E, -c, -e), with four (anti-C, -D, -E, -c) and with three (anti-C, -D, -E). The same probabilities at other levels of testing can be calculated by applying the method and tables presented in this paper. Since the tables presented herein include calculations for chromosomes which appear to occur but rarely as far as known to date, these tables can be applied to all populations with different chromosome frequencies.

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References

Elbel, H. & Sellier, K.: Beitrag zum 'positiven' Vaterschaftsnachweis mit den bekannten Blutgruppensystemen. Dtsch. Z. gerichtl. Med., 44, 196, 1955.

Essen-Möller, E.: Die Beweiskraft der Aehnlichkeit im Vaterschaftsnachweis. Mitt. Anthrop. Ges. Wien, 68, 9, 1938.

— & Quensel, C.-E.: Zur Theorie des Vaterschaftsnachweises auf Grund von Aehnlichkeitsbefunden. Dtsch. Z. gerichtl. Med., 31, 70, 1939.

Komatu, Y.: Hanzaigaku-Zasshi., 13, 485, 1939 (Japanese).

— Probability-theoretic Investigations on Inheritance. IV_1. Mother-Child Combinations. Proc. Jap. Acad., 27, 587, 1951.

— Probability-theoretic Investigations on Inheritance. XII_1 & XII_2. Probability of Paternity. Proc. Jap. Acad., 28, 359 & 365, 1952.

Ludwig, W.: Die wahrscheinlichkeitstheoretische Argumentation bei der Vaterschaftsdiagnose. Wissensch. Z. Karl-Marx-Univ. Leipzig. Mathem.-Naturwissensch. Reihe., 4, 13, 1954/55.

— & Wartmann, R.: Kritische Betrachtungen zum erbbiologischen Vaterschaftstest. Dtsch. Z. gerichtl. Med., 41, 289, 1952.

Okajima, M.: On the Probability of Paternity and its Application on Rh Blood Groups. Jap. J. Leg. Med., 9, 414, 1955 (Japanese).

Race, R. R. & Sanger, R.: Blood Groups in Man. 1954, Oxford.

Uetake, M. & Yokoyama, M.: Jap. J. Leg. Med. & Criminol., 20, 98, 1954 (Japanese).

Wichmann, D. & Tuppa, K.: Zur Anwendung von Rechenverfahren im Vaterschaftsgutachten. Dtsch. Z. gerichtl. Med., 43, 54, 1954.

Tab. 1 - Frequency of chromosome (From Race and Sanger, 1954)

CDe = R^1 = s	cdE = r'' = w
cde = r = t	Cde = r' = x
cDE = R^2 = u	CDE = R^z = y
cDe = R^0 = v	CdE = r^y = z

Tab. 2

Phenotype tested with 5 antisera		Genotype			Frequency of phenotype
	Anti- C D E c e				
Ph-1	— — — + +	cde/cde			t^2
Ph-2	— + — + +	cDe/cde	cDe/cDe		$v(2t + v)$
Ph-3	— — + + +	cdE/cde			$2tw$
Ph-4	— — + + —	cdE/cdE			w^2
Ph-5	— + + + —	cDE/cDE	cDE/cdE		$u(u + 2w)$
Ph-6	— + + + +	cDE/cDe	cDE/cde	cDe/cdE	$2(tu + uv + vw)$
Ph-7	+ — — + +	Cde/cde			$2tx$
Ph-8	+ + — + +	CDe/cDe	CDe/cde	cDe/Cde	$2(st + sv + vx)$
Ph-9	+ — + + +	cdE/Cde	CdE/cde		$2(tz + wx)$
Ph-10	+ — + + —	CdE/cdE			$2wz$
Ph-11	+ + + + +	CDe/cDE	cDe/CDE	CDe/cdE	$2(su + sw + ty + ux + vy + vz)$
		cDE/Cde	CDE/cde	CdE/cDe	
Ph-12	+ + + + —	cDE/CDE	cdE/CDE	CdE/cDE	$2(uy + uz + wy)$
Ph-13	+ — — — +	Cde/Cde			x^2
Ph-14	+ + — — +	CDe/CDe	CDe/Cde		$s(s + 2x)$
Ph-15	+ + + — +	CDe/CDE	Cde/CDE	CdE/CDe	$2(sy + sz + xy)$
Ph-16	+ + + — —	CDE/CDE	CdE/CDE		$y(y + 2z)$
Ph-17	+ — + — +	CdE/Cde			$2xz$
Ph-18	+ — + — —	CdE/CdE			z^2

Tab. 3 - Probability of mother-child-father combination ($H_{l \cdot m \cdot n}$)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
1	1	1	t^4	1	9	10	t^2wz
1	1	2	t^3v	1	9	11	t^2vz
1	1	3	t^3w	1	9	12	t^2uz
1	1	6	t^3u	1	9	15	st^2z
1	1	7	t^3x	1	9	16	t^2yz
1	1	8	st^3	1	9	17	t^2xz
1	1	9	t^3z	1	9	18	t^2z^2
1	1	11	t^3y	1	11	11	$t^2y\alpha_2$
1	2	2	$t^2v\alpha_2$	1	11	12	$t^2y\alpha_3$
1	2	6	$t^2v\alpha_3$	1	11	15	$t^2y\alpha_1$
1	2	8	$t^2v\alpha_1$	1	11	16	$t^2y\alpha_4$
1	2	11	$t^2v\alpha_4$	2	1	1	= 1 1 2
1	3	3	t^3w	2	1	2	t^2v^2
1	3	4	t^2w^2	2	1	3	t^2vw
1	3	5	t^2uw	2	1	6	t^2uv
1	3	6	t^2vw	2	1	7	t^2vx
1	3	9	t^2wx	2	1	8	st^2v
1	3	10	t^2wz	2	1	9	t^2vz
1	3	11	st^2w	2	1	11	t^2vy
1	3	12	t^2wy	2	2	1	= 1 2 2
1	6	5	$t^2u\alpha_3$	2	2	2	$v^2\alpha_2(\alpha_2 + 2t)$
1	6	6	$t^2u\alpha_2$	2	2	3	$tvw\alpha_2$
1	6	11	$t^2u\alpha_1$	2	2	6	$tv^2\alpha_3 + v\alpha_2\beta_4$
1	6	12	$t^2u\alpha_4$	2	2	7	$tvx\alpha_2$
1	7	7	t^3x	2	2	8	$tv^2\alpha_1 + v\alpha_2\beta_1$
1	7	8	t^2vx	2	2	9	$tvz\alpha_2$
1	7	9	t^2wx	2	2	11	$tv^2\alpha_4 + v\alpha_2\beta_5$
1	7	11	t^2ux	2	3	3	t^2vw
1	7	13	t^2x^2	2	3	4	tvw^2
1	7	14	st^2x	2	3	5	$tuvw$
1	7	15	t^2xy	2	3	6	tv^2w
1	7	17	t^2xz	2	3	9	$tvwx$
1	8	8	$st^2\alpha_2$	2	3	10	$tvwz$
1	8	11	$st^2\alpha_3$	2	3	11	$stvw$
1	8	14	$st^2\alpha_1$	2	3	12	$tvwy$
1	8	15	$st^2\alpha_4$	2	6	3	$tvw\alpha_2$
1	9	9	t^3z	2	6	4	$vw^2\alpha_2$

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Tabl. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
2	6	5	$tuv\alpha_3 + uv\alpha_2(\alpha_3 + w)$	2	11	18	$vz^2\alpha_2$
2	6	6	$tuv\alpha_2 + v\alpha_2\beta_4$	3	1	1	= 1 1 3
2	6	9	$vwx\alpha_2$	3	1	2	= 2 1 3
2	6	10	$vwz\alpha_2$	3	1	3	t^2w^2
2	6	11	$tuv\alpha_1 + v\alpha_2\beta_2$	3	1	6	t^2uw
2	6	12	$tuv\alpha_4 + v\alpha_2\beta_6$	3	1	7	t^2xw
2	7	7	t^2vx	3	1	8	st^2w
2	7	8	tv^2x	3	1	9	t^2wz
2	7	9	$tvwx$	3	1	11	t^2wy
2	7	11	$tuvx$	3	2	2	= 2 2 3
2	7	13	tvx^2	3	2	6	$tvw\alpha_3$
2	7	14	$stvx$	3	2	8	$tvw\alpha_1$
2	7	15	$tvxy$	3	2	11	$tvw\alpha_4$
2	7	17	$tvxz$	3	3	1	= 1 3 3
2	8	7	$tvx\alpha_2$	3	3	2	= 2 3 3
2	8	8	$stv\alpha_2 + v\alpha_2\beta_1$	3	3	3	$2t^2w^2$
2	8	9	$vwx\alpha_2$	3	3	4	tw^3
2	8	11	$stv\alpha_3 + v\alpha_2\beta_2$	3	3	5	tuw^2
2	8	13	$vx^2\alpha_2$	3	3	6	$tw(tu + vw)$
2	8	14	$stv\alpha_1 + sv\alpha_2(\alpha_1 + x)$	3	3	7	t^2wx
2	8	15	$stv\alpha_4 + v\alpha_2\beta_3$	3	3	8	st^2w
2	8	17	$vxz\alpha_2$	3	3	9	$tw(tz + wx)$
2	9	9	t^2vz	3	3	10	tw^2z
2	9	10	$tvwz$	3	3	11	$tw(sw + ty)$
2	9	11	tv^2z	3	3	12	tw^2y
2	9	12	$tuvz$	3	4	3	t^2w^2
2	9	15	$stvz$	3	4	4	tw^3
2	9	16	$tvyz$	3	4	5	tuw^2
2	9	17	$tvxz$	3	4	6	tvw^2
2	9	18	tvz^2	3	4	9	tw^2x
2	11	9	$tvz\alpha_2$	3	4	10	tw^2z
2	11	10	$vwz\alpha_2$	3	4	11	stw^2
2	11	11	$tvy\alpha_2 + v\alpha_2\beta_5$	3	4	12	tw^2y
2	11	12	$tvy\alpha_3 + v\alpha_2\beta_6$	3	5	5	$tuw\alpha_3$
2	11	15	$tvy\alpha_1 + v\alpha_2\beta_3$	3	5	6	$tuw\alpha_2$
2	11	16	$tvy\alpha_4 + vy\alpha_2(\alpha_4 + z)$	3	5	11	$tuw\alpha_1$
2	11	17	$vxz\alpha_2$	3	5	12	$tuw\alpha_4$

Tab. 3 - (continued)

l, m and n of $H_{l\cdot m\cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
3	6	2	= 2 6 3
3	6	5	$tuw\alpha_3$
3	6	6	$tuw\alpha_2 + tvw\alpha_3$
3	6	8	$tvw\alpha_1$
3	6	11	$tuw\alpha_1 + tvw\alpha_4$
3	6	12	$tuw\alpha_4$
3	7	7	t^2wx
3	7	8	$tvwx$
3	7	9	tw^2x
3	7	11	$tuwx$
3	7	13	twx^2
3	7	14	$stwx$
3	7	15	$twxy$
3	7	17	$twxz$
3	8	8	$stw\alpha_2$
3	8	11	$stw\alpha_3$
3	8	14	$stw\alpha_1$
3	8	15	$stw\alpha_4$
3	9	7	t^2wx
3	9	8	$tvwx$
3	9	9	$tw(tz + wx)$
3	9	10	tw^2z
3	9	11	$tw(ux + vz)$
3	9	12	$tuwz$
3	9	13	twx^2
3	9	14	$stwx$
3	9	15	$tw(sz + xy)$
3	9	16	$twyz$
3	9	17	$2twxz$
3	9	18	twz^2
3	10	9	t^2wz
3	10	10	tw^2z
3	10	11	$tvwz$
3	10	12	$tuwz$
3	10	15	$stwz$
3	10	16	$twyz$
3	10	17	$twxz$
3	10	18	twz^2
3	11	8	$stw\alpha_2$
3	11	11	$stw\alpha_3 + twy\alpha_2$
3	11	12	$twy\alpha_3$
3	11	14	$stw\alpha_1$
3	11	15	$stw\alpha_4 + twy\alpha_1$
3	11	16	$twy\alpha_4$
3	12	11	$twy\alpha_2$
3	12	12	$twy\alpha_3$
3	12	15	$twy\alpha_1$
3	12	16	$twy\alpha_4$
4	3	1	= 1 3 4
4	3	2	= 2 3 4
4	3	3	= 3 3 4
4	3	6	tuw^2
4	3	7	txw^2
4	3	8	stw^2
4	3	9	tw^2z
4	3	11	tw^2y
4	4	3	= 3 4 4
4	4	4	w^4
4	4	5	uw^3
4	4	6	vw^3
4	4	9	w^3x
4	4	10	w^3z
4	4	11	sw^3
4	4	12	w^3y
4	5	5	$uw^2\alpha_3$
4	5	6	$uw^2\alpha_2$
4	5	11	$uw^2\alpha_1$
4	5	12	$uw^2\alpha_4$
4	6	2	= 2 6 4
4	6	6	$vw^2\alpha_3$
4	6	8	$vw^2\alpha_1$
4	6	11	$vw^2\alpha_4$
4	9	7	tw^2x
4	9	8	vw^2x

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
4	9	9	w^3x
4	9	11	uw^2x
4	9	13	w^2x^2
4	9	14	sw^2x
4	9	15	w^2xy
4	9	17	w^2xz
4	10	9	tw^2z
4	10	10	w^3z
4	10	11	vw^2z
4	10	12	uw^2z
4	10	15	sw^2z
4	10	16	w^2yz
4	10	17	w^2xz
4	10	18	w^2z^2
4	11	8	$sw^2\alpha_2$
4	11	11	$sw^2\alpha_3$
4	11	14	$sw^2\alpha_1$
4	11	15	$sw^2\alpha_4$
4	12	11	$w^2y\alpha_2$
4	12	12	$w^2y\alpha_3$
4	12	15	$w^2y\alpha_1$
4	12	16	$w^2y\alpha_4$
5	3	1	= 1 3 5
5	3	2	= 2 3 5
5	3	3	= 3 3 5
5	3	6	tu^2w
5	3	7	$tuwx$
5	3	8	$stuw$
5	3	9	$tuwz$
5	3	11	$tuwy$
5	4	3	= 3 4 5
5	4	4	= 4 4 5
5	4	5	u^2w^2
5	4	6	uvw^2
5	4	9	uw^2x
5	4	10	uw^2z
5	4	11	suw^2
5	4	12	uw^2y
5	5	3	= 3 5 5
5	5	4	= 4 5 5
5	5	5	$u^2\alpha_3(\alpha_3 + 2w)$
5	5	6	$u^2w\alpha_2 + u\alpha_3\beta_4$
5	5	9	$uwx\alpha_3$
5	5	10	$uwz\alpha_3$
5	5	11	$u^2w\alpha_1 + u\alpha_3\beta_2$
5	5	12	$u^2w\alpha_4 + u\alpha_3\beta_6$
5	6	1	= 1 6 5
5	6	2	= 2 6 5
5	6	3	= 3 6 5
5	6	6	$uvw\alpha_3 + u\alpha_3\beta_4$
5	6	7	$tux\alpha_3$
5	6	8	$uvw\alpha_1 + u\alpha_3\beta_1$
5	6	9	$tuz\alpha_3$
5	6	11	$uvw\alpha_4 + u\alpha_3\beta_5$
5	9	7	$tuwx$
5	9	8	$uvwx$
5	9	9	uw^2x
5	9	11	u^2wx
5	9	13	uwx^2
5	9	14	$suwx$
5	9	15	$uwxy$
5	9	17	$uwxz$
5	10	9	$tuwz$
5	10	10	uw^2z
5	10	11	$uvwz$
5	10	12	u^2wz
5	10	15	$suwz$
5	10	16	$uwyz$
5	10	17	$uwxz$
5	10	18	uwz^2
5	11	7	$tux\alpha_3$
5	11	8	$suw\alpha_2 + u\alpha_3\beta_1$
5	11	9	$uwx\alpha_3$
5	11	11	$suw\alpha_3 + u\alpha_3\beta_2$

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
5	11	13	$ux^2\alpha_3$	6	3	10	tuwz
5	11	14	$suw\alpha_1 + su\alpha_3(\alpha_1 + x)$	6	3	11	tw(su + vy)
5	11	15	$suw\alpha_4 + u\alpha_3\beta_3$	6	3	12	tuwy
5	11	17	$uxz\alpha_3$	6	4	3	= 3 4 6
5	12	9	$tuz\alpha_3$	6	4	4	= 4 4 6
5	12	10	$uwz\alpha_3$	6	4	5	= 5 4 6
5	12	11	$uwy\alpha_2 + u\alpha_3\beta_5$	6	4	6	v^2w^2
5	12	12	$uwy\alpha_3 + u\alpha_3\beta_6$	6	4	9	vw^2x
5	12	15	$uwy\alpha_1 + u\alpha_3\beta_3$	6	4	10	vw^2z
5	12	16	$uwy\alpha_4 + uy\alpha_3(\alpha_4 + z)$	6	4	11	svw^2
5	12	17	$uxz\alpha_3$	6	4	12	vw^2y
5	12	18	$uz^2\alpha_3$	6	5	3	= 3 5 6
6	1	1	= 1 1 6	6	5	4	= 4 5 6
6	1	2	= 2 1 6	6	5	5	= 5 5 6
6	1	3	= 3 1 6	6	5	6	$uvw\alpha_2 + u\alpha_2\beta_4$
6	1	6	t^2u^2	6	5	9	$uwx\alpha_2$
6	1	7	t^2ux	6	5	10	$uwz\alpha_2$
6	1	8	st^2u	6	5	11	$suw\alpha_2 + u\alpha_1\beta_4$
6	1	9	t^2uz	6	5	12	$uwy\alpha_2 + u\alpha_4\beta_4$
6	1	11	t^2uy	6	6	1	= 1 6 6
6	2	1	= 1 2 6	6	6	2	= 2 6 6
6	2	2	= 2 2 6	6	6	3	= 3 6 6
6	2	3	= 3 2 6	6	6	4	= 4 6 6
6	2	6	$tuv\alpha_3 + v\alpha_3\beta_4$	6	6	5	= 5 6 6
6	2	7	$tvx\alpha_3$	6	6	6	$tu^2\alpha_2 + v^2w\alpha_3 + \beta_4(\beta_4 + uv)$
6	2	8	$stv\alpha_3 + v\alpha_1\beta_4$	6	6	7	$tux\alpha_2$
6	2	9	$tvz\alpha_3$	6	6	8	$stu\alpha_2 + v\alpha_1\beta_4$
6	2	11	$tvy\alpha_3 + v\alpha_4\beta_4$	6	6	9	$tuz\alpha_2 + vwx\alpha_3$
6	3	1	= 1 3 6	6	6	10	$vwz\alpha_2$
6	3	2	= 2 3 6	6	6	11	$svw\alpha_3 + tuy\alpha_2 + \beta_4(u\alpha_1 + v\alpha_4)$
6	3	3	= 3 3 6	6	6	12	$vwy\alpha_3 + u\alpha_4\beta_4$
6	3	4	= 4 3 6	6	7	7	t^2ux
6	3	5	= 5 3 6	6	7	8	tuvx
6	3	6	2tuvw	6	7	9	tuwx
6	3	7	tvwx	6	7	11	tu^2x
6	3	8	stvw	6	7	13	tux^2
6	3	9	tw(ux + vz)	6	7	14	stux

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
6	7	15	tuxy
6	7	17	tuxz
6	8	7	$tvx\alpha_3$
6	8	8	$v^2x\alpha_3 + s\alpha_2\beta_4$
6	8	9	$vwx\alpha_3$
6	8	11	$uvx\alpha_3 + s\alpha_3\beta_4$
6	8	13	$vx^2\alpha_3$
6	8	14	$svx\alpha_3 + s\alpha_1\beta_4$
6	8	15	$vxy\alpha_3 + s\alpha_4\beta_4$
6	8	17	$vxz\alpha_3$
6	9	7	tvwx
6	9	8	v^2wx
6	9	9	$vw^2x + t^2uz$
6	9	10	tuwz
6	9	11	uv(tz + wx)
6	9	12	tu^2z
6	9	13	vwx^2
6	9	14	svwx
6	9	15	stuz + vwxy
6	9	16	tuyz
6	9	17	xz(tu + vw)
6	9	18	tuz^2
6	10	9	tvwz
6	10	10	vw^2z
6	10	11	v^2wz
6	10	12	uvwz
6	10	15	svwz
6	10	16	vwyz
6	10	17	vwxz
6	10	18	vwz^2
6	11	7	$tux\alpha_2$
6	11	8	$uvx\alpha_2 + s\alpha_2\beta_4$
6	11	9	$tvz\alpha_3 + uwx\alpha_2$
6	11	10	$vwz\alpha_3$
6	11	11	$u^2x\alpha_2 + v^2z\alpha_3 + \beta_4(s\alpha_3 + y\alpha_2)$
6	11	12	$uvz\alpha_3 + y\alpha_3\beta_4$
6	11	13	$ux^2\alpha_2$
6	11	14	$sux\alpha_2 + s\alpha_1\beta_4$
6	11	15	$svz\alpha_3 + uxy\alpha_2 + \beta_4(\beta_3 + sy)$
6	11	16	$vyz\alpha_3 + y\alpha_4\beta_4$
6	11	17	$xz(\beta_4 + uv)$
6	11	18	$vz^2\alpha_3$
6	12	9	$tuz\alpha_2$
6	12	10	$uwz\alpha_2$
6	12	11	$uvz\alpha_2 + y\alpha_2\beta_4$
6	12	12	$u^2z\alpha_2 + y\alpha_3\beta_4$
6	12	15	$suz\alpha_2 + y\alpha_1\beta_4$
6	12	16	$uyz\alpha_2 + y\alpha_4\beta_4$
6	12	17	$uxz\alpha_2$
6	12	18	$uz^2\alpha_2$
7	1	1	= 1 1 7
7	1	2	= 2 1 7
7	1	3	= 3 1 7
7	1	6	= 6 1 7
7	1	7	t^2x^2
7	1	8	st^2x
7	1	9	t^2xz
7	1	11	t^2xy
7	2	2	= 2 2 7
7	2	6	= 6 2 7
7	2	8	$tvx\alpha_1$
7	2	11	$tvx\alpha_4$
7	3	3	= 3 3 7
7	3	4	= 4 3 7
7	3	5	= 5 3 7
7	3	6	= 6 3 7
7	3	9	twx^2
7	3	10	twxz
7	3	11	stwx
7	3	12	twxy
7	6	5	= 5 6 7
7	6	6	= 6 6 7
7	6	11	$tux\alpha_1$
7	6	12	$tux\alpha_4$

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father: l	m	n	Probability of mother-child-father combination
7	7	1	= 1 7 7
7	7	2	= 2 7 7
7	7	3	= 3 7 7
7	7	6	= 6 7 7
7	7	7	$2t^2x^2$
7	7	8	$tx(st + vx)$
7	7	9	$tx(tz + wx)$
7	7	11	$tx(ty + ux)$
7	7	13	tx^3
7	7	14	stx^2
7	7	15	tx^2y
7	7	17	tx^2z
7	8	2	= 2 8 7
7	8	6	= 6 8 7
7	8	8	$tx(\beta_1 + sv)$
7	8	11	$stx\alpha_3 + tvx\alpha_4$
7	8	14	$stx\alpha_1$
7	8	15	$stx\alpha_4$
7	9	3	= 3 9 7
7	9	4	= 4 9 7
7	9	5	= 5 9 7
7	9	6	= 6 9 7
7	9	9	$tx(tz + wx)$
7	9	10	$2twxz$
7	9	11	$tx(sw + vz)$
7	9	12	$tx(uz + wy)$
7	9	15	$stxz$
7	9	16	$txyz$
7	9	17	tx^2z
7	9	18	txz^2
7	11	5	= 5 11 7
7	11	6	= 6 11 7
7	11	11	$tux\alpha_1 + txy\alpha_2$
7	11	12	$tux\alpha_4 + txy\alpha_3$
7	11	15	$txy\alpha_1$
7	11	16	$txy\alpha_4$
7	13	7	t^2x^2
7	13	8	tvx^2
7	13	9	twx^2
7	13	11	tux^2
7	13	13	tx^3
7	13	14	stx^2
7	13	15	tx^2y
7	13	17	tx^2z
7	14	8	$stx\alpha_2$
7	14	11	$stx\alpha_3$
7	14	14	$stx\alpha_1$
7	14	15	$stx\alpha_4$
7	15	11	$txy\alpha_2$
7	15	12	$txy\alpha_3$
7	15	15	$txy\alpha_1$
7	15	16	$txy\alpha_4$
7	17	9	t^2xz
7	17	10	$twxz$
7	17	11	$tvxz$
7	17	12	$tuxz$
7	17	15	$stxz$
7	17	16	$txyz$
7	17	17	tx^2z
7	17	18	txz^2
8	1	1	= 1 1 8
8	1	2	= 2 1 8
8	1	3	= 3 1 8
8	1	6	= 6 1 8
8	1	7	= 7 1 8
8	1	8	s^2t^2
8	1	9	st^2z
8	1	11	st^2y
8	2	1	= 1 2 8
8	2	2	= 2 2 8
8	2	3	= 3 2 8
8	2	6	= 6 2 8
8	2	7	= 7 2 8
8	2	8	$stv\alpha_1 + v\alpha_1\beta_1$

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination
l	m	n		l	m	n	
8	2	9	$tvz\alpha_1$	8	8	11	$sty\alpha_2 + uvx\alpha_1 + \beta_1(s\alpha_3 + v\alpha_4)$
8	2	11	$tvy\alpha_1 + v\alpha_4\beta_1$	8	8	13	$vx^2\alpha_1$
8	3	3	= 3 3 8	8	8	14	$svx\alpha_1 + s\alpha_1\beta_1$
8	3	4	= 4 3 8	8	8	15	$vxy\alpha_1 + s\alpha_4\beta_1$
8	3	5	= 5 3 8	8	8	17	$vxz\alpha_1$
8	3	6	= 6 3 8	8	9	3	= 3 9 8
8	3	9	stwx	8	9	4	= 4 9 8
8	3	10	stwz	8	9	5	= 5 9 8
8	3	11	s^2tw	8	9	6	= 6 9 8
8	3	12	stwy	8	9	9	$st^2z + vx^2w$
8	6	3	= 3 6 8	8	9	10	$wz(st + vx)$
8	6	4	= 4 6 8	8	9	11	$sv(tz + wx)$
8	6	5	= 5 6 8	8	9	12	$stuz + vwxy$
8	6	6	= 6 6 8	8	9	15	s^2tz
8	6	9	$vwx\alpha_1$	8	9	16	styz
8	6	10	$vwz\alpha_1$	8	9	17	stxz
8	6	11	$svw\alpha_1 + u\alpha_1\beta_1$	8	9	18	stz^2
8	6	12	$vwy\alpha_1 + u\alpha_4\beta_1$	8	11	3	= 3 11 8
8	7	1	= 1 7 8	8	11	4	= 4 11 8
8	7	2	= 2 7 8	8	11	5	= 5 11 8
8	7	3	= 3 7 8	8	11	6	= 6 11 8
8	7	6	= 6 7 8	8	11	9	$swx\alpha_2 + tvz\alpha_1$
8	7	7	= 7 7 8	8	11	10	$wz(\beta_1 + sv)$
8	7	8	2stvx	8	11	11	$s^2w\alpha_2 + v^2z\alpha_1 + \beta_1(u\alpha_1 + y\alpha_2)$
8	7	9	$tx(sw + vz)$	8	11	12	$swy\alpha_2 + uvz\alpha_1 + \beta_1(\beta_6 + uy)$
8	7	11	$tx(su + vy)$	8	11	15	$svz\alpha_1 + y\alpha_1\beta_1$
8	7	13	stx^2	8	11	16	$vyz\alpha_1 + y\alpha_4\beta_1$
8	7	14	s^2tx	8	11	17	$vxz\alpha_1$
8	7	15	stxy	8	11	18	$vz^2\alpha_1$
8	7	17	stxz	8	13	7	= 7 13 8
8	8	1	= 1 8 8	8	13	8	v^2x^2
8	8	2	= 2 8 8	8	13	9	vx^2w
8	8	3	= 3 8 8	8	13	11	uvx^2
8	8	6	= 6 8 8	8	13	13	vx^3
8	8	7	= 7 8 8	8	13	14	svx^2
8	8	8	$s^2t\alpha_2 + v^2x\alpha_1 + \beta_1(\beta_1 + sv)$	8	13	15	vx^2y
8	8	9	$stz\alpha_2 + vwx\alpha_1$	8	13	17	vx^2z

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
8	14	7	= 7 14 8	9	3	2	= 2 3 9
8	14	8	$svx\alpha_2 + s\alpha_2\beta_1$	9	3	3	= 3 3 9
9	14	9	$swx\alpha_2$	9	3	4	= 4 3 9
8	14	11	$sux\alpha_2 + s\alpha_3\beta_1$	9	3	5	= 5 3 9
8	14	13	$sx^2\alpha_2$	9	3	6	= 6 3 9
8	14	14	$s^2x\alpha_2 + s\alpha_1\beta_1$	9	3	7	= 7 3 9
8	14	15	$sxy\alpha_2 + s\alpha_4\beta_1$	9	3	8	= 8 3 9
8	14	17	$sxz\alpha_2$	9	3	9	$2twxz$
8	15	9	$stz\alpha_2$	9	3	10	twz^2
8	15	10	$swz\alpha_2$	9	3	11	$tw(sz + xy)$
8	15	11	$svz\alpha_2 + y\alpha_2\beta_1$	9	3	12	$twyz$
8	15	12	$suz\alpha_2 + y\alpha_3\beta_1$	9	4	3	= 3 4 9
8	15	15	$s^2z\alpha_2 + y\alpha_1\beta_1$	9	4	4	= 4 4 9
8	15	16	$syz\alpha_2 + y\alpha_4\beta_1$	9	4	5	= 5 4 9
8	15	17	$sxz\alpha_2$	9	4	6	= 6 4 9
8	15	18	$sz^2\alpha_2$	9	4	9	w^2x^2
8	17	9	$tvxz$	9	4	10	w^2xz
8	17	10	$vxwz$	9	4	11	sw^2x
8	17	11	v^2xz	9	4	12	w^2xy
8	17	12	$uvxz$	9	5	5	= 5 5 9
8	17	15	$svxz$	9	5	6	= 6 5 9
8	17	16	$vxyz$	9	5	11	$uwx\alpha_1$
8	17	17	vx^2z	9	5	12	$uwx\alpha_4$
8	17	18	vxz^2	9	6	2	= 2 6 9
9	1	1	= 1 1 9	9	6	5	= 5 6 9
9	1	2	= 2 1 9	9	6	6	= 6 6 9
9	1	3	= 3 1 9	9	6	8	= 8 6 9
9	1	6	= 6 1 9	9	6	11	$tuz\alpha_1 + vwx\alpha_4$
9	1	7	= 7 1 9	9	6	12	$tuz\alpha_4$
9	1	8	= 8 1 9	9	7	1	= 1 7 9
9	1	9	t^2z^2	9	7	2	= 2 7 9
9	1	11	t^2yz	9	7	3	= 3 7 9
9	2	2	= 2 2 9	9	7	6	= 6 7 9
9	2	6	= 6 2 9	9	7	7	= 7 7 9
9	2	8	= 8 2 9	9	7	8	= 8 7 9
9	2	11	$tvz\alpha_4$	9	7	9	$2twxz$
9	3	1	= 1 3 9	9	7	11	$tx(uz + wy)$

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
9	7	13	tx^2z	5	10	16	$wxyz$
9	7	14	$stxz$	9	10	17	wx^2z
9	7	15	$txyz$	9	10	18	wxz^2
9	7	17	txz^2	9	11	2	= 2 11 9
9	8	2	= 2 8 9	9	11	5	= 5 11 9
9	8	6	= 6 8 9	9	11	6	= 6 11 9
9	8	8	= 8 8 9	9	11	8	= 8 11 9
9	8	11	$stz\alpha_3 + vwx\alpha_4$	9	11	11	$swx\alpha_3 + tvz\alpha_4 + tyz\alpha_2 + uwx\alpha_1$
9	8	14	$stz\alpha_1$	9	11	12	$tyz\alpha_3 + uwx\alpha_4$
9	8	15	$stz\alpha_4$	9	11	14	$swx\alpha_1$
9	9	1	= 1 9 9	9	11	15	$swx\alpha_4 + tyz\alpha_1$
9	9	2	= 2 9 9	9	11	16	$tyz\alpha_4$
9	9	3	= 3 9 9	9	12	5	= 5 12 9
9	9	4	= 4 9 9	9	12	6	= 6 12 9
9	9	5	= 5 9 9	9	12	11	$tuz\alpha_1 + wxy\alpha_2$
9	9	6	= 6 9 9	9	12	12	$tuz\alpha_4 + wxy\alpha_3$
9	9	7	= 7 9 9	9	12	15	$wxy\alpha_1$
9	9	8	= 8 9 9	9	12	16	$wxy\alpha_4$
9	9	9	$2t^2z^2 + 2w^2x^2$	9	13	7	= 7 13 9
9	9	10	$wz(tz + wx)$	9	13	8	= 8 13 9
9	9	11	$tz(ty + vz) + wx(sw + ux)$	9	13	9	w^2x^2
9	9	12	$tuz^2 + w^2xy$	9	13	11	uwx^2
9	9	13	wx^3	9	13	13	wx^3
9	9	14	swx^2	9	13	14	swx^2
9	9	15	$stz^2 + wx^2y$	9	13	15	wx^2y
9	9	16	tyz^2	9	13	17	wx^2z
9	9	17	$xz(tz + wx)$	9	14	8	= 8 14 9
9	9	18	tz^3	9	14	11	$swx\alpha_3$
9	10	3	= 3 10 9	9	14	14	$swx\alpha_1$
9	10	4	= 4 10 9	9	14	15	$swx\alpha_4$
9	10	5	= 5 10 9	9	15	8	= 8 15 9
9	10	6	= 6 10 9	9	15	11	$stz\alpha_3 + wxy\alpha_2$
9	10	9	$2twxz$	9	15	12	$wxy\alpha_3$
9	10	10	$wz(tz + wx)$	9	15	14	$stz\alpha_1$
9	10	11	$wz(st + vx)$	9	15	15	$stz\alpha_4 + wxy\alpha y_1$
9	10	12	$wz(ty + ux)$	9	15	16	$wxy\alpha_4$
9	10	15	$swxz$	9	16	11	$tyz\alpha_2$

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination
l	m	n		l	m	n	
9	16	12	$tyz\alpha_3$	10	4	11	sw^2z
9	16	15	$tyz\alpha_1$	10	4	12	w^2yz
9	16	16	$tyz\alpha_4$	10	5	5	= 5 5 10
9	17	7	= 7 17 9	10	5	6	= 6 5 10
9	17	8	= 8 17 9	10	5	11	$uwz\alpha_1$
9	17	9	$2twxz$	10	5	12	$uwz\alpha_4$
9	17	10	w^2xz	10	6	2	= 2 6 10
9	17	11	$xz(tu + vw)$	10	6	6	= 6 6 10
9	17	12	$uwxz$	10	6	8	= 8 6 10
9	17	13	tx^2z	10	6	11	$vwz\alpha_4$
9	17	14	$stxz$	10	9	1	= 1 9 10
9	17	15	$xz(sw + ty)$	10	9	2	= 2 9 10
9	17	16	$wxyz$	10	9	3	= 3 9 10
9	17	17	$xz(tz + wx)$	10	9	6	= 6 9 10
9	17	18	wxz^2	10	9	7	= 7 9 10
9	18	9	t^2z^2	10	9	8	= 8 9 10
9	18	10	twz^2	10	9	9	= 9 9 10
9	18	11	tvz^2	10	9	11	$wz(ty + ux)$
9	18	12	tuz^2	10	9	13	wx^2z
9	18	15	stz^2	10	9	14	$swxz$
9	18	16	tyz^2	10	9	15	$wxyz$
9	18	17	txz^2	10	9	17	wxz^2
9	18	18	tz^3	10	10	3	= 3 10 10
10	3	1	= 1 3 10	10	10	4	= 4 10 10
10	3	2	= 2 3 10	10	10	5	= 5 10 10
10	3	3	= 3 3 10	10	10	6	= 6 10 10
10	3	6	= 6 3 10	10	10	9	= 9 10 10
10	3	7	= 7 3 10	10	10	10	$2w^2z^2$
10	3	8	= 8 3 10	10	10	11	$wz(sw + vz)$
10	3	9	= 9 3 10	10	10	12	$wz(uz + wy)$
10	3	11	$twyz$	10	10	15	swz^2
10	4	3	= 3 4 10	10	10	16	wyz^2
10	4	4	= 4 4 10	10	10	17	wxz^2
10	4	5	= 5 4 10	10	10	18	wz^3
10	4	6	= 6 4 10	10	11	2	= 2 11 10
10	4	9	= 9 4 10	10	11	6	= 6 11 10
10	4	10	w^2z^2	10	11	8	= 8 11 10

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
10	11	11	$swz\alpha_3 + vwz\alpha_4$	11	1	11	= 7 1 11
10	11	14	$swz\alpha_1$	11	1	8	= 8 1 11
10	11	15	$swz\alpha_4$	11	1	9	= 9 1 11
10	12	5	= 5 12 10	11	1	11	t^2y^2
10	12	6	= 6 12 10	11	2	1	= 1 2 11
10	12	11	$uwz\alpha_1 + wyz\alpha_2$	11	2	2	= 2 2 11
10	12	12	$wz(\beta_6 + uy)$	11	2	3	= 3 2 11
10	12	15	$wyz\alpha_1$	11	2	6	= 6 2 11
10	12	16	$wyz\alpha_4$	11	2	7	= 7 2 11
10	15	8	= 8 15 10	11	2	8	= 8 2 11
10	15	11	$swz\alpha_3$	11	2	9	= 9 2 11
10	15	14	$swz\alpha_1$	11	2	11	$tvy\alpha_4 + v\alpha_4\beta_5$
10	15	15	$swz\alpha_4$	11	3	1	= 1 3 11
10	16	11	$wyz\alpha_2$	11	3	2	= 2 3 11
10	16	12	$wyz\alpha_3$	11	3	3	= 3 3 11
10	16	15	$wyz\alpha_1$	11	3	4	= 4 3 11
10	16	16	$wyz\alpha_4$	11	3	5	= 5 3 11
10	17	7	= 7 17 10	11	3	6	= 6 3 11
10	17	8	= 8 17 10	11	3	7	= 7 3 11
10	17	9	= 9 17 10	11	3	8	= 8 3 11
10	17	11	$uwxz$	11	3	9	= 9 3 11
10	17	13	wx^2z	11	3	10	= 10 3 11
10	17	14	$swxz$	11	3	11	$2stwy$
10	17	15	$wxyz$	11	3	12	twy^2
10	17	17	wxz^2	11	4	3	= 3 4 11
10	18	9	= 9 18 10	11	4	4	= 4 4 11
10	18	10	w^2z^2	11	4	5	= 5 4 11
10	18	11	vwz^2	11	4	6	= 6 4 11
10	18	12	uwz^2	11	4	9	= 9 4 11
10	18	15	swz^2	11	4	10	= 10 4 11
10	18	16	wyz^2	11	4	11	s^2w^2
10	18	17	wxz^2	11	4	12	sw^2y
10	18	18	wz^3	11	5	3	= 3 5 11
11	1	1	= 1 1 11	11	5	4	= 4 5 11
11	1	2	= 2 1 11	11	5	5	= 5 5 11
11	1	3	= 3 1 11	11	5	6	= 6 5 11
11	1	6	= 6 1 11	11	5	9	= 9 5 11

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
11	5	10	= 10 5 11	11	8	15	$vxy\alpha_4 + s\alpha_4\beta_5$
11	5	11	$suw\alpha_1 + u\alpha_1\beta_2$	11	8	17	$vxz\alpha_4$
11	5	12	$uwy\alpha_1 + u\alpha_4\beta_2$	11	9	1	= 1 9 11
11	6	1	= 1 6 11	11	9	2	= 2 9 11
11	6	2	= 2 6 11	11	9	3	= 3 9 11
11	6	3	= 3 6 11	11	9	4	= 4 9 11
11	6	4	= 4 6 11	11	9	5	= 5 9 11
11	6	5	= 5 6 11	11	9	6	= 6 9 11
11	6	6	= 6 6 11	11	9	7	= 7 9 11
11	6	7	= 7 6 11	11	9	8	= 8 9 11
11	6	8	= 8 6 11	11	9	9	= 9 9 11
11	6	9	= 9 6 11	11	9	10	= 10 9 11
11	6	10	= 10 6 11	11	9	11	$2suwx + 2tvyz$
11	6	11	$2svw\alpha_4 + 2u\alpha_1\beta_5$	11	9	12	$uy(tz + wx)$
11	6	12	$vwy\alpha_4 + u\alpha_4\beta_5$	11	9	13	swx^2
11	7	1	= 1 7 11	11	9	14	s^2wx
11	7	2	= 2 7 11	11	9	15	$sy(tz + wx)$
11	7	3	= 3 7 11	11	9	16	ty^2z
11	7	6	= 6 7 11	11	9	17	$xz(sw + ty)$
11	7	7	= 7 7 11	11	9	18	tyz^2
11	7	8	= 8 7 11	11	10	3	= 3 10 11
11	7	9	= 9 7 11	11	10	4	= 4 10 11
11	7	11	$2tuxy$	11	10	5	= 5 10 11
11	7	13	tx^2y	11	10	6	= 6 10 11
11	7	14	$stxy$	11	10	9	= 9 10 11
11	7	15	txy^2	11	10	10	= 10 10 11
11	7	17	$txyz$	11	10	11	$2svwz$
11	8	1	= 1 8 11	11	10	12	$wz(su + vy)$
11	8	2	= 2 8 11	11	10	15	s^2wz
11	8	3	= 3 8 11	11	10	16	$swyz$
11	8	6	= 6 8 11	11	10	17	$swxz$
11	8	7	= 7 8 11	11	10	18	swz^2
11	8	8	= 8 8 11	11	11	1	= 1 11 11
11	8	9	= 9 8 11	11	11	2	= 2 11 11
11	8	11	$sty\alpha_3 + uvx\alpha_4 + s\alpha_3\beta_5 + v\alpha_4\beta_2$	11	11	3	= 3 11 11
11	8	13	$vx^2\alpha_4$	11	11	4	= 4 11 11
11	8	14	$svx\alpha_4 + s\alpha_1\beta_5$	11	11	5	= 5 11 11

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination
l	m	n	
11	11	6	= 6 11 11
11	11	7	= 7 11 11
11	11	8	= 8 11 11
11	11	9	= 9 11 11
11	11	10	= 10 11 11
11	11	11	$2u^2x\alpha_1+2v^2z\alpha_4+2s\alpha_3\beta_2+y\alpha_2\beta_5$
11	11	12	$swy\alpha_3 + uvz\alpha_4 + u\alpha_4\beta_2 + y\alpha_3\beta_5$
11	11	13	$ux^2\alpha_1$
11	11	14	$sux\alpha_1 + s\alpha_1\beta_2$
11	11	15	$svz\alpha_4 + uxy\alpha_1 + s\alpha_4\beta_2 + y\alpha_1\beta_5$
11	11	16	$vyz\alpha_4 + y\alpha_4\beta_5$
11	11	17	$uxz\alpha_1 + vxz\alpha_4$
11	11	18	$vz^2\alpha_4$
11	12	3	= 3 12 11
11	12	4	= 4 12 11
11	12	5	= 5 12 11
11	12	6	= 6 12 11
11	12	9	= 9 12 11
11	12	10	= 10 12 11
11	12	11	$2swy\alpha_2 + 2u\alpha_1\beta_5$
11	12	12	$u^2z\alpha_1 + wy^2\alpha_2 + u\alpha_4\beta_5 + y\alpha_3\beta_2$
11	12	15	$suz\alpha_1 + y\alpha_1\beta_2$
11	12	16	$uyz\alpha_1 + y\alpha_4\beta_2$
11	12	17	$uxz\alpha_1$
11	12	18	$uz^2\alpha_1$
11	13	7	= 7 13 11
11	13	8	= 8 13 11
11	13	9	= 9 13 11
11	13	11	u^2x^2
11	13	13	ux^3
11	13	14	sux^2
11	13	15	ux^2y
11	13	17	ux^2z
11	14	7	= 7 14 11
11	14	8	= 8 14 11
11	14	9	= 9 14 11
11	14	11	$sux\alpha_3 + s\alpha_3\beta_2$
11	14	13	$sx^2\alpha_3$
11	14	14	$s^2x\alpha_3 + s\alpha_1\beta_2$
11	14	15	$sxy\alpha_3 + s\alpha_4\beta_2$
11	14	17	$sxz\alpha_3$
11	15	7	= 7 15 11
11	15	8	= 8 15 11
11	15	9	= 9 15 11
11	15	10	= 10 15 11
11	15	11	$2svz\alpha_3 + 2y\alpha_2\beta_2$
11	15	12	$suz\alpha_3 + y\alpha_3\beta_2$
11	15	13	$x^2y\alpha_2$
11	15	14	$sxy\alpha_2 + s\alpha_1\beta_5$
11	15	15	$s^2z\alpha_3 + xy^2\alpha_2 + s\alpha_4\beta_5 + y\alpha_1\beta_2$
11	15	16	$syz\alpha_3 + y\alpha_4\beta_2$
11	15	17	$sxz\alpha_3 + xyz\alpha_2$
11	15	18	$sz^2\alpha_3$
11	16	9	= 9 16 11
11	16	10	= 10 16 11
11	16	11	$vyz\alpha_2 + y\alpha_2\beta_5$
11	16	12	$uyz\alpha_2 + y\alpha_3\beta_5$
11	16	15	$syz\alpha_2 + y\alpha_1\beta_5$
11	16	16	$y^2z\alpha_2 + y\alpha_4\beta_5$
11	16	17	$xyz\alpha_2$
11	16	18	$yz^2\alpha_2$
11	17	7	= 7 17 11
11	17	8	= 8 17 11
11	17	9	= 9 17 11
11	17	10	= 10 17 11
11	17	11	$2uvxz$
11	17	12	u^2xz
11	17	13	vx^2z
11	17	14	$svxz$
11	17	15	$xz(su + vy)$
11	17	16	$uxyz$

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
11	17	17	$sz(ux + vz)$
11	17	18	uxz^2
11	18	9	= 9 18 11
11	18	10	= 10 18 11
11	18	11	v^2z^2
11	18	12	uvz^2
11	18	15	svz^2
11	18	16	vyz^2
11	18	17	vxz^2
11	18	18	vz^3
12	3	1	= 1 3 12
12	3	2	= 2 3 12
12	3	3	= 3 3 12
12	3	6	= 6 3 12
12	3	7	= 7 3 12
12	3	8	= 8 3 12
12	3	9	= 9 3 12
12	3	11	= 11 3 12
12	4	3	= 3 4 12
12	4	4	= 4 4 12
12	4	5	= 5 4 12
12	4	6	= 6 4 12
12	4	9	= 9 4 12
12	4	10	= 10 4 12
12	4	11	= 11 4 12
12	4	12	w^2y^2
12	5	3	= 3 5 12
12	5	4	= 4 5 12
12	5	5	= 5 5 12
12	5	6	= 6 5 12
12	5	9	= 9 5 12
12	5	10	= 10 5 12
12	5	11	= 11 5 12
12	5	12	$uwy\alpha_4 + u\alpha_4\beta_6$
12	6	1	= 1 6 12
12	6	2	= 2 6 12
12	6	3	= 3 6 12
12	6	6	= 6 6 12
12	6	7	= 7 6 12
12	6	8	= 8 6 12
12	6	9	= 9 6 12
12	6	11	= 11 6 12
12	9	1	= 1 9 12
12	9	2	= 2 9 12
12	9	3	= 3 9 12
12	9	6	= 6 9 12
12	9	7	= 7 9 12
12	9	8	= 8 9 12
12	9	9	= 9 9 12
12	9	11	= 11 9 12
12	9	13	wx^2y
12	9	14	$swxy$
12	9	15	wxy^2
12	9	17	$wxyz$
12	10	3	= 3 10 12
12	10	4	= 4 10 12
12	10	5	= 5 10 12
12	10	6	= 6 10 12
12	10	9	= 9 10 12
12	10	10	= 10 10 12
12	10	11	= 11 10 12
12	10	12	$2uwyz$
12	10	15	$swyz$
12	10	16	wy^2z
12	10	17	$wxyz$
12	10	18	wyz^2
12	11	1	= 1 11 12
12	11	2	= 2 11 12
12	11	3	= 3 11 12
12	11	6	= 6 11 12
12	11	7	= 7 11 12
12	11	8	= 8 11 12
12	11	9	= 9 11 12
12	11	11	= 11 11 12

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination
l	m	n	
12	11	13	$ux^2\alpha_4$
12	11	14	$sux\alpha_4 + s\alpha_1\beta_6$
12	11	15	$uxy\alpha_4 + s\alpha_4\beta_6$
12	11	17	$uxz\alpha_4$
12	12	3	= 3 12 12
12	12	4	= 4 12 12
12	12	5	= 5 12 12
12	12	6	= 6 12 12
12	12	9	= 9 12 12
12	12	10	= 10 12 12
12	12	11	= 11 12 12
12	12	12	$u^2z\alpha_4 + wy^2\alpha_3 + \beta_6(\beta_6 + uy)$
12	12	15	$suz\alpha_4 + y\alpha_1\beta_6$
12	12	16	$uyz\alpha_4 + y\alpha_4\beta_6$
12	12	17	$uxz\alpha_4$
12	12	18	$uz^2\alpha_4$
12	15	7	= 7 15 12
12	15	8	= 8 15 12
12	15	9	= 9 15 12
12	15	11	= 11 15 12
12	15	13	$x^2y\alpha_3$
12	15	14	$sxy\alpha_3 + s\alpha_1\beta_6$
12	15	15	$xy^2\alpha_3 + s\alpha_4\beta_6$
12	15	17	$xyz\alpha_3$
12	16	9	= 9 16 12
12	16	10	= 10 16 12
12	16	11	= 11 16 12
12	16	12	$uyz\alpha_3 + y\alpha_3\beta_6$
12	16	15	$syz\alpha_3 + y\alpha_1\beta_6$
12	16	16	$y^2z\alpha_3 + y\alpha_4\beta_6$
12	16	17	$xyz\alpha_3$
12	16	18	$yz^2\alpha_3$
12	17	7	= 7 17 12
12	17	8	= 8 17 12
12	17	9	= 9 17 12
12	17	11	= 11 17 12
12	17	13	ux^2z

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination
l	m	n	
12	17	14	$suxz$
12	17	15	$uxyz$
12	17	17	uxz^2
12	18	9	= 9 18 12
12	18	10	= 10 18 12
12	18	11	= 11 18 12
12	18	12	u^2z^2
12	18	15	suz^2
12	18	16	uyz^2
12	18	17	uxz^2
12	18	18	uz^3
13	7	1	= 1 7 13
13	7	2	= 2 7 13
13	7	3	= 3 7 13
13	7	6	= 6 7 13
13	7	7	= 7 7 13
13	7	8	= 8 7 13
13	7	9	= 9 7 13
13	7	11	= 11 7 13
13	8	2	= 2 8 13
13	8	6	= 6 8 13
13	8	8	= 8 8 13
13	8	11	= 11 8 13
13	9	3	= 3 9 13
13	9	4	= 4 9 13
13	9	5	= 5 9 13
13	9	6	= 6 9 13
13	9	9	= 9 9 13
13	9	10	= 10 9 13
13	9	11	= 11 9 13
13	9	12	= 12 9 13
13	11	5	= 5 11 13
13	11	6	= 6 11 13
13	11	11	= 11 11 13
13	11	12	= 12 11 13
13	13	7	= 7 13 13
13	13	8	= 8 13 13

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
13	13	9	= 9 13 13
13	13	11	= 11 13 13
13	13	13	x^4
13	13	14	sx^3
13	13	15	x^3y
13	13	17	x^3z
13	14	8	= 8 14 13
13	14	11	= 11 14 13
13	14	14	$sx^2\alpha_1$
13	14	15	$sx^2\alpha_4$
13	15	11	= 11 15 13
13	15	12	= 12 15 13
13	15	15	$x^2y\alpha_1$
13	15	16	$x^2y\alpha_4$
13	17	9	= 9 17 13
13	17	10	= 10 17 13
13	17	11	= 11 17 13
13	17	12	= 12 17 13
13	17	15	sx^2z
13	17	16	x^2yz
13	17	17	x^3z
13	17	18	x^2z^2
14	7	1	= 1 7 14
14	7	2	= 2 7 14
14	7	3	= 3 7 14
14	7	6	= 6 7 14
14	7	7	= 7 7 14
14	7	8	= 8 7 14
14	7	9	= 9 7 14
14	7	11	= 11 7 14
14	8	1	= 1 8 14
14	8	2	= 2 8 14
14	8	3	= 3 8 14
14	8	6	= 6 8 14
14	8	7	= 7 8 14
14	8	8	= 8 8 14
14	8	9	= 9 8 14
14	8	11	= 11 8 14
14	9	3	= 3 9 14
14	9	4	= 4 9 14
14	9	5	= 5 9 14
14	9	6	= 6 9 14
14	9	9	= 9 9 14
14	9	10	= 10 9 14
14	9	11	= 11 9 14
14	9	12	= 12 9 14
14	11	3	= 3 11 14
14	11	4	= 4 11 14
14	11	5	= 5 11 14
14	11	6	= 6 11 14
14	11	9	= 9 11 14
14	11	10	= 10 11 14
14	11	11	= 11 11 14
14	11	12	= 12 11 14
14	13	7	= 7 13 14
14	13	8	= 8 13 14
14	13	9	= 9 13 14
14	13	11	= 11 13 14
14	13	13	= 13 13 14
14	13	14	s^2x^2
14	13	15	sx^2y
14	13	17	sx^2z
14	14	7	= 7 14 14
14	14	8	= 8 14 14
14	14	9	= 9 14 14
14	14	11	= 11 14 14
14	14	13	= 13 14 14
14	14	14	$s^2\alpha_1(\alpha_1 + 2x)$
14	14	15	$s^2x\alpha_4 + s\alpha_1\beta_3$
14	14	17	$sxz\alpha_1$
14	15	9	= 9 15 14
14	15	10	= 10 15 14
14	15	11	= 11 15 14
14	15	12	= 12 15 14

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Tab. 3 - (continued)

l, m n and of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
14	15	15	$sxy\alpha_1 + s\alpha_1\beta_3$	15	9	10	= 10 9 15
14	15	16	$sxy\alpha_4 + sy\alpha_1(\alpha_4 + z)$	15	9	11	= 11 9 15
14	15	17	$sxz\alpha_1$	15	9	12	= 12 9 15
14	15	18	$sz^2\alpha_1$	15	10	3	= 3 10 15
14	17	9	= 9 17 14	15	10	4	= 4 10 15
14	17	10	= 10 17 14	15	10	5	= 5 10 15
14	17	11	= 11 17 14	15	10	6	= 6 10 15
14	17	12	= 12 17 14	15	10	9	= 9 10 15
14	17	15	s^2xz	15	10	10	= 10 10 15
14	17	16	sxyz	15	10	11	= 11 10 15
14	17	17	sx^2z	15	10	12	= 12 10 15
14	17	18	sxz^2	15	11	1	= 1 11 15
15	7	1	= 1 7 15	15	11	2	= 2 11 15
15	7	2	= 2 7 15	15	11	3	= 3 11 15
15	7	3	= 3 7 15	15	11	4	= 4 11 15
15	7	6	= 6 7 15	15	11	5	= 5 11 15
15	7	7	= 7 7 15	15	11	6	= 6 11 15
15	7	8	= 8 7 15	15	11	7	= 7 11 15
15	7	9	= 9 7 15	15	11	8	= 8 11 15
15	7	11	= 11 7 15	15	11	9	= 9 11 15
15	8	1	= 1 8 15	15	11	10	= 10 11 15
15	8	2	= 2 8 15	15	11	11	= 11 11 15
15	8	3	= 3 8 15	15	11	12	= 12 11 15
15	8	6	= 6 8 15	15	12	3	= 3 12 15
15	8	7	= 7 8 15	15	12	4	= 4 12 15
15	8	8	= 8 8 15	15	12	5	= 5 12 15
15	8	9	= 9 8 15	15	12	6	= 6 12 15
15	8	11	= 11 8 15	15	12	9	= 9 12 15
15	9	1	= 1 9 15	15	12	10	= 10 12 15
15	9	2	= 2 9 15	15	12	11	= 11 12 15
15	9	3	= 3 9 15	15	12	12	= 12 12 15
15	9	4	= 4 9 15	15	13	7	= 7 13 15
15	9	5	= 5 9 15	15	13	8	= 8 13 15
15	9	6	= 6 9 15	15	13	9	= 9 13 15
15	9	7	= 7 9 15	15	13	11	= 11 13 15
15	9	8	= 8 9 15	15	13	13	= 13 13 15
15	9	9	= 9 9 15	15	13	14	= 14 13 15

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination
15	13	15	x^2y^2
15	13	17	x^2yz
15	14	7	= 7 14 15
15	14	8	= 8 14 15
15	14	9	= 9 14 15
15	14	11	= 11 14 15
15	14	13	= 13 14 15
15	14	14	= 14 14 15
15	14	15	$sxy\alpha_4 + s\alpha_4\beta_3$
15	14	17	$sxz\alpha_4$
15	15	7	= 7 15 15
15	15	8	= 8 15 15
15	15	9	= 9 15 15
15	15	10	= 10 15 15
15	15	11	= 11 15 15
15	15	12	= 12 15 15
15	15	13	= 13 15 15
15	15	14	= 14 15 15
15	15	15	$s^2z\alpha_4 + xy^2\alpha_1 + \beta_3(\beta_3 + sy)$
15	15	16	$syz\alpha_4 + y\alpha_4\beta_3$
15	15	17	$sxz\alpha_4 + xyz\alpha_1$
15	15	18	$sz^2\alpha_4$
15	16	9	= 9 16 15
15	16	10	= 10 16 15
15	16	11	= 11 16 15
15	16	12	= 12 16 15
15	16	15	$syz\alpha_1 + y\alpha_1\beta_3$
15	16	16	$syz\alpha_4 + y^2\alpha_1(\alpha_4 + z)$
15	16	17	$xyz\alpha_1$
15	16	18	$yz^2\alpha_1$
15	17	7	= 7 17 15
15	17	8	= 8 17 15
15	17	9	= 9 17 15
15	17	10	= 10 17 15
15	17	11	= 11 17 15
15	17	12	= 12 17 15
15	17	13	= 13 17 15
15	17	14	= 14 17 15
15	17	15	$2sxyz$
15	17	16	xy^2z
15	17	17	$xz(sz + xy)$
15	17	18	xyz^2
15	18	9	= 9 18 15
15	18	10	= 10 18 15
15	18	11	= 11 18 15
15	18	12	= 12 18 15
15	18	15	s^2z^2
15	18	16	syz^2
15	18	17	sxz^2
15	18	18	sz^3
16	9	1	= 1 9 16
16	9	2	= 2 9 19
16	9	3	= 3 9 16
16	9	6	= 6 9 16
16	9	7	= 7 9 16
16	9	8	= 8 9 16
16	9	9	= 9 9 16
16	9	11	= 11 9 16
16	10	3	= 3 10 16
16	10	4	= 4 10 16
16	10	5	= 5 10 16
16	10	6	= 6 10 16
16	10	9	= 9 10 16
16	10	10	= 10 10 16
16	10	11	= 11 10 16
16	10	12	= 12 10 16
16	11	1	= 1 11 16
16	11	2	= 2 11 16
16	11	3	= 3 11 16
16	11	6	= 6 11 16
16	11	7	= 7 11 16
16	11	8	= 8 11 16
16	11	9	= 9 11 16

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father			Probability of mother-child-father combination
l	m	n		l	m	n	
16	11	11	= 11 11 16	16	18	15	= 15 18 16
16	12	3	= 3 12 16	16	18	16	y^2z^2
16	12	4	= 4 12 16	16	18	17	xyz^2
16	12	5	= 5 12 16	16	18	18	yz^3
16	12	6	= 6 12 16	17	7	1	= 1 7 17
16	12	9	= 9 12 16	17	7	2	= 2 7 17
16	12	10	= 10 12 16	17	7	3	= 3 7 17
16	12	11	= 11 12 16	17	7	6	= 6 7 17
16	12	12	= 12 12 16	17	7	7	= 7 7 17
16	15	7	= 7 15 16	17	7	8	= 8 7 17
16	15	8	= 8 15 16	17	7	9	= 9 7 17
16	15	9	= 9 15 16	17	7	11	= 11 7 17
16	15	11	= 11 15 16	17	8	2	= 2 8 17
16	15	13	= 13 15 16	17	8	6	= 6 8 17
16	15	14	= 14 15 16	17	8	8	= 8 8 17
16	15	15	= 15 15 16	17	8	11	= 11 8 17
16	15	17	$xyz\alpha_4$	17	9	1	= 1 9 17
16	16	9	= 9 16 16	17	9	2	= 2 9 17
16	16	10	= 10 16 16	17	9	3	= 3 9 17
16	16	11	= 11 16 16	17	9	4	= 4 9 17
16	16	12	= 12 16 16	17	9	5	= 5 9 17
16	16	15	= 15 16 16	17	9	6	= 6 9 17
16	16	16	$y^2\alpha_4(\alpha_4 + 2z)$	17	9	7	= 7 9 17
16	16	17	$xyz\alpha_4$	17	9	8	= 8 9 17
16	16	18	$yz^2\alpha_4$	17	9	9	= 9 9 17
16	17	7	= 7 17 16	17	9	10	= 10 9 17
16	17	8	= 8 17 16	17	9	11	= 11 9 17
16	17	9	= 9 17 16	17	9	12	= 12 9 17
16	17	11	= 11 17 16	17	10	3	= 3 10 17
16	17	13	= 13 17 16	17	10	4	= 4 10 17
16	17	14	= 14 17 16	17	10	5	= 5 10 17
16	17	15	= 15 17 16	17	10	6	= 6 10 17
16	17	17	xyz^2	17	10	9	= 9 10 17
16	18	9	= 9 18 16	17	10	10	= 10 10 17
16	18	10	= 10 18 16	17	10	11	= 11 10 17
16	18	11	= 11 18 16	17	10	12	= 12 10 17
16	18	12	= 12 18 16	17	11	2	= 2 11 17

Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l	m	n	Probality of mother-child-father combination
17	11	5	= 5 11 17	17	17	13	= 13 17 17
17	11	6	= 6 11 17	17	17	14	= 14 17 17
17	11	8	= 8 11 17	17	17	15	= 15 17 17
17	11	11	= 11 11 17	17	17	16	= 16 17 17
17	11	12	= 12 11 17	17	17	17	$2x^2z^2$
17	12	5	= 5 12 17	17	17	18	zx^3
17	12	7	= 7 12 17	17	18	9	= 9 18 17
17	12	11	= 11 12 17	17	18	10	= 10 18 17
17	12	12	= 12 12 17	17	18	11	= 11 18 17
17	13	7	= 7 13 17	17	18	12	= 12 18 17
17	13	8	= 8 13 17	17	18	15	= 15 18 17
17	13	9	= 9 13 17	17	18	16	= 16 18 17
17	13	11	= 11 13 17	17	18	17	x^2z^2
17	13	13	= 13 13 17	17	18	18	xz^3
17	13	14	= 14 13 17	18	9	1	= 1 9 18
17	13	15	= 15 13 17	18	9	2	= 2 9 18
17	13	17	x^2z^2	18	9	3	= 3 9 18
17	14	8	= 8 14 17	18	9	6	= 6 9 18
17	14	11	= 11 14 17	18	9	7	= 7 9 18
17	14	14	= 14 14 17	18	9	8	= 8 9 18
17	14	15	= 15 14 17	18	9	9	= 9 9 18
17	15	8	= 8 15 17	18	9	11	= 11 9 18
17	15	11	= 11 15 17	18	10	3	= 3 10 18
17	15	12	= 12 15 17	18	10	4	= 4 10 18
17	15	14	= 14 15 17	18	10	5	= 5 10 18
17	15	15	= 15 15 17	18	10	6	= 6 10 18
17	15	16	= 16 15 17	18	10	9	= 9 10 18
17	16	11	= 11 16 17	18	10	10	= 10 10 18
17	16	12	= 12 16 17	18	10	11	= 11 10 18
17	16	15	= 15 16 17	18	10	12	= 12 10 18
17	16	16	= 16 16 17	18	11	2	= 2 11 18
17	17	7	= 7 17 17	18	11	6	= 6 11 18
17	17	8	= 8 17 17	18	11	8	= 8 11 18
17	17	9	= 9 17 17	18	11	11	= 11 11 18
17	17	10	= 10 17 17	18	12	5	= 5 12 18
17	17	11	= 11 17 17	18	12	7	= 7 12 18
17	17	12	= 12 17 17	18	12	11	= 11 12 18

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Tab. 3 - (continued)

l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l m n	Probability of mother-child-father combination	l, m and n of $H_{l \cdot m \cdot n}$ which reveal phenotypes of mother, child and father l m n	Probability of mother-child-father combination
18 12 12	= 12 12 18	18 17 11	= 11 17 18
18 15 8	= 8 15 18	18 17 13	= 13 17 18
18 15 11	= 11 15 18	18 17 14	= 14 17 18
18 15 14	= 14 15 18	18 17 15	= 15 17 18
18 15 15	= 15 15 18	18 17 17	= 17 17 18
18 16 11	= 11 16 18	18 18 9	= 9 18 18
18 16 12	= 12 16 18	18 18 10	= 10 18 18
18 16 15	= 15 16 18	18 18 11	= 11 18 18
18 16 16	= 16 16 18	18 18 13	= 13 18 18
		18 18 15	= 15 18 18
18 17 7	= 7 17 18	18 18 16	= 16 18 18
18 17 8	= 8 17 18	18 18 17	= 17 18 18
18 17 9	= 9 17 18	18 18 18	z^4

$$
\begin{aligned}
\alpha_1 &= s + x \\
\alpha_2 &= t + v \\
\alpha_3 &= u + w \\
\alpha_4 &= y + z \\
\beta_1 &= st + sv + vx \\
\beta_2 &= su + sw + ux \\
\beta_3 &= sy + sz + xy \\
\beta_4 &= tu + uv + vw \\
\beta_5 &= ty + vy + vz \\
\beta_6 &= uy + uz + wy
\end{aligned}
$$

Tab. 4 - Probability of mother-child combination ($K_{l \cdot m}$)

l and m of $K_{l \cdot m}$ which reveal phenotypes of mother and child		Probability of mother-child combination	l and m of $K_{l \cdot m}$ which reveal phenotypes of mother and child		Probability of mother-child combination
l or m	m l		l or m	m l	
1	1	t^3	5	11	$suw + u(s + x)(u + w)$
1	2	t^2v	5	12	$uwy + u(u + w)(y + z)$
1	3	t^2w	6	6	$tu^2+v^2w+u(t+v)^2+v(u+w)^2$
1	6	t^2u	6	7	tux
1	7	t^2x	6	8	$stu + v(s + x)(u + w)$
1	8	st^2	6	9	$tuz + vwx$
1	9	t^2z	6	10	vwz
1	11	t^2y	6	11	$svw + tuy + u(s + x)(t + v) + v(u + w)(y + z)$
2	2	$tv^2 + v(t + v)^2$	6	12	$vwy + u(t + v)(y + z)$
2	3	tvw	7	7	$tx(t + x)$
2	6	$tuv + v(t + v)(u + w)$	7	8	$tx(s + v)$
2	7	tvx	7	9	$tx(w + z)$
2	8	$stv + v(s + x)(t + v)$	7	11	$tx(u + y)$
2	9	tvz	7	13	tx^2
2	11	$tvy + v(t + v)(y + z)$	7	14	stx
3	3	$tw(t + w)$	7	15	txy
3	4	tw^2	7	17	txz
3	5	tuw	8	8	$s^2t+v^2x+s(t+v)^2+v(s+x)^2$
3	6	$tw(u + v)$	8	9	$stz + vwx$
3	7	twx	8	11	$sty + uvx + s(t + v)(u + w) + v(s + x)(y + z)$
3	8	stw	8	13	vx^2
3	9	$tw(x + z)$	8	14	$svx + s(s + x)(t + v)$
3	10	twz	8	15	$vxy + s(t + v)(y + z)$
3	11	$tw(s + y)$	8	17	vxz
3	12	twy	9	9	$tz(t + z) + wx(w + x)$
4	4	w^3	9	10	$wz(t + x)$
4	5	uw^2	9	11	$tz(v + y) + wx(s + u)$
4	6	vw^2	9	12	$tuz + wxy$
4	9	w^2x	9	13	wx^2
4	10	w^2z	9	14	swx
4	11	sw^2	9	15	$stz + wxy$
4	12	w^2y	9	16	tyz
5	5	$u^2w + u(u + w)^2$	9	17	$xz(t + w)$
5	6	$uvw + u(t + v)(u + w)$			
5	9	uwx			
5	10	uwz			

Tab. 4 - (continued)

l and m of $K_{l \cdot m}$ which reveal phenotypes of mother and child: l / m or m / l		Probability of mother-child combination
9	18	tz^2
10	10	$wz(w + z)$
10	11	$wz(s + v)$
10	12	$wz(u + y)$
10	15	swz
10	16	wyz
10	17	wxz
10	18	wz^2
11	11	$s^2w+ty^2+u^2x+v^2z+s(u+w)^2+u(s+x)^2+v(y+z)^2+y(t+v)^2$
11	12	$swy + uvz + u(s + x)(y + z) + y(t + v)(u + w)$
11	13	ux^2
11	14	$sux + s(s + x)(u + w)$
11	15	$svz + uxy + s(u + w)(y + z) + y(s + x)(t + v)$
11	16	$vyz + y(t + v)(y + z)$
11	17	$xz(u + v)$
11	18	vz^2
12	12	$u^2z+wy^2+u(y+z)^2+y(u+w)^2$
12	15	$suz + y(s + x)(u + w)$
12	16	$uyz + y(u + w)(y + z)$
12	17	uxz
12	18	uz^2
13	13	x^2
13	14	sx^2
13	15	x^2y
13	17	x^2z
14	14	$s^2x + s(s + x)^2$
14	15	$sxy + s(s + x)(y + z)$
14	17	sxz
15	15	$s^2z+xy^2+s(y+z)^2+y(s+x)^2$
15	16	$syz + y(s + x)(y + z)$
15	17	$xz(s + y)$
15	18	sz^2
16	16	$y^2z + y(y + z)^2$
16	17	xyz
16	18	yz^2
17	17	$xz(x + z)$
17	18	xz^2
18	18	z^3

Tab. 5

Phenotype tested with 4 antisera	Anti- C D E c	Genetic constitution of phenotype	Frequency of phenotype
Ph-1′	— — — +	Ph-1	t^2
Ph-2′	— + — +	Ph-2	$v(2t + v)$
Ph-3′	— — + +	Ph-3 + Ph-4	$w(2t + w)$
Ph-4′	— + + +	Ph-5 + Ph-6	$u^2 + 2(tu + uv + uw + vw)$
Ph-5′	+ — — +	Ph-7	$2tx$
Ph-6′	+ + — +	Ph-8	$2(st + sv + vx)$
Ph-7′	+ — + +	Ph-9 + Ph-10	$2(tz + wx + wz)$
Ph-8′	+ + + +	Ph-11 + Ph-12	$2(su+sw+ty+ux+uy+uz+vy+vz+wy)$
Ph-9′	+ — — —	Ph-13	x^2
Ph-10′	+ + — —	Ph-14	$s(s + 2x)$
Ph-11′	+ + + —	Ph-15 + Ph-16	$y^2 + 2(sy + sz + xy + yz)$
Ph-12′	+ — + —	Ph-17 + Ph-18	$z(2x + z)$

Tab. 6 - Probability of mother-child combination ($K_{l'\cdot m'}$)

l' and m' of $K_{l'\cdot m'}$ which reveal phenotypes of mother and child l'	m'	Probability of mother-child combination	l' and m' of $K_{l'\cdot m'}$ which reveal phenotypes of mother and child l'	m'	Probability of mother-child combination
1'	1'	$K_{1\cdot 1}$	5'	5'	$K_{7\cdot 7}$
1'	2'	$K_{1\cdot 2}$	5'	6'	$K_{7\cdot 8}$
1'	3'	$K_{1\cdot 3}$	5'	7'	$K_{7\cdot 9}$
1'	4'	$K_{1\cdot 6}$	5'	8'	$K_{7\cdot 11}$
1'	5'	$K_{1\cdot 7}$	5'	9'	$K_{7\cdot 13}$
1'	6'	$K_{1\cdot 8}$	5'	10'	$K_{7\cdot 14}$
1'	7'	$K_{1\cdot 9}$	5'	11'	$K_{7\cdot 15}$
1'	8'	$K_{1\cdot 11}$	5'	12'	$K_{7\cdot 17}$
2'	1'	$K_{2\cdot 1}$	6'	1'	$K_{8\cdot 1}$
2'	2'	$K_{2\cdot 2}$	6'	2'	$K_{8\cdot 2}$
2'	3'	$K_{2\cdot 3}$	6'	3'	$K_{8\cdot 3}$
2'	4'	$K_{2\cdot 6}$	6'	4'	$K_{8\cdot 6}$
2'	5'	$K_{2\cdot 7}$	6'	5'	$K_{8\cdot 7}$
2'	6'	$K_{2\cdot 8}$	6'	6'	$K_{8\cdot 8}$
2'	7'	$K_{2\cdot 9}$	6'	7'	$K_{8\cdot 9}$
2'	8'	$K_{2\cdot 11}$	6'	8'	$K_{8\cdot 11}$
3'	1'	$K_{3\cdot 1}$	6'	9'	$K_{8\cdot 13}$
3'	2'	$K_{3\cdot 2}$	6'	10'	$K_{8\cdot 14}$
3'	3'	$K_{3\cdot 3} + K_{3\cdot 4} + K_{4\cdot 3} + K_{4\cdot 4}$	6'	11'	$K_{8\cdot 15}$
3'	4'	$K_{3\cdot 5} + K_{3\cdot 6} + K_{4\cdot 5} + K_{4\cdot 6}$	6'	12'	$K_{8\cdot 17}$
3'	5'	$K_{3\cdot 7}$	7'	1'	$K_{9\cdot 1}$
3'	6'	$K_{3\cdot 8}$	7'	2'	$K_{9\cdot 2}$
3'	7'	$K_{3\cdot 9} + K_{3\cdot 10} + K_{4\cdot 9} + K_{4\cdot 10}$	7'	3'	$K_{9\cdot 3} + K_{9\cdot 4} + K_{10\cdot 3} + K_{10\cdot 4}$
3'	8'	$K_{3\cdot 11} + K_{3\cdot 12} + K_{4\cdot 11} + K_{4\cdot 12}$	7'	4'	$K_{9\cdot 5} + K_{9\cdot 6} + K_{10\cdot 5} + K_{10\cdot 6}$
4'	1'	$K_{6\cdot 1}$	7'	5'	$K_{9\cdot 7}$
4'	2'	$K_{6\cdot 2}$	7'	6'	$K_{9\cdot 8}$
4'	3'	$K_{5\cdot 3} + K_{5\cdot 4} + K_{6\cdot 3} + K_{6\cdot 4}$	7'	7'	$K_{9\cdot 9} + K_{9\cdot 10} + K_{10\cdot 9} + K_{10\cdot 10}$
4'	4'	$K_{5\cdot 5} + K_{5\cdot 6} + K_{6\cdot 5} + K_{6\cdot 6}$	7'	8'	$K_{9\cdot 11} + K_{9\cdot 12} + K_{10\cdot 11} + K_{10\ 12}$
4'	5'	$K_{6\cdot 7}$	7'	9'	$K_{9\ 13}$
4'	6'	$K_{6\cdot 8}$	7'	10'	$K_{9\cdot 14}$
4'	7'	$K_{5\cdot 9} + K_{5\cdot 10} + K_{6\cdot 9} + K_{6\cdot 10}$	7'	11'	$K_{9\cdot 15} + K_{9\cdot 16} + K_{10\cdot 15} + K_{10\cdot 16}$
4'	8'	$K_{5\cdot 11} + K_{5\cdot 12} + K_{6\cdot 11} + K_{6\cdot 12}$	7'	12'	$K_{9\cdot 17} + K_{9\cdot 18} + K_{10\cdot 17} + K_{10\cdot 18}$
5'	1'	$K_{7\cdot 1}$	8'	1'	$K_{11\cdot 1}$
5'	2'	$K_{7\cdot 2}$	8'	2'	$K_{11\cdot 2}$
5'	3'	$K_{7\cdot 3}$	8'	3'	$K_{11\cdot 3} + K_{11\cdot 4} + K_{12\cdot 3} + K_{12\cdot 4}$
5'	4'	$K_{7\cdot 6}$	8'	4'	$K_{11\cdot 5} + K_{11\cdot 6} + K_{12\cdot 5} + K_{12\cdot 6}$

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Tab. 6 - (continued)

l' and m' of $K_{l'\cdot m'}$ wchih reveal phenotypes of mother and child l'	m'	Probability of mother-child combination	l' and m' $K_{l'\cdot m'}$ which reveal phenotypes of mother and child l'	m'	Probability of mother-child combination
8'	5'	$K_{11\cdot 7}$	10'	9'	$K_{14\cdot 13}$
8'	6'	$K_{11\cdot 8}$	10'	10'	$K_{14\cdot 14}$
8'	7'	$K_{11\cdot 9} + K_{11\cdot 10} + K_{12\cdot 9} + K_{12\cdot 10}$	10'	11'	$K_{14\cdot 15}$
8'	8'	$K_{11\cdot 11} + K_{11\cdot 12} + K_{12\cdot 11} + K_{12\cdot 12}$	10'	12'	$K_{14\cdot 17}$
8'	9'	$K_{11\cdot 13}$	11'	5'	$K_{15\cdot 7}$
8'	10'	$K_{11\cdot 14}$	11'	6'	$K_{15\cdot 8}$
8'	11'	$K_{11\cdot 15} + K_{11\cdot 16} + K_{12\cdot 15} + K_{12\cdot 16}$	11'	7'	$K_{15\cdot 9} + K_{15\cdot 10} + K_{16\cdot 9} + K_{16\cdot 10}$
8'	12'	$K_{11\cdot 17} + K_{11\cdot 18} + K_{12\cdot 17} + K_{12\cdot 18}$	11'	8'	$K_{15\cdot 11} + K_{15\cdot 12} + K_{16\cdot 11} + K_{16\cdot 12}$
9'	5'	$K_{13\cdot 7}$	11'	9'	$K_{15\cdot 13}$
9'	6'	$K_{13\cdot 8}$	11'	10'	$K_{15\cdot 14}$
9'	7'	$K_{13\cdot 9}$	11'	11'	$K_{15\cdot 15} + K_{15\cdot 16} + K_{16\cdot 15} + K_{16\cdot 16}$
9'	8'	$K_{13\cdot 11}$	11'	12'	$K_{15\cdot 17} + K_{15\cdot 18} + K_{16\cdot 17} + K_{16\cdot 18}$
9'	9'	$K_{13\cdot 13}$	12'	5'	$K_{17\cdot 7}$
9'	10'	$K_{13\cdot 14}$	12'	6'	$K_{17\cdot 8}$
9'	11'	$K_{13\cdot 15}$	12'	7'	$K_{17\cdot 9} + K_{17\cdot 10} + K_{18\cdot 9} + K_{18\cdot 10}$
9'	12'	$K_{13\cdot 17}$	12'	8'	$K_{17\cdot 11} + K_{17\cdot 12} + K_{18\cdot 11} + K_{18\cdot 12}$
10'	5'	$K_{14\cdot 7}$	12'	9'	$K_{17\cdot 13}$
10'	6'	$K_{14\cdot 8}$	12'	10'	$K_{17\cdot 14}$
10'	7'	$K_{14\cdot 9}$	12'	11'	$K_{17\cdot 15} + K_{17\cdot 16} + K_{18\cdot 15} + K_{18\cdot 16}$
10'	8'	$K_{14\cdot 11}$	12'	12'	$K_{17\cdot 17} + K_{17\cdot 18} + K_{18\cdot 17} + K_{18\cdot 18}$

Tab. 7

Phenotype tested with 3 antisera	Anti- C D E	Genetic constitution of phenotype	Frequency of phenotype
Ph-1″	— — —	Ph-1	t^2
Ph-2″	— + —	Ph-2	$v(2t + v)$
Ph-3″	— — +	Ph-3 + Ph-4	$w(2t + w)$
Ph-4″	— + +	Ph-5 + Ph-6	$u^2 + 2(tu + uv + uw + vw)$
Ph-5″	+ — —	Ph-7 + Ph-13	$x(2t + x)$
Ph-6″	+ + —	Ph-8 + Ph-14	$s^2 + 2(st + sv + sx + vx)$
Ph-7″	+ — +	Ph-9 + Ph-10 + Ph-17 + Ph-18	$z^2 + 2(tz + wx + wz + xz)$
Ph-8″	+ + +	Ph-11 + Ph 12 + Ph-15 + Ph-16	$y^2 + 2(su+sw+sy+sz+ty+ux+uy+uz + vy + vz + wy + xy + yz)$

Tab. 8 - Probability of mother-child combination ($K_{l''\cdot m''}$)

l'' and m'' of $K_{l''\cdot m''}$ which reveal phenotyps of mother and child l'' m''	Probabability of mother-child combination
1'' 1''	$K_{1\cdot1}$
1'' 2''	$K_{1\cdot2}$
1'' 3''	$K_{1\cdot3}$
1'' 4''	$K_{1\cdot6}$
1'' 5''	$K_{1\cdot7}$
1'' 6''	$K_{1\cdot8}$
1'' 7''	$K_{1\cdot9}$
1'' 8''	$K_{1\cdot11}$
2'' 1''	$K_{2\cdot1}$
2'' 2''	$K_{2\cdot2}$
2'' 3''	$K_{2\cdot3}$
2'' 4''	$K_{2\cdot6}$
2'' 5''	$K_{2\cdot7}$
2'' 6''	$K_{2\cdot8}$
2'' 7''	$K_{2\cdot9}$
2'' 8''	$K_{2\cdot11}$
3'' 1''	$K_{3\cdot1}$
3'' 2''	$K_{3\cdot2}$
3'' 3''	$K_{3\cdot3} + K_{3\cdot4} + K_{4\cdot3} + K_{4\cdot4}$
3'' 4''	$K_{3\cdot5} + K_{3\cdot6} + K_{4\cdot5} + K_{4\cdot6}$
3'' 5''	$K_{3\cdot7}$
3'' 6''	$K_{3\cdot8}$
3'' 7''	$K_{3\cdot9} + K_{3\cdot10} + K_{4\cdot9} + K_{4\cdot10}$
3'' 8''	$K_{3\cdot11} + K_{3\cdot12} + K_{4\cdot11} + K_{4\cdot12}$
4'' 1''	$K_{6\cdot1}$
4'' 2''	$K_{6\cdot2}$
4'' 3''	$K_{5\cdot3} + K_{5\cdot4} + K_{6\cdot3} + K_{6\cdot4}$
4'' 4''	$K_{5\cdot5} + K_{5\cdot6} + K_{6\cdot5} + K_{6\cdot6}$
4'' 5''	$K_{6\cdot7}$
4'' 6''	$K_{6\cdot8}$
4'' 7''	$K_{5\cdot9} + K_{5\cdot10} + K_{6\cdot9} + {}_{6\cdot10}$
4'' 8''	$K_{5\cdot11} + {}_{5\cdot12} + K_{6\cdot11} + {}_{6\cdot12}$
5'' 1''	$K_{7\cdot1}$
5'' 2''	$K_{7\cdot2}$
5'' 3''	$K_{7\cdot3}$
5'' 4''	$K_{7\cdot6}$
5'' 5''	$K_{7\cdot7} + K_{7\cdot13} + K_{13\cdot7} + K_{13\cdot13}$
5'' 6''	$K_{7\cdot8} + K_{7\cdot14} + K_{13\cdot8} + K_{13\cdot14}$
5'' 7''	$K_{7\cdot9} + K_{7\cdot17} + K_{13\cdot9} + K_{13\cdot17}$
5'' 8''	$K_{7\cdot11} + K_{7\cdot15} + K_{13\cdot11} + K_{13\cdot15}$
6'' 1''	$K_{8\cdot1}$
6'' 2''	$K_{8\cdot2}$
6'' 3''	$K_{8\cdot3}$
6'' 4''	$K_{8\cdot6}$
6'' 5''	$K_{8\cdot7} + K_{8\cdot13} + K_{14\cdot7} + K_{14\cdot13}$
6'' 6''	$K_{8\cdot8} + K_{8\cdot14} + K_{14\cdot8} + K_{14\cdot14}$
6'' 7''	$K_{8\cdot9} + K_{8\cdot17} + K_{14\cdot9} + K_{14\cdot17}$
6'' 8''	$K_{8\cdot11} + K_{8\cdot15} + K_{14\cdot11} + K_{14\cdot15}$
7'' 1''	$K_{9\cdot1}$
7'' 2''	$K_{9\cdot2}$
7'' 3''	$K_{9\cdot3} + K_{9\cdot4} + K_{10\cdot3} + K_{10\cdot4}$
7'' 4''	$K_{9\cdot5} + K_{9\cdot6} + K_{10\cdot5} + K_{10\cdot6}$
7'' 5''	$K_{9\cdot7} + K_{9\cdot13} + K_{17\cdot7} + K_{17\cdot13}$
7'' 6''	$K_{9\cdot8} + K_{9\cdot14} + K_{17\cdot8} + K_{17\cdot14}$
7'' 7''	$K_{9\cdot9} + K_{9\cdot10} + K_{9\cdot17} + K_{9\cdot18}$
	$+K_{10\cdot9}+K_{10\cdot10}+K_{10\cdot17}+K_{10\cdot18}$
	$+K_{17\cdot9}+K_{17\cdot10}+K_{17\cdot17}+K_{17\cdot18}$
	$+K_{18\cdot9}+K_{18\cdot10}+K_{18\cdot17}+K_{18\cdot18}$
7'' 8''	$K_{9\cdot11} + K_{9\cdot12} + K_{9\cdot15} + K_{9\cdot16}$
	$+K_{10\cdot11}+K_{10\cdot12}+K_{10\cdot15}+K_{10\cdot16}$
	$+K_{17\cdot11}+K_{17\cdot12}+K_{17\cdot15}+K_{17\cdot16}$
	$+K_{18\cdot11}+K_{18\cdot12}+K_{18\cdot15}+K_{18\cdot16}$
8'' 1''	$K_{11\cdot1}$
8'' 2''	$K_{11\cdot2}$
8'' 3''	$K_{11\cdot3} + K_{11\cdot4} + K_{12\cdot3} + K_{12\cdot4}$
8'' 4''	$K_{11\cdot5} + K_{11\cdot6} + K_{12\cdot5} + K_{12\cdot6}$
8'' 5''	$K_{11\cdot7} + K_{11\cdot13} + K_{15\cdot7} + K_{15\cdot13}$
8'' 6''	$K_{11\cdot8} + K_{11\cdot14} + K_{15\cdot8} + K_{15\cdot14}$
8'' 7''	$K_{11\cdot9} + K_{11\cdot10} + K_{11\cdot17} + {}_{11\cdot18}$
	$+{}_{12\cdot9}+K_{12\cdot10}+K_{12\cdot17}+K_{12\cdot18}$
	$+K_{15\cdot9}+K_{15\cdot10}+K_{15\cdot17}+{}_{15\cdot18}$
	$+{}_{16\cdot9}+K_{16\cdot10}+K_{16\cdot17}+{}_{16\cdot18}$
8'' 8''	$K_{11\cdot11} + K_{11\cdot12} + K_{11\cdot15} + K_{11\cdot16}$
	$+K_{12\cdot11}+K_{12\cdot12}+K_{12\cdot15}+K_{12\cdot16}$
	$+K_{15\cdot11}+K_{15\cdot12}+K_{15\cdot15}+K_{15\cdot16}$
	$+K_{16\cdot11}+K_{16\cdot12}+K_{16\cdot15}+K_{16\cdot16}$

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SOMMARIO

Un metodo di calcolare la probabilità della paternità, basato sul teorema di Bayes, è stato sviluppato per i gruppi di sangue Rh. Sono state preparate delle tabelle per il calcolo della probabilità di paternità per i livelli più comuni di prove, cioè, con cinque antisieri (anti-C, -D, -E, -c, -e), con quattro (anti-C, -D, -E, -c) e con tre (anti-C, -D, -E). Le stesse probabilità negli altri livelli di prove possono essere calcolate applicando il metodo e le tabelle presentati in questa relazione.

Poichè le tabelle qui presentate includono i calcoli per i cromosomi che appaiono molto raramente, per quanto si sa al presente, queste tabelle si possono applicare a tutte le popolazioni con differenti frequenze cromatiche.