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# COMBINING UNMERGEABLES: A METHODOLOGICAL FRAMEWORK FOR AXIOMATIC FUSION OF QUALITATIVE DESIGN THEORIES

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## ABSTRACT

The proposed methodological framework concerns axiomatic theory fusion (ATF) of non-additive engineering design theories. ATF includes seven steps: (i) semantic discretization of the composite theories, (ii) deriving epistemological entities by logical/semantic analysis, (iii) establishing and representation of relations among all relevant epistemological entities, (iv) combining the inter-theoretical epistemological entities of the component theories, (v) deriving propositions based on the combined set of epistemological entities, (vi) transcription of the epistemological entities and propositions into a textual/visual theory description, and (vii) validation of the resultant theory in application contexts. The proposed framework makes ATF an effective, content independent methodology for fusing component theories, no matter if they are descriptive, explanatory, predictive or controlling in nature. ATF methodology requires professional comprehension and rigor from the researchers. It is necessary to justify the logical correctness and practical validity of the target theory in the specific application context.

**Keywords**: Design theory, Research methodologies and methods, Systems Engineering (SE), Axiomatic theory fusion, Postulates and propositions

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# 1 INTRODUCTION

The scientific knowledge space of engineering design can be seen as a rough topological space populated by (the sets of knowledge of) engineering design theories (theories, for short). An engineering design theory is a logical construct that includes a system of assertions concerning facts and their relationships. Typically, they: (i) capture the existence of a natural or artificial phenomenon and provide an initial familiarity with it, (ii) provide a complete and accurate description based on deliberate observations and careful inspections, (iii) account for the influential factors (reasons and causalities) of why a phenomenon behaves as it does in a given context, (iv) forecast other probable phenomena based on the relationships among two or more phenomena, or (v) transform knowledge in order to enable the regulation of a phenomenon in various contexts and to provide dependable solutions for practical problems. The existing theories do not necessarily form a continuous topological space. There may be gaps between the knowledge sets of bounded formal theories, while certain knowledge sets of theories may overlap. The reasons of this theoretical discontinuity are that the majority of theories (i) is mono-disciplinary (or at most interdisciplinary), (ii) has different perspectives at dealing with a phenomenon, and (iii) is derived in differing contexts. As a consequence of the existence of gaps, not all observable or supposed phenomena can be described, explained, predicted or controlled sufficiently by the existing theories. Some of the knowledge gaps can be eliminated only by new theories explored by scientific research (Badino, 2015). However, some others may be treated differently. Namely, the complementing sets and the overlapping knowledge sets (theories) might be combined in a given perspective. This may facilitate not only the description and explanation of phenomena in a holistic manner from multiple perspectives, but may also lend itself to a transdisciplinary knowledge development without long-lasting constructionist research (Okhuysen and Bonardi, 2011).

The practical reason of the effort to combine theories is the abundance of already available theories. However, the current lack of non-heuristic (systematic) methodologies hinders their orderly and dependable combination. The framework proposed in the rest of the paper for combining seemingly unmergeable theories is underpinned by the idea of 'bricolage'. The common meaning of this word is creating something in a "do-it-yourself" manner from existing things that are readily available. Bricolage has been considered a guiding principle for developing theories by logical and sematic combination. Methodologically, it is a goal-driven and resource-constrained approach of artefact and knowledge synthesis (Boxenbaum and Rouleau, 2011). It can be adopted to steer the process of constructing novel theories from existing component theories. From the viewpoint of its logics, bricolage is deductive approach. This is an issue for theory synthesis since a bricolage of theories needs not only passing interfaces, but also adapting the components to each other. The potential of solving theory synthesis problems by bricolage depends on the combinability (compositionality) of the considered theories. This will be addressed in the rest of this paper, which is structured as follows: Section 2 provides a concise overview of axiomatic knowledge systems and the research efforts concerning a procedural framework for axiomatic combination of theories. Section 3 describes the procedural steps of axiomatic theory fusion (ATF), the information processing methods and the computational support tools. Section 4 provides information about some observations and possible future work.

# 2 RECENT RESULTS IN UNDERSTANDING AXIOMATIC KNOWLEDGE SYSTEMS AND COMBINING THEORIES

## 2.1 Axiomatic knowledge systems

Though often used as synonyms, the terms 'definition', 'assumption', 'theorem', 'theory', 'axiom', 'postulate' and 'proposition' should be differentiated. A theory is a chunk of formal human knowledge or understanding (Achinstein, 1977). As defined in classical philosophy, an axiom is an assertion that formulates obvious or well-established (self-evident) truth that can be accepted without a proof, controversy or question (Schlimm, 2006). The term 'axiom' has subtle definitional differences when used in different fields and contexts. It may be used in two related, but distinguishable senses: 'logical axioms' and 'non-logical axioms'. In modern logic, an axiom is simply a premise or starting point for reasoning and a foundational element of a formal deductive system (Hintikka, 2007). It is a proposition or a formula that is stipulated to be true for the purpose of a chain of reasoning. Non-logical axioms are substantive assertions about the entities of a specific knowledge domain. Usually, they are expressed as language


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formulas that capture theory-specific assumptions (e.g. the whole is greater than the part). Non-logical axioms aim at capturing what is special about a particular structure (or set of structures, such as groups). Thus, unlike logical axioms, non-logical axioms are not tautologies. Another name that is also frequently used for a non-logical axiom is postulate. A postulate is a proposition that is requested or supposed to be true without proof for the sake of studying the consequences that follow from it. We insist on this differentiation between axioms and postulates in the rest of this paper.

Axioms and postulates are non-composable epistemic elements of a theory and lend themselves to a sound logical (deductive) reasoning, which assumes syllogism and other rigorous rules of inference. Relations, principles, laws or rules established by axioms are not necessary truths, but sanctioned by experience (Dimarogonas, 1993). An axiomatic system (AS) is a logical system, which possesses an explicitly stated set of axioms, from which theorems can be derived. In principle, every theory could be axiomatized by considering axioms, postulates and their logical relationships, and formalized down to the bare language of logical formulas (Simon, 1979). Axiomatization involves reduction of some knowledge system, theory or concept to a finite set of axioms, as well as the process of defining a formal system by a set of axioms. Maintaining the basic properties of an AS, such as (i) completeness, (ii) consistency, (iii) independence, and (iv) non-redundancy, is crucial for axiomatic theory fusion. A complete AS is a special kind of formal system. An AS is called complete if every assertion or its negation is derivable. An AS is said to be consistent if it lacks contradiction, i.e. if it is impossible to derive a contradictory proposition (both an assertion and its denial) from it. An axiom is independent if it cannot be derived from other axioms. An AS is called independent if each of its underlying axioms is independent. While independence is not a necessary requirement for a knowledge system, consistency is. Of course, it is requested that the component theories are axiomatizable in terms of epistemic elements that are true in them. Eventually, a system of axioms is a framework for capturing the knowledge conveyed by a theory. A set of axioms should be non-redundant. This implies that axioms, which can be deduced from other axioms, should not be regarded as axiom.

Various forms of axiomatization-based theory construction have been applied in the different disciplinary fields. For instance, axiomatization has been successfully used to create a robust knowledge platform for a trust-based recommendation system (Andersen *et al.*, 2008). First, an epistemological description of basic concepts and their relations was provided by using appropriate existing theories in the given context. Then, a formal theoretical system was constructed by a deductive process, starting from the available axioms. In the end, additional axioms and postulates were defined to make the epistemological basis of a new theory consistent and complete. The literature is rich of specific research results. For instance, the relationship of the axiomatic method and the accounting science was investigated in (Spencer, 1963). An axiomatic theory of organization/environment interaction was developed in (Ganey, 1979). A specific axiomatization was used to construct a theory of accounting in (Carlson and Lamb, 1981). To improve systems, axiomatic design, quality control tools, and designed experiments were combined in (Engelhardt, 2000). Axiomatic approach was used in the conceptual phase of product design in (Sozo *et al.*, 2001). The applicability of axiomatic design equations in variant design was investigated in (Marston *et al.*, 1997).

### 2.2 Procedural frameworks for axiomatic theory fusion

The exploration and critical analysis of the above findings led us to the following reasoning concerning the research issue at hand: Traditionally, theories are derived in retrospective, inductive and deductive ways (Giere, 2000). Current data-driven science attempts to formulate theories based on patterns extracted from massive data sets, and to convert the patterns into coherent knowledge frameworks of theories. The issue of direct combination of theories with the goal of establishing more comprehensive and powerful theories is known in the literature for years (Dey, 1995). 'Metaphorical bricolage' was described as a specific approach to combining implicit assumptions from multiple bodies of knowledge in (Boxenbaum and Rouleau, 2011). This obeys the constructivist paradigm and features a systematic decomposition and re-composition of composite theories. It is both evolutionary (regulated by facts) and combinatory (blending parts), but also has some intuitive (human interpretation dependent) process elements. These intuitive elements cannot be avoided in the framework, since (i) different component theories may not be completely coherent and consistent, and (ii) human construal may be needed to combine the different theories if facts, evidences, logic and relationships are missing, Combination of theories is expected to result in an epistemic compliance among the representational, explanatory and transformational dimensions of the component theories. These major principles of the approach were operationalized in our research.


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In line with the 'dogmas' discussed in (Badino, 2010) concerning scientific theories, namely that: (i) their knowledge content is stored in a handful of fundamental laws, (ii) they have a core of not-disprovable assertions that makes them what they are, and (iii) they are taken as historically and structurally given, axiomatization was applied to capture the knowledge contents of component theories. Axioms and postulates represented elementary pieces of knowledge. Thus, both the decomposition and the re-composition part of the fusion process elaborate on sets of interrelated axioms and postulates. Since the theory as a whole is taken as proven, its epistemic elements (the, are supposed to be units of 'relative truth'. Note that trueness of a derived theory will not be better than that of the lowest of the component theories. At the same time, hypothetical reasoning is inevitable in cases where: (i) the internal morphology of theories is presented only as loose assemblage of propositions or models, (ii) the boundaries are only hazily and porously defined, and (iii) the limitations and constraints are not explored and described exhaustively. As discussed in (Modell *et al.*, 2017), theories are eventually cultural products whose epistemic dynamics and normativity can be illuminated only by keeping their inherent historicity in mind. The procedural framework of the (ATF) methodology is shown in Figure 1.

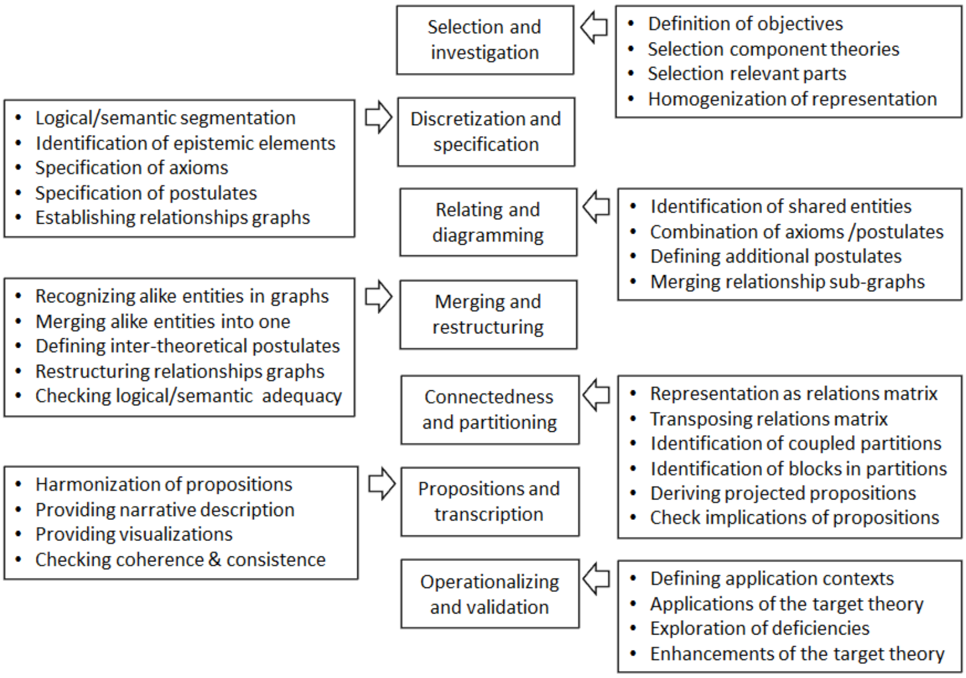


*Figure1. Overview of the process of axiomatic fusion of engineering design theories*

As a lesson learnt from the forerunning experimental investigations, the overall process has been decomposed into seven sub-processes. These are as follows: (i) selection and semantic investigation of the composite theories according to the purpose of theory fusion, (ii) discretization of the component theories, and deriving and visualization of epistemic elements, (iii) combination of the sets of epistemic elements and exploring inter-theoretical relationships, (iv) reducing the alike subject entities and restructuring the relationships graph, (v) investigation of the connectedness with regards to subject entities and partitioning based on a matrix representation, (vi) formulation of propositions and textual transcription of the fused theory, and (vii) operationalization and validation of the fused theory in application contexts. Different manual or computational methods are used in the process, for instance, to: (i) discretize qualitative theories, (ii) define primitive entities, (iii) represent semantic/relational structures, (iv) merge component theories based on inter-theoretical relations, (v) justify the resultant theory, and (vi) provide a narrative for the interpretation of the resultant theory. The specific activities belonging to each of the above sub-processes will be discussed below.

# 3 THE METHODOLOGICAL FRAMEWORK OF AXIOMATIC THEORY FUSION

## 3.1 Selection and semantic investigation of the composite theories

Axiomatic fusion creates an inferential connection between elements of the component theories and the new synthetic theory derived based on them. The former ones are alternatively called as source



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theories, while the latter one as target theory. In our conceptualization, a target theory is a purposeful combination of a finite number of component theories. The objective of combining them depends on the objectives of dealing with (investigating) a phenomenon and on the purpose application of the target theory. Thus, the proposed methodological framework does not advise on which component theories should be selected from the pool of possible theories for a particular application and purpose. Consequently, the decision about these should be made by the researcher/engineer before starting the procedure of theory fusion. Likewise, the number of the needed (individually partially insufficient) source theories can only be decided upon when the decision on the objectives of ATF has been made.

In terms of component theory selection, the recommended principle is parsimony, which implies that the simplest possible theories with the highest relevance to the addressed phenomenon and fit for purpose are to be selected. An optimal set of component theories provides a sufficient level of expressiveness and requests only a limited amount of cognitive effort. For the reason that formulation of source theories does typically not happen in the context of the phenomenon addressed by theory fusion, it is conceivable that some of them can actually be not used. The relevant set of source theories is referred to as 'kernel'. This is the starting point at processing component theories. The larger the kernel formed by source theories, the higher the chance of constructing a target theory that fits for the purpose. As a summary, the abovementioned (i) content relevance, (ii) parsimony, and (iii) the size of the kernel jointly establish the criteria for considering, or not, a particular source theory. The decision on choosing and extracting the knowledge contents of component theories should be based on the above aspects and on critical systems thinking.

## 3.2 Discretization of component theories and specification of epistemic elements

An important sub-process of ATF is discretization of the component theories. The method of discretization is intuitive logical/semantic analysis. Discretization means transferring the component theories into constructs comprising finite sets of epistemic elements. These elements are axioms and postulates, which represent the lowest-level meaningful rudiments of the component theories. They may be expressed by symbolic constructs, mathematical formulas, declarative texts, visual images, data tables, information diagrams, etc. as parts of the component theories. The epistemic elements are axioms and postulates. They refer to one or more things, called subject entities, and include assertions which express some logical/semantic relationships. Since more than one epistemic element may refer to the same subject entity, there is a natural connectivity between them. Based on this, the epistemic elements of a component theory form a network-type structure that can graphically be represented as a graph. Thus, after discretization, each component theory is given as an inner composition of finite number of epistemic elements and is represented as a planar graph.

Mentioned above, the axioms and the postulates convey specific assertions about the subject entities. An axiom is an intuitively provable elementary assertion of the component theory. Postulates have a kind of dual nature and can therefore be captured in two somewhat different ways. First, a postulate can be an explicit assertion of a component theory, which is however not deemed to be intuitively evidential. As such, a postulate claims what is required to be true in a particular context without arriving at a contradiction. Second, a postulate can be an assertion that is not explicitly included in the specification of a component theory. Such postulates are also required to claim believed truth in the particular context. The assertions of the axioms and the postulates may express specific attributes, reflexive relations, and mutual relations of the subject entities. Orientation of reflexive relations is evidential. However, mutual relations must be orientated in order to avoid semantic conflicts and to allow for transitivity. It is assumed that other propositions can be derived based on the axioms and postulates of a component theory as their logical consequences.

In terms of defining epistemic elements, the starting assumption is that every component theory (i) provides a finite set of subject entities, (ii) assigns a finite set of attributes to them, (iii) specifies assertions entailing reflexive relations of the subject entities with themselves, and (iv) specifies assertions entailing mutual relations between two subject entities. The first step of deriving the epistemic elements is finding all substantial subject entities in the component theories. The subject entities may be physical, virtual or abstract things, and may be expressed explicitly or implicitly in the theory. Obviously, their recognition depends on their original representation within the theory. If the subject entities are uniquely and consistently named, then finding them is straightforward. Otherwise they need to be identified by logical reasoning. In addition, certain subject entities may only be pointed at or implied implicitly by the assertions of the epistemic elements. These have to be extracted



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by logical/semantic reasoning and then identified by concrete names. Same applies to the interrelationships of these subject entities with other ones. In addition to the names, it is also useful to assign codes to each subject entity and to the assertions in order to support computational processing.

### 3.3 Relating the epistemic elements and diagramming their arrangement

As a result of the above step of the ATF process, separate lists of axioms and postulates are available. This step of processing focuses on capturing all specific interrelationships of the axioms and postulates, and providing a graphical representation for each component theory. The axioms and postulates are logically/semantically interrelated if their assertions concern the same subject entity of the component theory. Based on their assertive interrelations, the axioms and postulates form a network, which can graphically be represented as an orientated planar graph. Such graphs can be constructed by using any graph visualization software. Figure 2 shows an example for the graph representation of a discretization of a (simple) component theory. The nodes are the subject entities and the edges represent the interrelations created by the assertions of the concerned axioms/postulates. The annotations attached to the nodes and the edges can convey information about the subject entities and the assertions, respectively. This displayable graph representation shows the logical/semantic skeleton of a component theory. If all axioms and postulates of a component theory are included, then the skeleton is said to be complete. Completeness is however a subjective concept since it is influenced by the intentions and considerations of the researcher. For the time being, no general principles, formal rules, or quantitative/qualitative measures have been defined to evaluate completeness. This is important since implicit interrelations may also be 'hiding' between subject entities, in particular, when the whole of a component theory is considered. These hiding or implicit interrelations need to be captured by additional postulates. As a conclusion, it is necessary and useful, but also challenging to explore and represent all interrelationships by epistemic entities. If it is not done with sufficient care, then there is a chance of having logical/semantic inconsistences in the fused theory.

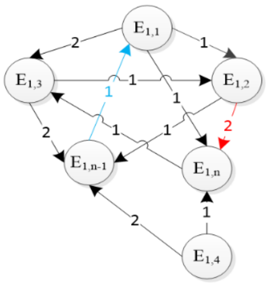


*Figure 2. Relating subject entities and diagramming their arrangement*

It can be observed based on the graphical visualization if the graph of a given component theory: (i) consists of multiple comparable parts, (ii) has some minor disconnected parts, or (iii) is single-pieced (monolithic). If there is any disconnected sub-graph in the graph, then the relevance of the disconnected part needs to be further investigated considering the objective of theory synthesis. If a disconnected subgraph is relevant, then it should have some logical/semantic relations with the main graph. These relations can be established through the specification of one or more postulates. An additional issue is that certain subject entities may be named differently in the axioms and postulates belonging to the main graphs or to the disjoint sub-graphs. It can be decided upon by applying the assertions of the concerned axioms and postulates to an assumed 'common' subject entity. If the assertions apply, then the differently named entities can be merged into one generic subject entity and identified as such. In terms of the graph, it means unifying the nodes representing these variously named entities and rearranging the connecting edges accordingly. It is advisable to check the logical/semantic consistency after merging the concerned subject entities and the rearrangement of the connections.

### 3.4 Merging common subject entities and restructuring inter-theoretical relationships

The previous step elaborated on the conversion of one component theory. This step deals with multiple component theories concurrently and combines them (actually, their graphs). The starting assumption is that the graphs of all component theories are available in a graphically visualized form. With


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regards to the actions: (i) merging subject entities, (ii) rearranging their interrelations, and (iii) establishing new logical/semantic interrelations are completed as before. The next step is merging these graphs into one composite graph. An example of combining graphs is shown in Figure 3. The merging process starts with placing all graphs into a reference frame. After this, the subject entities with (i) similar or resembling names, (ii) identical meaning, or (iii) similar roles should be identified considering all graphs. The assertions of the existing axioms or postulates inform about whether the entities found in the different graphs have identical meaning or similar roles. If entities of these characteristics are found in the graphs of the component theories, then they can be used as the starting points of combining the graphs. By merging the entities in the way described above, the graphs will have shared nodes. The edges representing the assertions of the concerned axioms and postulates should be rearranged according to the shared subject entities. This processing should be completed for all shared entities (and thus, for all concerned assertion of axioms and postulates).

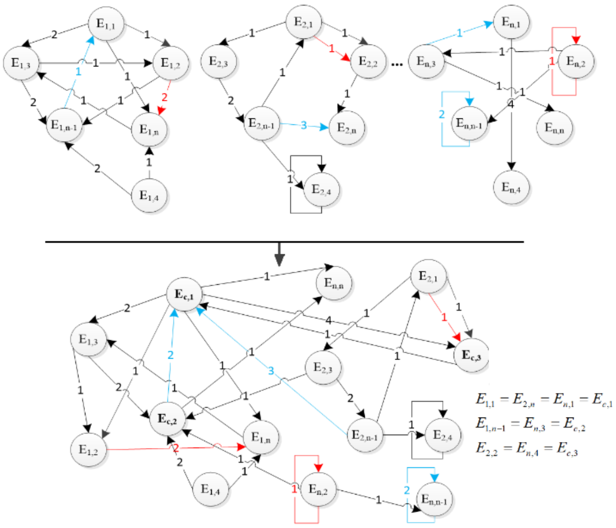


*Figure 3. Merging common entities and restructuring inter-theoretical relationships*

As a consequence of the previous step, no disjoint subgraphs will remain. That is, the combined graphs form one single composite graph in the end. Nevertheless, it may be possible to establish additional meaningful inter-theoretical associations even if no merge-able subject entities are found in the graphs of the component theories. Logical/semantic interrelations may arise based on imposing the purpose of the target theory. These interrelations can be captured by introducing new inter-theoretical postulates concerning subject entities of different graphs. The number of shared entities expresses the scale of the overlap of the component theories. As mentioned above, the overlap is larger when the component theories share an extensive logical/semantic kernel. The number of the original relations and the additional relations is an indication of the strength of adhesion of the component theories. The whole of the graph merge is an intricate process and calls for logical/semantic justification. The bottom line is that the established inter-theoretical postulates should in one way or other be implied by the concerned component theories.

### 3.5 Analysis of connectedness and structural partitioning

As part of the first stage of theory fusion, the previous steps resulted in a system of epistemic elements. Represented by the composite graph, this combination of the axioms and postulates established the logical/semantic skeleton of the target theory. In order to explore higher-level semantic associations among the component theories and the phenomenon, the next steps further elaborate on this system of epistemic elements. Specifically, the combinatorial connectedness is analyzed and a structural partitioning is applied. The discussion of these is the objective of the rest of this sub-section. Based on the graphical representation of the composite graphs, the analysis of the combinatorial connectedness can be done manually. However, it may be complicated in the case of a complex graph. Therefore, it is practical to consider computational support. It was found that the effectiveness of computational processing can be facilitated by converting the composite graph into a matrix representation and applying a specific matrix transformation. For this purpose, the tool offered in the MatLab package was used, but many other matrix management tools are suitable.


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Through matrix transformation, the connectedness of the sets of axioms and postulates can be made more tangible. The analysis of connectedness starts out from the included subject entities. A connectivity matrix is generated, which includes: (i) the identifiers of all subject entities (in the cells of its first row and column), (ii) the reflexive relations of the entities (in the cells of its main diagonal), and (iii) the orientated interrelations (in its other cells). By a systematic rearrangement of the rows and columns of the connectivity matrix, a partitioned matrix can be obtained, in which the partitions display the connectedness of the sets of epistemic elements. The connectivity matrix can be an uncoupled one, when there are mutually exclusive partitions alongside the main diagonal. It shows which sets of epistemic elements can be processed and interpreted independently from each other. The connectivity matrix is decoupled, if it has partitions only either in its upper or lower triangular matrix part. The connectivity matrix is coupled, if the in-diagonal and off-diagonal partitions overlap with each other. In this case, the connectedness of the sets of axioms and postulates should be considered not only within the partitions, but also between the partitions. An important fact is that the partitions may be strongly coupled internally. Such partitions are terminal, i.e. they cannot be decomposed any further. As such, they form one discrete semantic cluster.

| | $E_{1,1}$ | $E_{1,2}$ | $E_{1,3}$ | $E_{1,4}$ | $E_{1,5}$ | $E_{1,6}$ | $E_{1,7}$ | $E_{1,8}$ | $E_{1,9}$ | $E_{1,10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $E_{1,1}$ | $A_{1,6}$ | $A_{1,1}$ | $A_{1,2}$ | $A_{1,3}$; $A_{1,5}$; $A_{1,7}$ | 0 | 0 | 0 | 0 | $A_{1,12}$ | $A_{1,14}$; $A_{1,15}$ |
| $E_{1,2}$ | $A_{1,1}$ | $A_{1,13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_{1,3}$ | $A_{1,2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_{1,4}$ | $A_{1,3}$; $A_{1,5}$; $A_{1,7}$ | 0 | 0 | $A_{1,4}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_{1,5}$ | 0 | 0 | 0 | 0 | $A_{1,9}$ | 0 | 0 | 0 | 0 | 0 |
| $E_{1,6}$ | 0 | 0 | 0 | 0 | 0 | 0 | $A_{1,10}$ | $A_{1,11}$ | 0 | 0 |
| $E_{1,7}$ | 0 | 0 | 0 | 0 | 0 | $A_{1,10}$ | $A_{1,8}$ | 0 | 0 | 0 |
| $E_{1,8}$ | 0 | 0 | 0 | 0 | 0 | $A_{1,11}$ | 0 | 0 | 0 | 0 |
| $E_{1,9}$ | $A_{1,12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $P^{D}_{1,1}$; $P^{A}_{1,1}$ | 0 |
| $E_{1,10}$ | $A_{1,14}$; $A_{1,15}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Figure 4. Analysis of connectedness and structural partitioning*

The cluster-level connectedness determines which clusters should be processed and interpreted first, since they have implications on the ‘meaning’ of other clusters. This is an additional source of logical/semantic information. Within each cluster, the axioms and the postulates laying in a row or a column convey assertions about the same subject entity. Their assertions can typically be combined and concurrently projected to the addressed phenomenon. The constructs formed by the associable assertions is distinguished as a semantic block. Every cluster may include multiple blocks, the contents of which are pre-synthesized knowledge with regards to the concerned subject entities. At reasoning towards the target theory, they support higher level reasoning. What it means in the practice is that they hint at higher level (implied) assertions, called propositions, which facilitate tailoring the target theory to the investigated phenomenon, or even to the application context. This is discussed below.

### 3.6 Deriving propositions and textual transcription

The proposed approach of ATF intends to make the target theory strong in terms of its descriptive, explanatory or predictive power. The inclusion of additional within-theory postulates and inter-theoretical postulates points in this direction. As discussed in the preceding sub-sections, the inclusion is made possible by the logical/semantic connectedness of the component theories. This makes it possible to derive other epistemic elements, namely propositions, as mentioned above. Propositions are means of formulating and handling context orientated inter-theoretical assertions. In addition to the semantic content and the expectations, they also allow taking the essence of the addressed phenomenon and the purpose of theory development into consideration, as well as other aspects such as relevance, coherence and expressiveness in the specification of propositions. Stating propositions includes three actions: (i) composing proper propositions, (ii) checking their implications, and (iii) harmonization of the propositions. Propositions can be formulated in multiple forms, for instance, as


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(i) declarative assertions, (ii) conditional assertions (production rules), (iii) decision trees, or (iv) constraints networks. The amount of the propositions is influenced, but not determined, by the number of blocks of the semantic clusters. The larger the number of propositions, the more semantic knowledge is projected to the phenomenon or the application case. An issue is that the clusters and the blocks may imply propositions that are not, or only partly relevant. Therefore, the sub-process has to end with filtering the formulated propositions by simultaneously considering their relevance and suggestions in the context of the phenomenon and the target application.

As follows from the above discussion, the extracted axioms and postulates, the specified postulates, the concerned subject entities, and the derived propositions constitute the epistemic elements of the target theory. Recording these, however, is just an in-process representation of the axiomatically-fused theory. To facilitate better comprehension, they can be transcribed into a visually enriched textual form. Anyway, this form of presentation of qualitative engineering theories is typical, preferred and the most expressive in the practice. Thus, the last step of the ATF process is providing a textual specification of the target theory, which allows including supplementary explanations, interpretations and references. The structure and associations of the semantic blocks discussed above provide guidance for organizing the textual specification. When the target theory is exhaustively specified, it should go through a rigorous justification, which is aimed at verifying or falsifying its logical properness (i.e. its coherence and consistence). In our work it has been done by critical systems thinking, but formal methods should be considered (which is a self-contained future research on its own).

### 3.7 Operationalization and validation in application contexts

Validation of the theory should explore if it is able to describe, explain, predict or control what it is supposed to do. Validation may include multiple aspects such as sufficiency and efficiency, strength and weaknesses, and possibilities and limits of the theory in a specific application context. If the proposed theory contradicts itself logically or empirically, or if it is in conflict with other theoretical claims, then it must be scrutinized or discarded. Considering only these aspects, the validation of a fused theory is a complicated matter - sometimes a ‘mission impossible’ without experimentation and empirical testing. On the other hand, several authors argued that qualitative theories need their own procedures for attaining validity, which are different from those of quantitative approaches.

Since the background research is still in a booting up phase, the emphasis was put on the applicability testing of the proposed ATF methodology, rather than on the justification and validation of the generated new theories. The reader must know that our research concentrated on the construction of a theory that can underpin the development of a smart data analytics toolbox for supporting the enhancement of white goods based on exploitation of middle of life product data. The applicability of the methodology was tested in this particular case and the main findings are concisely summarized below. The phenomenon addressed in the above application context was the possible range of smart functionalities that support white goods designers to improve their products based on middle of life data. A first step towards validation of the resultant theory was made by examining the relevance and the implications of the propositions. The follow up research will consider a systematic exploration of the non-obvious deficiencies of the proposed theory, as well as of the ATF methodology, and introduce enhancements.

## 4 SOME CONCLUSIONS AND FUTURE WORK

The primary objective of our research was to study the principle of axiomatic theory fusion and to develop a procedural framework for a computer aided methodology. Such a solution provides multiple advantages such as: (i) saving time and costs needed for new explorative research projects and (ii) extending the range of real problems that can be covered by scientific theories. We found that the proposed framework and methodology are application neutral, i.e. independent of the theories considered for fusion. The only assumption was that they had to be decomposable to interrelated epistemic elements. The practical experiences obtained so far with the methodology showed that the knowledge fusion procedure is time consuming, in particular in the case of combining multiple extensive theories. Our experience shows that combining five component theories into a specific target theory needed approximately 120 hours from an insightful PhD student. In addition, ATF needs considerate semantic interpretation and rational decisions. Therefore, it needs computational support.



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On the other hand, it offers methods for monitoring the coherence and consistency of the outcome and thereby helps avoid possible errors in the whole process. A crucial element of the methodology is identification and interpretation of the epistemic elements and the logical/semantic relation patterns implied by them in the component theories.
Human interpretation and intuition seem to be indispensable concerning the proposed procedures, methods and instruments of the AFT methodology. The level of experience can significantly influence the efficiency and correctness of knowledge processing. Computational transposition of entity-interrelation matrices often creates opportunity for 'out-of-the-box thinking' (i.e. may result in not trivial configurations). Many strands of future research can be conceived. The sort terms objectives include investigation of enhancement opportunities, with regards to: (i) the efforts needed for utilization, (ii) the efficiency of manual application, and (iii) involvement of additional traditional or novel computer support means. The longer-tem inquiries can be sorted into three categories: (i) performance testing and limit analysis of the methodology considering complex application cases, (ii) development of a dedicated computational toolbox to support effective application, and (iii) development of a smart reasoning mechanism to support semantic interpretation and processing.

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