A RE-CONSIDERATION OF THE EVOLUTION HYPOTHESIS OF THE ORIGIN OF THE RESONANCES AMONG SATURN'S SATELLITES

A.T. Sinclair
Royal Greenwich Observatory
Hailsham, Sussex, England.

Abstract

The hypothesis that the origin of the resonances among Saturn's satellites is due to orbital evolution is critically reviewed. It is concluded that the hypothesis provides a plausible explanation of the origin of the Mimas-Tethys resonance, but it is unsatisfactory for Enceladus-Dione, since their resonance is having little effect on the relative evolution rate of these satellites at present.

1. INTRODUCTION

It was shown by Roy & Ovenden (1954) that there are more occurrences of pairs of satellites with mean motions close to the ratio of two small integers than would be expected by chance. These are referred to as commensurabilities of mean motions or, more simply, as resonances. Goldreich (1965) suggested that these resonances were formed as the result of orbital evolution caused by dissipation of the tidal energy generated in the planet by the satellite. He suggested that the satellite orbits would evolve until a resonance was encountered, and showed that once a resonance had been entered by some means it would not be disrupted by subsequent tidal evolution. Allan (1969) worked out further details of the evolution of a resonance, and Sinclair (1972) and Greenberg (1973) described mechanisms whereby a resonance would be entered as the satellite orbits evolved towards it. At this stage it appeared that the tidal evolution hypothesis would satisfactorily explain most of the features of the three resonances in the Saturn system, namely those of Mimas-Tethys, Enceladus-Dione and Titan-Hyperion. Peale (1976) has written a comprehensive review of this subject.

In this paper we review the tidal hypothesis as applied to Mimas-Tethys and Enceladus-Dione. For Mimas-Tethys it appears to be a satifactory explanation, and is consistent with their anomalously high inclinations. For Enceladus-Dione it is not a satisfactory explanation of the present ratio of the mean motions.

V. V. Markellos and Y. Kozai (eds.), Dynamical Trapping and Evolution in the Solar System, 19–25.

For Titan-Hyperion the tidal hypothesis is dynamically possible (Greenberg 1973), but there is some doubt (Goldreich 1965) as to whether tidal dissipation is large enough to cause signficant evolution of Titan's orbit. An alternative explanation (Sinclair 1972) is that Hyperion is one of many small bodies that were formed near Titan, and only Hyperion has remained, as it happened to be formed in a resonance that protected it from close approaches to Titan.

2. RESONANCES AT A 2:1 COMMENSURABILITY

Suppose that a pair of satellites is close to a 2:1 resonance, so that the mean motion of the inner satellite is approximately twice that of the outer satellite. We take the equatorial plane of the planet as the reference plane, and denote the orbital elements of the inner satellite by $a,e,i, \lambda, \tilde{\omega}, \Omega$, where a is the semi-major axis, e is the eccentricity, i is the inclination to the equatorial plane, λ is the mean longitude, $\tilde{\omega}$ is the longitude of the pericentre, and Ω is the longitude of the ascending node on the equatorial plane. The orbital elements of the outer satellite are denoted by similar primed symbols. We denote by n,n' the mean motions, and by m,m' the ratios of the masses of the satellites to the mass of the planet.

At a 2:1 commensurability the following critical combinations of angular elements occur in the disturbing functions of the satellites, and can cause resonance effects:-

$$\theta = 2\lambda' - \lambda - \tilde{\omega}$$

$$\theta' = 2\lambda' - \lambda - \tilde{\omega}'$$

$$\phi = 2\lambda' - \lambda - \Omega$$

$$\phi' = 2\lambda' - \lambda - \Omega'$$

The oblateness of the planet causes $\tilde{\omega}, \tilde{\omega}', \Omega, \Omega'$ to vary at significantly different rates, and so the arguments $\theta, \theta', \phi, \phi'$ become critical (ie have a very small rate of change) for a range of values of $2n' - n$. Combinations of these arguments also occur in the disturbing functions, but the only combination that can cause significant resonance effects is $\phi + \phi'$. The resonances of the arguments ϕ, ϕ' and $\phi + \phi'$ involve the inclinations and nodes of the orbits, and are referred to as inclination-type. The resonances of θ and θ' involve the eccentricities and apses, and are referred to as eccentricity-type.

It has been shown by Yoder (1973: see Peale, 1976 p 236) that as a pair of satellites evolves towards a resonance it is only possible for them to be trapped in the resonance if the direction of evolution is such that a/a' is increasing. If so the resonance arguments would normally be encountered in the order $\phi, \phi + \phi', \phi', \theta', \theta$, assuming that the oblateness of Saturn is the only significant contribution to the rates of change of the nodes and apses (Sinclair 1972). However it is possible for $\phi + \phi'$ to be encountered earlier in the sequence if $i \ll i'$, for θ' to be encountered earlier if e' is small, and for θ to be encountered earlier if e is small. This is because in these circumstances the resonances will have a significant effect on the rates of change of the nodes and apses. However resonances have very little effect on the evolution rate when encountered far from their normal position in this way, and can essentially be ignored until the normal position is approached.

Before a resonance is encountered the argument concerned will have a negative rate of change (since the direction of evolution is such that $2n' - n$ is increasing from negative values). This rate of change becomes zero at the exact resonance, and positive if the resonance is passed through. If the satellites become trapped in the resonance then the argument will have on average a zero rate of change, and will oscillate, or librate, about a certain value. There are two mechanisms by which the satellites can enter a libration. One of these is a capture process, with usually a fairly low capture probability, and leads to a large amplitude libration. It occurs very close to the normal position of the resonance, and there is an abrupt change in the evolution rates of the mean motions as the libration commences, so that $d/dt\ (n - 2n')$ drops virtually to zero. The second mechanism operates if one of the eccentricities or inclinations is very small, when entry into the libration is automatic, and results in a small amplitude libration. Entry into libration can occur some distance from the normal position of the resonance, and initially the libration has little effect on the evolution rates of n and n'. Instead it forces the apse or node of one of the orbits to vary at such a rate as to maintain the resonance condition. As the exact resonance is approached the libration has an increasing effect on the evolution rates of n and n'. This second mechanism can operate at the $\phi + \phi'$, θ' and θ resonances only.

3. MIMAS-TETHYS RESONANCE

Mimas and Tethys are in a libration at the $\phi + \phi'$ resonance, in which the argument $\phi + \phi'$ oscillates about 0° with an amplitude of 97°. This is the only inclination-type resonance known in the solar system, whereas there are several eccentricity-type resonances.

The hypothesis that this resonance is due to tidal evolution (Allan 1969, Sinclair 1972) supposes that initially Mimas' orbit lay inside the position of resonance with Tethys. The tidal forces would cause Mimas to evolve outwards relatively faster than Tethys, so that a/a' increased. The ϕ resonance was encountered first, where the probability of capture into resonance was 7%. Capture did not occur, and the evolution continued until the $\phi + \phi'$ resonance was encountered. The probability of capture here was 4%, and capture did occur. A libration was formed with amplitude very close to the limiting value of 180°. At this stage the inclinations were $i = 0^\circ.42$ and $i' = 1^\circ.05$, and these would have been the approximate original inclinations as the evolution up to this point would not have affected them significantly. After the formation of the libration the rate of change of n - 2n' would drop to about 10^{-5} of its value before entering the libration. The libration amplitude would gradually decrease, and the inclinations increase, so that after a time of about 2 x 10^8 years the present situation would be reached, with amplitude 97° and i = 1°.52, i' = 1°.09. (The quantity $(a'/a)^{\frac{1}{2}} m'i'^2/m - i^2$ remains constant during the evolution (Sinclair 1974)).

This appears to be a plausible explanation of the origin of the resonance, but there are a few unsatisfactory features. The probability involved is very small, but this is perhaps acceptable since this is the only resonance for which we have to invoke a probability mechanism. The recent formation of the resonance 2 x 10^8 years ago is worrying. This is a lower estimate of the age, derived by assuming that Mimas was formed just above synchronous height, but it must lie close to this limit if we suppose that appreciable evolution has occurred. However Smith et al (1982) deduce from the crater density observed on Iapetus from Voyager 2 that the inner satellites must have suffered far greater impact rates, and may have possibly been disrupted and re-accreted several times. If so, then a recent formation of the resonance would be reasonable.

The present inclinations of Mimas and Tethys (1°.52 and 1°.09) are rather large compared with those of the other inner satellites (Rhea - 0°.34, Titan - 0°.31, Enceladus and Dione ~ 0°.03). The tidal hypothesis suggests that the initial inclination of Mimas was a more typical value of 0°.42, and only Tethys had a somewhat anomalously large initial value of 1°.05. The probability of capture into the $\phi + \phi'$ resonance is larger for small values of i/i', and so the anomalously high value of i' resulted in a reasonable probability of capture into the resonance.

Any satisfactory theory of the origin of this resonance must relate the present anomalously high inclinations of Mimas and Tethys to the existence of this unique inclination resonance, and we see that the tidal hypothesis meets this condition.

4. ENCELADUS-DIONE RESONANCE

Enceladus and Dione are in a libration at the θ resonance, in which the argument θ oscillates about 0° with amplitude 1°.4 (Kozai, 1957, p. 95. An erroneous value of 20' is often quoted for the amplitude). The hypothesis that the resonance is due to tidal evolution (Sinclair 1972, 1974, Greenberg 1973) supposes that initially the orbit of Enceladus was inside the resonance with Dione. The tidal forces would cause Enceladus to evolve outwards relatively faster than Dione, so that a/a' increased. The ϕ, $\phi + \phi'$, ϕ' and θ' resonances would be encountered first, in this order. For suitable initial values of i, i' and e' there would be a low probability of capture into each of these resonances. Having avoided capture into any of these resonances the system would approach the θ resonance with a value of e (the eccentricity of Enceladus) smaller than a critical value 0.019 so that entry into the libration was certain. A very small initial value of e of about 0.0001 would explain the present libration amplitude.

Peale (1976, p 243) argues that this small value of e would mean that the θ resonance would be encountered before the other resonances, and this would explain the failure to enter the other resonances. This is not so. If θ were in a libration a long way from the exact resonance then the only quantities affected by the resonance would be e and $\tilde{\omega}$. These are not involved in the other four resonances, and so the θ resonance would not influence the process of passing through or being captured in the other resonances. In fact it is possible for more than one resonance to exist simultaneously between a pair of satellites, as occurs for Io and Europa (Sinclair 1975).

There is a serious problem with the above description of the origin of the θ resonance. Suppose that in the absence of the libration the rate of evolution due to tidal forces would be $d/dt\ (n - 2n') = -K$. Then it can be shown from the equations given by Sinclair (1972, p 178) that, if the system is in a small amplitude libration of the argument θ, then as the system evolves into resonance the variations of $n - 2n'$ and e are given by

$$de/dt = Kq\, e^2/(pe^3 + q^2) \qquad (1)$$

$$d/dt\ (n - 2n') = -\, Kq^2/(pe^3 + q^2) \qquad (2)$$

where
$$p = (3n^2\alpha m' + 12n'^2\, m)\, A$$

$$q = n\alpha m' A$$

where $A = \frac{1}{2}\ (4 + \alpha D) b_{\frac{1}{2}}^{(2)}\ (\alpha)$, $b_{\frac{1}{2}}^{(2)}$ is a Laplace coefficient, and D denotes differentation wrt α. For Enceladus-Dione, $\alpha = 0.63$, $A = 1.19$, $n = 2n'$ (on RHS of equations), $m = 1.27 \times 10^{-7}$, $m' = 1.82 \times 10^{-6}$, $e = 0.0044$. Hence

$$d/dt\ (n - 2n') = -\ 0.83\ K.$$

So at present the resonance is having little effect on the rate of evolution of the system. An idea of the time scale involved can be obtained by assuming a value $Q = 6.7 \times 10^4$ for the tidal dissipation function of Saturn, which was used by Allan (1969) and Sinclair (1972). (See these papers for a definition of Q). This gives $K = 1.55 \times 10^{-14}$ rad/(time unit)^2, where the time unit is such that $n' = 1$ rad/time unit (about 0.436 days).

Equations (1) and (2) have been integrated (equation 2 numerically), taking $n - 2n' = 0$ and $e = 0.0044$ at $t = 0$. The results are plotted in Figure 1. The approximate positions of the other resonances are marked relative to the present position of the system, although any

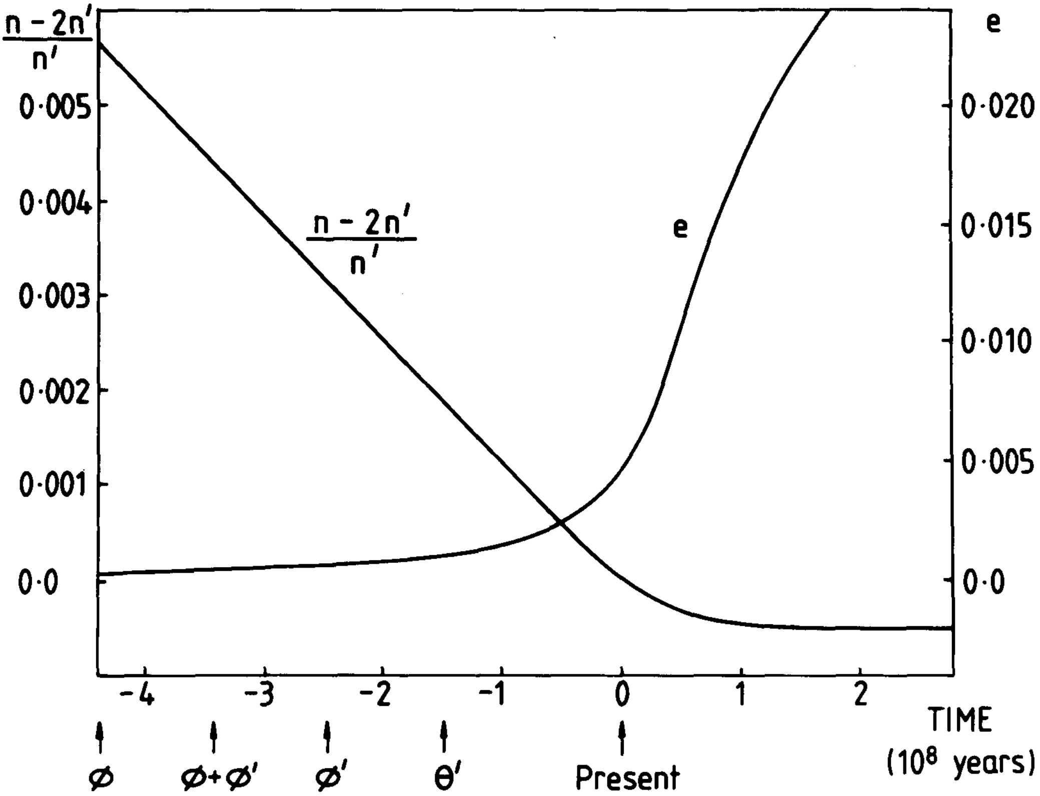

FIGURE 1. Variation of n - 2n' and e caused by θ resonance of Enceladus and Dione.

effects of these resonances are ignored in this calculation. It can be seen that according to the tidal hypothesis the satellites first encountered the general 2:1 resonance about 4.4 x 10^8 years ago. The θ resonance has had a significant effect on the eccentricity of Enceladus, but has had very little effect on the evolution rate of the system. However in about another 10^8 years time the resonance will have slowed the evolution rate virtually to a standstill.

This is clearly a most unlikely state in which to find the system. The idea of the tidal hypothesis is that we happen to see many objects in resonances because these are trapped states in which the objects spend a large proportion of their lifetimes. This can not be used to explain why we see the Enceladus-Dione system in a state through which it is passing at a virtually unhindered rate.

Hence we must conclude either that the tidal hypothesis does not provide an explanation of the origin of the Enceladus-Dione resonance, or that our model of tidal evolution is inadequate. The latter is probably the case. Yoder (1979) has considered the effect on orbital evolution of tidal dissipation within the satellite, generated by the forced eccentricity of an orbit caused by a resonance. He considers this effect in addition to the evolution caused by dissipation within the planet. He has applied this mechanism particularly to Io in order to explain the origin of the Laplace resonance among Io, Europa and Ganymede. He also considers the possible effect on Enceladus, which also has a significant forced eccentricity (0.0044) and a very small proper eccentricity (0.0001); such a state is indicative of tidal dissipation within the satellite. His calculations suggest that Enceladus would reach a forced eccentricity of 0.022 in a fully evolved state, and he concludes that it is unlikely that the pair have reached a steady-state configuration.

The Voyager 2 images of Saturn (Smith et al. 1982) show that Enceladus has had a complex geological history, with tidal heating caused by the resonance being the most likely cause. Hence it is probable that orbital evolution driven by dissipation in the planet has also acted, in order to maintain the satellites in a resonant situation. The complete description of the system is probably far more complex than the simple models so far proposed.

5. CONCLUSIONS

The hypothesis that orbital resonances between satellites are caused by evolution driven by tidal dissipation within the planet satisfactorily explains many of the features of the Mimas-Tethys resonance. The hypothesis fails to explain the present state of the Enceladus-Dione system, where the resonance is at present having very little effect on the evolution rate. It is probable that tidal dissipation with Enceladus is also involved.

6. REFERENCES

Allan, R.R. 1969. Astr. J., 74, pp 497-506.
Goldreich, P., 1965. MNRAS., 130, pp 159-181.
Greenberg, R., 1973. Astr. J., 78, pp 338-346.
Kozai, Y., 1957. Ann. Tokyo Obs., Ser. 2, 5, pp 73-106.
Peale, S.J. 1976. Annual Rev. Astron. & Astrophys., 14, pp 215-246.
Roy, A.E. & Ovenden, M.W., 1954. MNRAS., 114, pp 232-241.
Sinclair, A.T., 1972. MNRAS., 160, pp 169-187.
Sinclair, A.T., 1974. MNRAS., 166, pp 165-179.
Sinclair, A.T., 1975. MNRAS., 171, pp 59-72.
Smith, B.A., et al (29 authors). Science, 215 pp. 499-537.
Yoder, C.F., 1973. PhD Thesis. Univ. California, Santa Barbara, 303 pp.
Yoder, C.F., 1979. Nature, 279, pp 767-770.