DYNAMICAL PROBLEMS IN THE PREPARATION OF THE CATALOGUE OF ONE-APPARITION COMETS

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Subtle effects influencing the results of orbit determination have to be taken into consideration when recomputing the cometary orbits for the Catalogue. It is very important to apply the numerical integration method, taking into account the specific conditions of the comet's motion, which almost eliminates a cumulation of numerical errors of integration. In this point of view the recurrent power series integration is adapted for cometary orbital computations. The optimal value of step-size of integration according to the required accuracy is calculated at every integration step. The method gives excellent results for "normal" comets as well as in the cases of close approaches of a comet to the Sun and to the planets.

The idea of recomputing all the orbits of long-period comets arose about ten years ago. The work was undertaken at the Warsaw Astronomical Observatory with the cooperation of Slovakian astronomers at the Astronomical Institute in Bratislava and Tatranska Lomnica. To make reasonable the idea of the new catalogue, the following problems have been taken into consideration when recomputing the cometary orbits:

1. to collect all the observations of one-apparition comets and to reduce them to the one system of star catalogues;
2. to define precisely various types of observations and to include in them the corrections of precession, aberration, etc.;
3. to determine the mathematical criteria for eliminating and weighting the observations; and
4. to take into account nongravitational effects in the comet's motion.

Some of these problems were solved or mathematically interpreted by Dr. M. Bielicki (1970), Bielicki and Ziolkowski (1976), who have a great deal of experience in orbital computations having worked more than forty years on the orbits of the periodic comets Wolf 1 and Kopff.

R. L. Duncombe (ed.), Dynamics of the Solar System, 289–292.

There is also a problem which we cannot avoid in the cometary investigations: numerical integration of equations of motion of a comet. It is important to take this problem into consideration carefully since the construction of the catalogue including many subtle effects in the comet's motion could be destroyed by the simple cumulation of numerical errors when integrating the equations of motion.

In the last few years a number of papers were published in Celestial Mechanics on the method of recurrence power series expansion applied to the integration of equations of motion (Schanzle, 1971)(Broucke, 1971) (Black, 1973). It was shown that by using this method we can avoid a cumulation of truncation errors in the process of numerical integration. The method was successfully applied to the orbit computation of artificial satellites, in the restricted three-body problem, in the general n-body problem and in the planetary heliocentric motion. I adapted this method to the practical computations in cometary investigations and found it to be very convenient.

The equations describing the heliocentric motion of a comet as perturbed by the planets, p, are:

$$\frac{d^2x}{dt^2} = - k^2 \frac{x}{r^3} - k^2 \sum_p m_p \left(\frac{x-x_p}{\rho_p^3} + \frac{x_p}{R_p^3}\right)$$

and similarly for y and z, where k is Gaussian gravitation constant, m_p is mass of a planet in terms of the solar mass, x,y,z are the rectangular coordinates of a comet and X_p, Y_p, Z_p those of a planet. r, ρ_p, R_p are the distances of a comet to the Sun, of a comet to the planet, and of a planet to the Sun, respectively.

Suppose that the dependence of time of a function f=f(t) is given in the form of power series

$$f = f_o + \sum_{n=1}^{N} f_n (t-t_o)^n$$

and that just in such a form we want to obtain the solution of equations of motion, i.e., $x=x_o + \sum_{n=1}^{N} x_n (t-t_o)^n$ and similarly for y and z.

Let us define

$$\xi_p = x-X_p, \quad \eta_p = y-Y_p, \quad \zeta_p = z-Z_p$$

and

$$s = -k^2/r^3, \quad \sigma_p = -k^2/\rho_p^3, \quad S_p = -k^2/R_p^3$$

where

$$r^2 = x^2 + y^2 + z^2 \;, \quad \rho_p^2 = \xi_p^2 + \eta_p^2 + \zeta_p^2 \;, \quad R = X_p^2 + Y_p^2 + Z_p^2 \;.$$

Then the equations of motion can be written in the form:

$$\ddot{x} = sx + \sum_p m_p \sigma_p \xi_p + \sum_p m_p S_p X_p$$

$$\ddot{y} = sy + \sum_p m_p \sigma_p \eta_p + \sum_p m_p S_p Y_p$$

$$\ddot{z} = sz + \sum_p m_p \sigma_p \zeta_p + \sum_p m_p S_p Z_p$$

If we complete these equations by the relations

$$r\dot{r} = x\dot{x} + y\dot{y} + z\dot{z}, \quad r\dot{s} = -3s\dot{r}$$

$$\rho_p\dot{\rho}_p = \xi_p\dot{\xi}_p + \eta_p\dot{\eta}_p + \zeta_p\dot{\zeta}_p \;, \quad \rho_p\dot{\sigma}_p = -3\sigma_p\dot{\rho}_p$$

$$R_p\dot{R}_p = X_p\dot{X}_p + Y_p\dot{Y}_p + Z_p\dot{Z}_p \;, \quad R_p\dot{S}_p = -3S_p\dot{R}_p$$

we are able to derive the recurrence relations which allow us to compute consecutively the values of coefficients x_n, y_n, z_n starting from the initial values of x_o, y_o, z_o and x_1, y_1, z_1 as given for $t = t_o$. The recurrence relations for the two-body problem, given by Schanzle (1971), may serve as an example.

The recurrence relations for a cometary motion were derived on the assumption that for each moment of integration we have ready the power expansions for the planetary coordinates since the motion of the planets we know very well. Thus we can regard the cometary problem as a special case of the n-body problem when the equations of motion of disturbing bodies do not have to be integrated.

The power series integration method allows us to determine an optimal value of integration step as dependent on the desired accuracy of results of computations. Let us take the function

$$A(t) = A_o + \sum_{n=1}^{N} A_n (t-t_o)^n$$

where $A_n = |x_n| + |y_n| + |z_n|$ for $n = 0,1,\ldots,N$. Suppose that for $h = t_h - t_o$ we want to obtain $A(t_h) = A_o + \sum_{n=1}^{N} A_n h^n$ with the accuracy ε. The value of h may be determined as follows: Taking $A_{N-1} h^{N-1} = \varepsilon$ we have $h_o = (\varepsilon/A_{N-1})^{1/(N-1)}$; h_o is the preliminary value for the size

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of the integration step. Let us assume now that

$$\frac{A_{N+1}}{A_N} = \frac{A_N}{A_{N-1}}$$

and that $(A_{N-1} + A_N h + A_{N+1} h^2) h^{N-1} = \varepsilon$. Hence we have

$$h = \left[\frac{\varepsilon}{A_{N-1} + A_N h_o \left(1 + \frac{A_N}{A_{N-1}} h_o\right)}\right]^{1/N-1}$$

Thus for every moment of integration we compute the values of coefficients of the N terms of power expansion and then we calculate the optimal value of step-size according to the defined function A(t).

This method was applied to the integration of equations of various types of cometary motion, including, e.g., the close approach of the comet to the Sun (the orbit with a very small value of the perihelion distance, q), the close approach of the comet to Jupiter, the long integration interval (several revolutions of a minor planet). In all the cases the method gave excellent results.

It is worth noticing that the computed values of coefficients of power series expansion can be readily used in an interpolation formula to compute cometary coordinates for the moments of observations; thus it is convenient in the orbit improvement process.

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