SUPERNOVAE IN BINARY SYSTEMS : PRODUCTION OF RUN-AWAY STARS AND PULSARS

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ABSTRACT

The effects of an instantaneous asymmetric supernova explosion in an eccentric binary system are analyzed, taking into account the mass loss out of the system, the influence of the impact of the supernova shell on the companion star and the extra "kick" velocity which a collapsed star might receive in an asymmetric supernova explosion. For a random orientation in space of this asymmetric kick velocity, the survival probability and the runaway velocities are derived and their properties discussed for an explosion occurring at a given position in the initial keplerian orbit and the mean and extreme values of these quantities over one orbit are derived. As an example, the outcome of a possible supernova explosion in the ten best known WR+OB binaries is studied and a comparison is made with the observed run-away OB stars, radio pulsars and binary X-ray pulsars.

1. INTRODUCTION

The effects of supernovae occurring in binaries were first studied by Blaauw (1960) in the context of run-away OB stars. In those days one was not yet aware of the great effects of mass exchange and mass transfer on the evolution of components of close binary systems and the idea that the most massive component exploded first as a supernova was invoked to explain the observed run-away OB stars as being the released companions of stars which had gone through the supernova stage. The main effect studied was the mass loss from the initially circular system, taking into account numerically the details of the mass distribution in the ejected shell (Boersma, 1960). Later on the mass exchange in a binary system was included and the effects of the explosion of the less massive star were studied in the same way (van den Heuvel, 1968; De Cuyper, de Loore and van den Heuvel, 1977). Analytic approximations were given by Savedorff and Vila (1964) and by Hut and Verhulst (1981). The extention to initial eccentric orbits was given by Hadjidemetriou (1966), which studied the time dependent mass loss problem numerically

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and gave an analytic solution for the instantaneous case. Analytic solutions for initially circular orbits in case of an instantaneous explosion were also given by Sofia (1967), Gott (1972) and Mitalas (1976).

The effects of the interaction of the supernova shell with the companion star were found to be unable (unless under very special conditions) to make systems unbound, which were revolving initially in circular orbits, and which had undergone a mass loss of less than half of the initial total mass (Colgate, 1970; McCluskey and Kondo, 1971; Cheng, 1974; Sutantyo, 1974,1975; Khabazin, 1975). Wheeler, Lecar and McKee (1975) made an analytic approximation taking into account the effects of the internal structure on the reduction of the effective cross section of the star and on the mass ablated by the subsequent heating. A fully hydrodynamical treatment of the impact problem was given by Fryxell and Arnett (1981).

An acceleration of the collapsed star due to an asymmetry in the explosion process was taken into account numerically by De Cuyper (1974); de Loore, De Grève, van den Heuvel and De Cuyper (1975) for a time dependent explosion in a circular orbit. The analytic treatment of an instantaneous asymmetric explosion neglecting the effects of impact was given by Sutantyo (1978).

We will generalize here the problem to an instantaneous asymmetric explosion occurring at a given position in an initially eccentric orbit taking into account the mass loss out of the system, the effects of the interaction of the supernova shell with the companion star and the asymmetric kick velocity the collapsed star gets due to the asymmetry of the explosion process.
In section 2 the supernova explosion of a component of a binary is formulated, using observational data and theoretical prospections, as a celestial mechanical problem and the effects of an instantaneous asymmetric supernova explosion on the orbital parameters are derived analytically.
In section 3 the survival condition of the post-supernova system is analyzed and the survival probability for a given probability distribution of the asymmetric velocity parameter, the collapsed star gets as a result of the asymmetry of the explosion, is defined for an explosion occurring in a given point of the initial orbit and for its mean value over one orbit. Analytic expressions for the run-away velocities of the remaining bound system or of both remaining components in case of disruption with respect to the center of mass of the pre-supernova system are given in section 4.
These formulae are used to study the outcome of the first supernova explosion in massive close binaries. In section 5 an evaluation of the explosion parameters is made using observational data and theoretical model results. In section 6 the initial systems are derived from the best observed WR+OB binaries and the outcome of the supernova explosion of the Wolf-Rayet star at the end of its nuclear evolution is analyzed. The results are discussed together with the concluding remarks in section 7.

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2. THE EFFECTS OF AN INSTANTANEOUS ASYMMETRIC SUPERNOVA EXPLOSION ON THE ORBITAL PARAMETERS

2.1. Formulation of the problem

One of the major events in the evolution of a binary system is the supernova explosion of one of its components. The supernova itself is characterized by the implosion of a gravitationally unstable stellar core forming a neutron star (or possibly a black hole), the ejection of the stellar envelope with an observed velocity of the order of 1-3 10^4 km.s^{-1} (Shklovsky, 1968; Schatzman, 1965; Zwicky, 1965; Minkowski, 1969). In case the explosion itself is not fully symmetric internal forces will accelerate the exploding star. Hence the collapsed star may get an extra asymmetric velocity (Shklovsky, 1970; Fryxell, 1979).

In a binary, part of the ejected supernova shell will impact on the companion star. This inelastic collision strips off the outer edges of the companion star (as seen from the supernova) and accelerates the remaining part by direct momentum transfer and subsequent anisotropic ablation of some mixed stellar and impacting material due to the heating behind the shockfront, which forms a bow shock around the stellar core (Fryxell and Arnett, 1981).
When the supernova shell has past the companion star it no longer exerts any significant attraction on the remaining system. This is due to the fact that the mass in the ejected shell is nearly isotropically distributed so that its inside gravitational field can be neglected.
As a consequence of this the orbital motion of the remaining components changes. First of all the mass loss out of the system will decrease the gravitational binding energy. Secondly the orbital velocity change, due to the effects of the impact and the asymmetric explosion, will modify the orbital kinetic energy.
As the orbital velocities ($\sim 10^2$ km.s^{-1}) are some orders of magnitude smaller than the supernova ejection velocities ($\sim 10^4$ km.s^{-1}) and the typical thickness of the supernova shell is about one third of its expansion distance (Colgate, 1970), the time needed for the supernova shell to pass the companion star is much shorter than the initial orbital period. Hence we can neglect the time dependence of the mass loss out of the system and assume that the explosion occurs instantaneously.

2.2. The pre-supernova system

We consider a binary system with component masses M_1^o (the pre-supernova star) and M_2^o (the companion star) revolving around their common center of gravity C_g^o in a keplerian orbit with period P^o, eccentricity e^o and barycentric semi-major axes a_1^o and a_2^o, respectively. The semi-major axis of the relative orbit one component describes around its companion is denoted a^o.

At the instant of the explosion the barycentric distances and velocities: r_1, respectively r_2, and $\bar{v}_1^o$, respectively $\bar{v}_2^o$; the separation between both components r and the relative velocity $\bar{v}^o$ of the companion

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star with respect to the pre-supernova star, can be determined from the eccentric anomaly E^o (figure 1a,b) (Whittaker, 1944; Roy, 1978) as :

$$r = (1 - e^o . \cos E^o) . a^o \tag{2.1}$$

with: $$M_1^o . r_1 = M_2^o . r_2 = \mu^o . r \tag{2.2}$$

where: $$\mu^o = \frac{M_1^o . M_2^o}{M_1^o + M_2^o} \tag{2.3}$$

denotes the reduced mass of the initial system,

and $$\bar{v}^{o2} = \left[\frac{1 + e^o . \cos E^o}{1 - e^o . \cos E^o}\right] . v_c^{o2} \tag{2.4}$$

with radial and tangental components given by, respectively:

$$v_r^o = \frac{e^o . \sin E^o}{1 - e^o . \cos E^o} . v_c^o \tag{2.5}$$

$$v_t^o = \frac{\sqrt{1 - e^{o2}}}{1 - e^o . \cos E^o} . v_c^o \tag{2.6}$$

Making use of the pre-supernova momentum relation with respect to the center of gravity C_g^o one finds:

$$- M_1^o . \bar{v}_1^o = M_2^o . \bar{v}_2^o = \mu^o . \bar{v}^o \tag{2.7}$$

whereas: $$v_c^{o2} = G \frac{M_1^o + M_2^o}{a^o} = - \frac{2 . E^o}{\mu^o} \tag{2.8}$$

(with G the universal constant of gravity), denotes the square of the constant relative velocity the pre-supernova system would have if it revolved in a circular orbit of radius a^o. More generally this quantity is identical to the mean over one orbit of the square of the relative orbital velocity of the pre-supernova system. It can be expressed as a function of the total orbital energy:

$$E^o = -G \frac{M_1^o . M_2^o}{2 a^o} \tag{2.9}$$

and the reduced mass μ^o of the initial system and is therefore independent of the eccentricity.

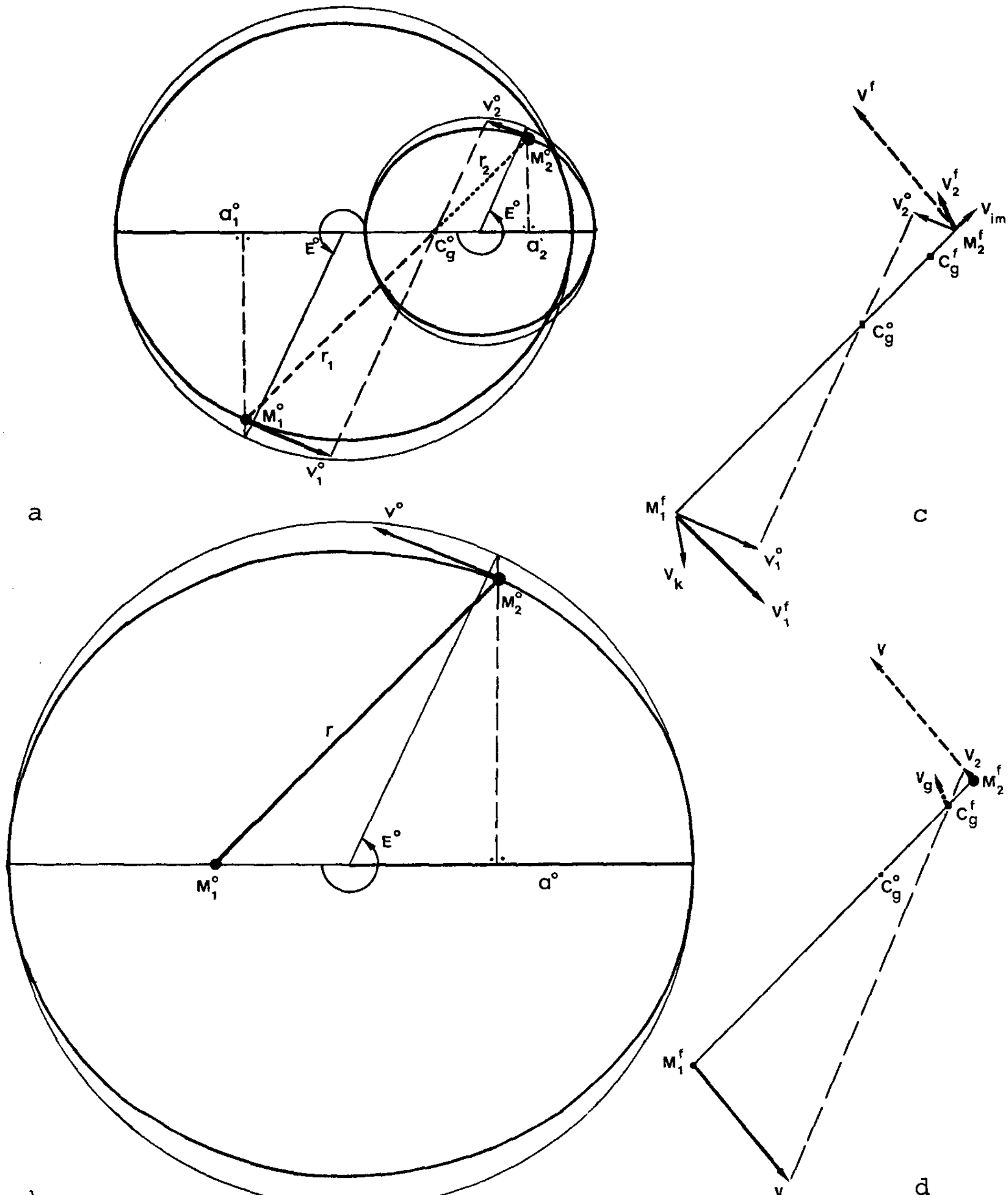

Figure 1. The orbital change due to an asymmetric instantaneous supernova explosion in an eccentric binary (e^o=.4). Part (a) and (b) show the position and relative velocities at the instant of the supernova event in the initial barycentric and relative orbits, respectively. The velocity change with respect to C_g^o is given in part (c). The new barycentric velocities with respect to C_g^f are given in part (d) together with the run-away velocity $\bar{v}_g$ of C_g^f with respect to C_g^o.

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We will make the problem dimensionless by expressing the distances in units of the semi-major axis of the initial relative orbit a^o and the velocities in units of the initial circular orbital velocity v_c^o.

2.3. The instantaneous supernova explosion

The instantaneous asymmetric supernova explosion of a component of a binary system will change the masses and velocities of both components. The collapsed star, a neutron star (or possibly a black hole) of mass M_1^f, receives an extra kick velocity $\bar{v}_k$, as a consequence of the assumed asymmetry of the explosion. The companion star loses a fraction of its mass by stripping and ablation and receives an extra radial velocity $\bar{v}_{im}$, due to the effects of the impact of the supernova shell. Hence, immediately after the supernova event (assumed to be of negligible duration) the collapsed star and its remaining companion of mass M_2^f revolve with barycentric velocities (cf. figure 1c) :

$$\bar{v}_1^f = \bar{v}_1^o + \bar{v}_k \tag{2.10}$$

and

$$\bar{v}_2^f = \bar{v}_2^o + \bar{v}_{im} \tag{2.11}$$

respectively, with respect to their initial center of gravity C_g^o, while the companion star has a relative velocity versus the collapsed star:

$$\bar{v}^f = \bar{v}^o + \bar{v}_{im} - \bar{v}_k \tag{2.12}$$

The new center of mass C_g^f of the remaining components has a run-away velocity $\bar{v}_g$ with respect to C_g^o (cf. figure 1d).

2.4. The final orbital parameters

From the vis-viva integral of the final relative orbit:

$$v^{f^2} = G \,.\, (M_1^f + M_2^f) \,.\, \left[\frac{2}{r} - \frac{1}{a^f} \right] \tag{2.13}$$

one finds, using eq. (2.1) and (2.8) that:

$$\frac{a^o}{a^f} = \frac{2}{1 - e^o \,.\, \cos E^o} - \frac{1}{\alpha} \,.\, \left[\frac{\bar{v}^f}{\bar{v}_c^o} \right]^2 \tag{2.14}$$

where: $$\alpha = \frac{M_1^f + M_2^f}{M_1^o + M_2^o} \tag{2.15}$$

denotes the fractional mass of the remaining system.
Hence the ratio of the semi-major axis of the pre- and post-supernova relative orbit is a function of the ratio of the final to the initial

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total mass of the system, the ratio of the post explosion relative velocity to the initial circular velocity and the relative separation in the initial orbit at the time of the supernova explosion. Using Kepler's third law we can express the ratio of the initial and final orbital period as a function of the ratios of the pre- and post-supernova total mass and relative semi-major axis, as:

$$\left(\frac{P^o}{P^f}\right)^2 = \alpha \cdot \left(\frac{a^o}{a^f}\right)^3 \qquad (2.16)$$

Hence the changes in the relative semi-major axis and orbital period are independent of the individual mass loss of each component, but depend only on the total fraction of mass loss.

The ratio of the total orbital energy of the pre- and post-supernova relative orbit is given (cf. eq. 2.9) as a function of the ratios of the initial and final mass of each component and relative semi-major axis as:

$$\frac{E^f}{E^o} = \frac{M_1^f . M_2^f}{M_1^o . M_2^o} \cdot \frac{a^o}{a^f} \qquad (2.17)$$

For the final eccentricity we find using Kepler's second law the equality:

$$1 - e^{f^2} = \frac{(r . v_t^f)^2}{G . (M_1^f + M_2^f) . a^f} \qquad (2.18)$$

which gives using eq. (2.1) and (2.8):

$$1 - e^{f^2} = \frac{(1 - e^o \cdot \cos E^o)^2}{\alpha} \cdot \frac{a^o}{a^f} \cdot \left(\frac{v_t^f}{v_c^o}\right)^2 \qquad (2.19)$$

The orbital angular momentum of the post explosion relative orbit is defined as:

$$\bar{h}^f = \mu^f \; \bar{r} \wedge \bar{v}^f \qquad (2.20)$$

Hence the ratio of the magnitude of the initial and final orbital angular momentum is given as a function of the ratio of the reduced mass of the pre- and post-supernova system and of the ratio of the tangental component of the initial and final relative orbital velocity, and using eq. (2.6) becomes:

$$\left\| \frac{\bar{h}^f}{\bar{h}^o} \right\| = \frac{\mu^f . v_t^f}{\mu^o . v_t^o} = \frac{1 - e^o \cdot \cos E^o}{\sqrt{1 - e^{o^2}}} \cdot \frac{\mu^f}{\mu^o} \cdot \frac{v_t^f}{v_c^o} \qquad (2.21)$$

3. THE SURVIVAL PROBABILITY

3.1. The survival condition

The condition for the system to remain bound, after the supernova explosion of one of its components, is that the final orbit is elliptic; i.e. the relative semi-major axis a^f must be positive. Hence, the condition for survival may be written using eq. (2.14) as:

$$\beta^2 \geq \left\{\frac{\bar{v}^f}{\bar{v}^o}\right\}^2 \tag{3.1}$$

with β depending on the fractional mass of the remaining system and the relative separation at the time of the explosion:

$$\beta = \left\{\frac{2\alpha}{1 - e^o \cdot \cos E^o}\right\}^{1/2} \tag{3.2}$$

In carthesian coordinates, for example with the X-axis connecting the exploding star to its companion at the instant of the explosion and the Y-axis lying in the orbital plane, we find that:

$$v^{f^2} = (v_{k_x} - v_r^o - v_{im})^2 + (v_{k_y} - v_t^o)^2 + v_{k_z}{}^2 \tag{3.3}$$

where subscripts x, y, z indicate the components along the X, Y, Z axes. From this we can state:

The survival condition for the remaining system requires that in the velocity space the endpoint of the kick velocity vector $\bar{v}_k$, the collapsar gets as a result of the asymmetry of the supernova explosion, is situated inside the sphere S of radius $\beta . v_c^o$ and center coinciding with the endpoint of the velocity vector $\bar{v}^o + \bar{v}_{im}$.

An explosion for which the endpoint of the vector $\bar{v}_k$ is situated on the surface of the sphere S gives a parabolic final orbit. Hereto the velocity vector $\bar{v}_k$ of magnitude $K.v_c^o$, should make an angle θ_S with respect to $\bar{v}^o + \bar{v}_{im}$, given by:

$$\cos \theta_s = \frac{K^2 + A^2 - \beta^2}{2.K.A} \tag{3.4}$$

where the value of β needed for this ranges from $\beta_{min} \leq \beta \leq \beta_{max}$, with:

$$\beta_{min} = |K - A| \tag{3.5}$$

$$\beta_{max} = K + A \tag{3.6}$$

and
$$A = \left\{\left[\frac{e^o \cdot \sin E^o}{1 - e^o \cdot \cos E^o} + I\right]^2 + \frac{1 - e^{o^2}}{(1 - e^o \cdot \cos E^o)^2}\right\}^{1/2} \tag{3.7}$$

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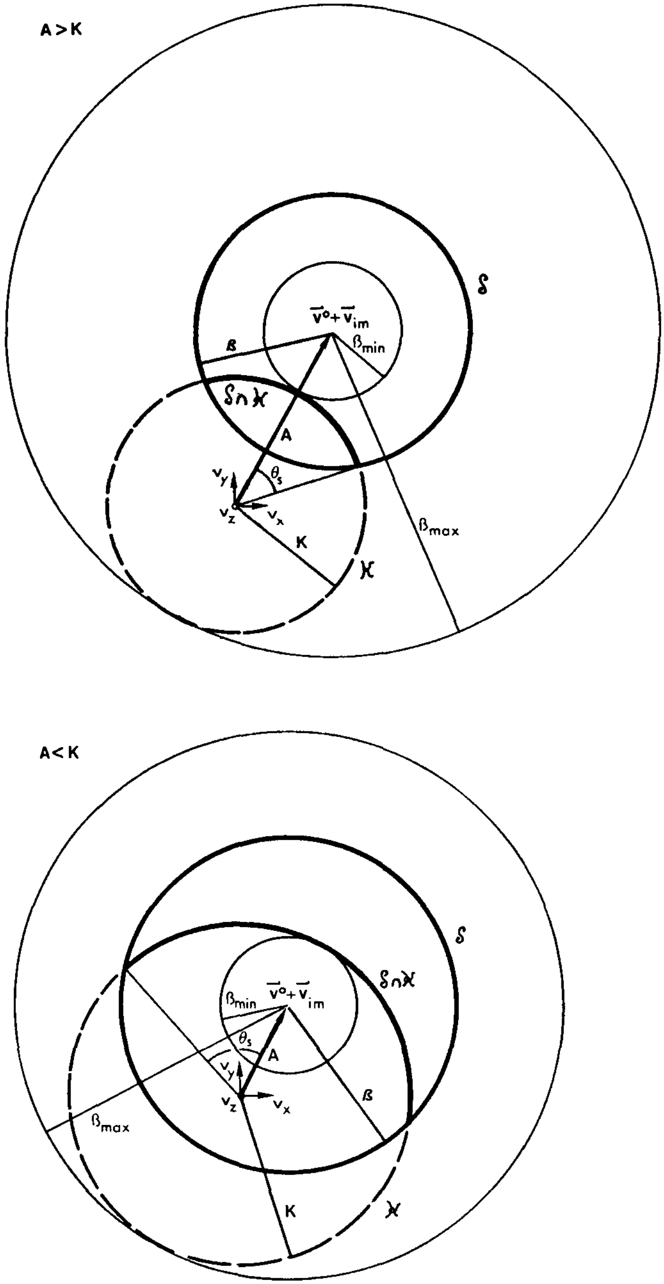

Figure 2. The intersection of the spheres S and K together with the critical values of the survival parameter β for the two possible cases $A > K$ and $A \leq K$.

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where K, I and A denote the magnitude of $\bar{v}_K$, $\bar{v}_{im}$ and $\bar{v}^o + \bar{v}_{im}$, respectively, in units of the circular orbital velocity v_c^o.

If $\beta \leq A$ a symmetric explosion disrupts the system whereas if $\beta > A$ the opposite is true.
Hence for the asymmetric survival condition three cases are to be distinguished according to the value of β (cf. figure 2), i.e.:

<u>case a</u> : $\beta < \beta_{min}$

Here the endpoint of the velocity vector $\bar{v}_K$ is situated outside the sphere S, i.e. the system is disrupted for any direction of the asymmetric kick velocity $\bar{v}_K$. If $K < A$ the origin lies outside the sphere S, hence a symmetric explosion would also disrupt the system. If $K > A$ the magnitude of the kick velocity is so large that it even disrupts the retrograde revolving final system.

<u>case b</u> : $\beta_{min} \leq \beta \leq \beta_{max}$

In this case the system will survive the supernova explosion for those directions of the asymmetric velocity vector $\bar{v}_K$, which are situated inside the solid angle centered at $\bar{v}^o + \bar{v}_{im}$ with semi-apex angle θ_S given by eq. (3.4).

<u>case c</u> : $\beta_{max} < \beta$

Here the velocity vector $\bar{v}_K$ lies entirely inside the sphere S, i.e. the system remains bound for any direction of the asymmetric velocity $\bar{v}_K$. As $\beta_{max} \geq A$, the origin is also situated inside the sphere S, so the system would survive a symmetric explosion ($\| \bar{v}_K \| = 0$) occurring at the same position in the initial orbit.

For an initially circular orbit ($e^o = 0$) the magnitude of the orbital velocity and the separation between both components remains constant, hence:

$$\beta = \sqrt{2.\alpha} \tag{3.8}$$

$$\beta_{\substack{min\\max}} = A = \sqrt{1 + I^2} \tag{3.9}$$

Here the survival condition is a function of the fractional mass and relative velocity of the remaining system. A symmetric explosion would disrupt the system if:

$$\alpha \leq \frac{1 + I^2}{2} \tag{3.10}$$

which is always fulfilled if more than half of the mass leaves the system. In terms of energy this is due to the fact that in a circular orbit the total orbital energy is one half of the constant gravitational energy. Taking away half of the total mass without changing the kinetic energy will unbound the system.

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3.2. The survival probability for an explosion occurring in a given point of the initial orbit

Assuming a certain probability disruption of the asymmetric velocity vector $\bar{v}_k$, one can define the survival probability for an instantaneous supernova explosion occurring in a given point with eccentric anomaly E^o of the initial orbit (of eccentricity e^o and circular velocity v_c^o), for a mass loss parameter α and impact parameter I.

As the supernova explosion itself is supposed to be unaffected by the presence of a companion star, we restrict ourselves here to a random orientation of the asymmetric velocity vector $\bar{v}_k$ with given magnitude $k.v_c^o$. Hence the endpoint of the asymmetric velocity vector $\bar{v}_k$ is randomly situated on the sphere K, centered at the origin with radius $k.v_c^o$. From the survival condition it follows that the corresponding survival probability $P\,(k;E^o)$ is equivalent to the fraction of the surface of the sphere K that is located inside the sphere S.
For $\beta_{min} \leq \beta \leq \beta_{max}$ the part of the surface of K inside S is a polar cap centered at $\bar{v}^o + \bar{v}_{im}$ with semi-apex angle θ_s, given by eq. (3.4). The survival probability is thus given by:

$$P(k;E^o) = \frac{1 - \cos\theta_s}{2} = \frac{\beta^2 - [k-A]^2}{4.k.A} = \frac{\beta^2 - \beta^2_{min}}{4.k.A} \qquad (3.11)$$

If $\beta < A$ the origin lies outside the sphere S and $P(k;E^o) < 1/2$.
The survival probability $P(k;E^o)$ of the remaining system after an instantaneous supernova explosion at a given point of eccentric anomaly E^o, for a randomly orientated asymmetric velocity $\bar{v}_k$ of given magnitude $k.v_c^o$, can be summarized as in Table I.

Table I

	$P(k;E^o)$
$\beta < \beta_{min}$	0
$\beta_{min} \leq \beta \leq \beta_{max}$	$\frac{\beta^2 - \beta^2_{min}}{4.k.A}$
$\beta_{max} < \beta$	1

For a circular pre-supernova orbit the dependence of the survival probability P on the total mass loss and magnitude of the asymmetric kick velocity is given in figure 3 for two values of the impact velocity.

3.3. The mean survival probability over one orbit

As the supernova explosion may equally likely occur at any instant of time during the revolution in the initial orbit, we define the mean survival probability $<P(k)>$ as the weighted average over one orbital

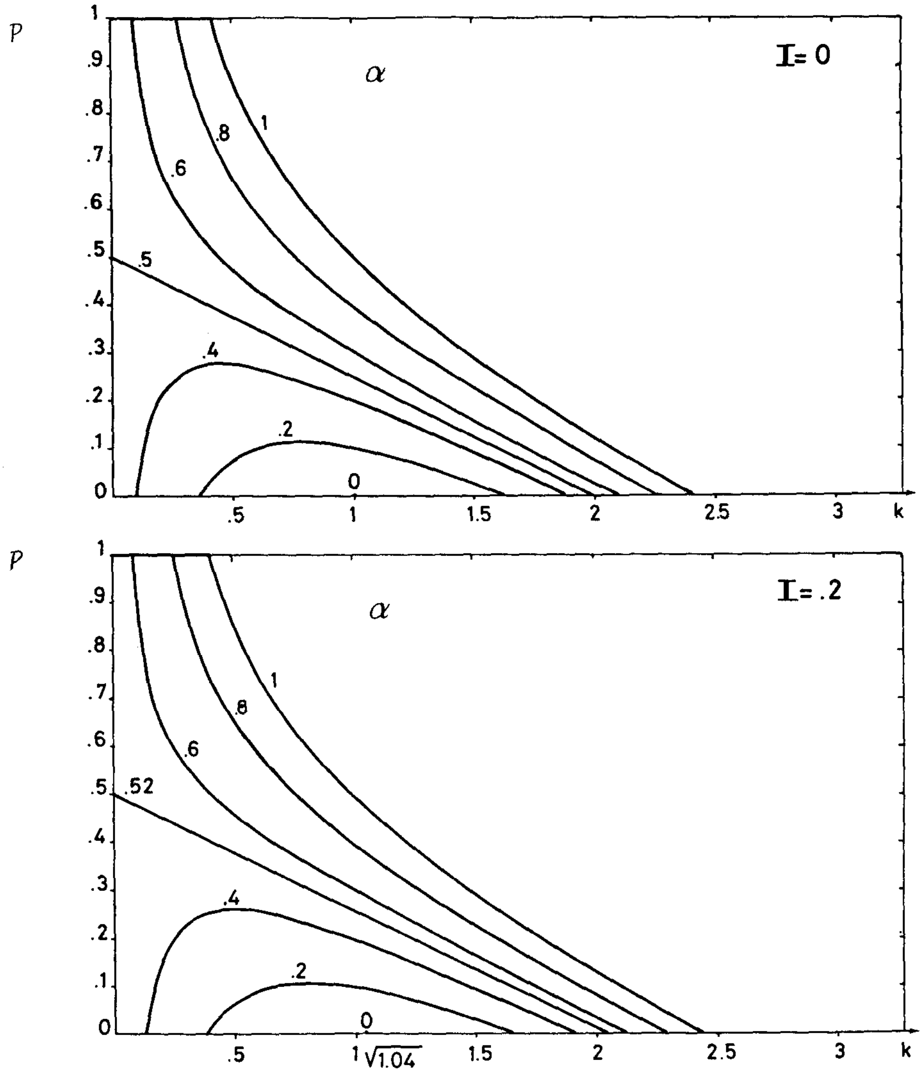

Figure 3. The dependence of the survival probability P on the magnitude of the asymmetric kick velocity for different constant values of the mass loss parameter and two values of the impact parameter in case of a circular pre-supernova orbit.

period of the survival probability $P(k;E^o)$ for an explosion that occurs in a given point of the initial orbit, with equal weight per unit of time; i.e. using Kepler's law of areas as weighting function:

$$\langle P(k) \rangle = \frac{1}{P^o} \int_o^{P^o} P(k;E^o)\, dt \tag{3.13}$$

Differentiating Kepler's parametric representation:

$$\frac{2.\pi.t}{P^o} = E^o - e^o.\ \sin E^o \qquad (3.14)$$

the time integral defining $<P(k)>$ can be replaced by an integral over the position in the initial orbit in function of the eccentric anomaly as:

$$<P(k)> = \frac{1}{\pi} \int_o^{\pi} P(k;E^o)\ .\ (1 - e^o.\ \cos E^o)\ dE^o \qquad (3.15)$$

4. THE RUN-AWAY VELOCITIES

4.1. The run-away velocity of the new center of gravity

The run-away velocity of the center of gravity C_g^f of the remaining system, after an instantaneous supernova explosion, with respect to the center of gravity C_g^o of the pre-supernova binary is given by the momentum equation of the post-supernova components in the initial center of mass system as a function of the velocities, at the instant after the instantaneous supernova explosion, and the masses of the remaining components. This equation takes the form using eq. (2.10, 2.11):

$$(M_1^f + M_2^f)\ .\ \bar{v}_g = M_1^f.\bar{v}_1^o + M_2^f.\bar{v}_2^o + M_2^f.\bar{v}_{im} + M_1^f.\bar{v}_k \qquad (4.1)$$

The first two terms on the right-hand side denote the momentum due to the mass loss from the system, the third term represents the momentum imparted by the supernova shell to the companion and the last term gives the momentum contributed by the asymmetry of the supernova explosion. Making use of the pre-supernova momentum relation with respect to C_g^o (cf. eq. 2.7) this can be written as:

$$\bar{v}_g = \frac{1}{M_1^f + M_2^f} \left\{ \left(\frac{M_1^o.M_2^f - M_2^o.M_1^f}{M_1^o + M_2^o}\right).\bar{v}^o + M_2^f.\bar{v}_{im} + M_1^f.\bar{v}_k \right\} \qquad (4.2)$$

or:

$$\bar{v}_g = \frac{1}{1 + q^f} \left\{ \left[\frac{q^f - q^o}{1 + q^o}\right].\bar{v}^o + q^f.\bar{v}_{im} + \bar{v}_k \right\} \qquad (4.3)$$

with:-

$$q^o = \frac{M_2^o}{M_1^o} \qquad (4.4)$$

and:

$$q^f = \frac{M_2^f}{M_1^f} \qquad (4.5)$$

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denoting the mass ratio of the secondary to the primary component before and after the supernova explosion, respectively.

Hence the extreme values of the magnitude of $\bar{v}_g$ as a function of the asymmetric velocity vector $\bar{v}_k$ with given magnitude $k.v_c^o$ coincide with $\bar{v}_k$ directed along or opposite to the vector $\left(\frac{q^f - q^o}{1 + q^o}\right).\bar{v}^o + q^f.\bar{v}_{im}$, and are given respectively by:

$$v_{g_{min}} = \frac{1}{1 + q^f}(k - B) \tag{4.6}$$

$$v_{g_{max}} = \frac{1}{1 + q^f}(k + B) \tag{4.7}$$

with:

$$B = \left\{\left[\left(\frac{q^f - q^o}{1 + q^o}\right) . \frac{e^o . \sin E^o}{1 - e^o . \cos E^o} + q^f.I\right]^2 + \left(\frac{q^f - q^o}{1 + q^o}\right)^2 \frac{1 - e^{o2}}{(1 - e^o.\cos E^o)^2}\right\}^{1/2} \tag{4.8}$$

denoting the magnitude of the vector $\left(\frac{q^f - q^o}{1 + q^o}\right).\bar{v}^o + q^f.\bar{v}_{im}$ in units of the circular orbital velocity v_c^o.

4.2. The run-away velocity of the bound systems

If the system survives the supernova explosion of one of its components, the newly formed remnant and its remaining companion revolve in elliptic orbits around their center of gravity C_g^f. This center of gravity will possess a run-away velocity $\bar{v}_b$ with respect to the center of gravity of the initial system as given by eq. (4.3) of the preceeding section. Here the extreme values of the magnitude of $\bar{v}_b$ as a function of the asymmetric velocity vector $\bar{v}_k$ with given magnitude $k.v_c^o$ also depend on the survival condition, i.e. on the extreme values of the distance of the endpoint of the vector: $-\left(\frac{q^f - q^o}{1 + q^o}\right).\bar{v}^o - q^f.\bar{v}_{im}$, to the part of the sphere K inside the sphere S, deduction of which will be given elsewhere.

4.3. The run-away velocities of the disrupted components

The magnitude of the relative velocity at infinity $\bar{v}^\infty$ of the remaining components of an unbound system is given by the vis-viva integral as:

$$\bar{v}^{\infty 2} = -G\,\frac{(M_1^f + M_2^f)}{a^f} \tag{4.9}$$

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Using the definition of the initial circular velocity v_c^o (eq. 2.8) together with the expression of the semi-major axes of the relative orbit (eq. 2.14) this becomes:

$$\bar{v}^{\infty 2} = -\alpha \cdot \frac{a^o}{a^f} \cdot v_c^{o^2} = \bar{v}^{f^2} - \beta^2 \cdot v_c^{o^2} \qquad (4.10)$$

Hence the run-away velocities of both components in case of disruption are, respectively :

$$\bar{v}_1^\infty = \bar{v}_g - \left[\frac{q^f}{1 + q^f}\right] \cdot \bar{v}^\infty \qquad (4.11)$$

$$\bar{v}_2^\infty = \bar{v}_g + \left[\frac{1}{1 + q^f}\right] \cdot \bar{v}^\infty \qquad (4.12)$$

where we used the momentum equation at infinity of the post-supernova system with respect to the new center of gravity C_g^f :

$$-M_1^f \cdot \bar{v}_1^\infty = M_2^f \cdot \bar{v}_2^\infty = \mu^f \cdot \bar{v}^\infty \qquad (4.13)$$

5. EVALUATION OF THE EXPLOSION PARAMETERS

Observations of white dwarfs in galactic clusters and theoretical stellar models indicate that for single stars the lower mass limit for becoming a supernova is around 6-8 $M_\odot$ (Gunn and Ostriker, 1970; van den Heuvel, 1975; Sugimoto and Nomoto, 1980). On the other hand, a component of a close binary loses before the end of its evolution more than two thirds of its mass by mass transfer to its companion and mass loss out of the system. As a consequence of this mass loss a much larger initial mass of a component of a binary is required for direct collapse of the stellar core to a relativistic star (van den Heuvel, 1981). Theoretical computations (van den Heuvel, 1974; de Loore and De Grève, 1975,1976) indicate that the lower mass limit for a component of a close binary to become a supernova by direct core collapse is at least 12-15 $M_\odot$. Mass loss by stellar wind during the main sequence and especially during the helium burning stage (presumably identified with Wolf-Rayet stars (Paczynski, 1971)) gives a lower limit of 15-20 $M_\odot$. Such stars leave helium cores more massive than 4 $M_\odot$. Helium stars of smaller masses evolve in close binaries through a second stage of mass exchange and are expected to end up as white dwarfs with masses below the Chandrasekhar limit (Arnett, 1974; Tutukov and Yungelson, 1973; van den Heuvel and Heise, 1972; de Loore and De Grève, 1975,1976). More massive helium components of a binary are expected to explode as supernovae (Arnett, 1975, 1978).

We assume here that the core of a supernova star will form a neutron star of mass 1.5 $M_\odot$ (Weaver and Woosley, 1978).

The initial stellar mass function (in the mass range 1 to 100 $M_\odot$) may be approximated by :

$$\psi(M) = C\ M^{-2.55}$$

(Limber, 1960; Taff, 1974; Miller and Scalo, 1979). As during the last 10^9 years the stellar birth rate in the galaxy was practically constant (Schmidt, 1959; Miller and Scalo, 1979) the number of the short lived stars of mass larger than 2 $M_\odot$ will be close to a steady state in which the birth rate equals the death rate. Assuming that 50% of the new born stars are in close binaries (Garmany, Conti and Massey, 1980; Abt and Levy, 1978) the ratio ζ of the birth rate of collapsing stars in close binaries and of collapsing single stars is given by :

$$\zeta \simeq .5 \int_{15}^{\infty} \psi(M)\ dM \Big/ \int_{5}^{\infty} \psi(M)\ dM = .09$$

Radio pulsars are thought to be neutron stars formed during a supernova explosion (Ruderman, 1972), hence most of them must have originated from low velocity single stars. As they are high velocity objects (up to $6\ 10^2$ km.s^{-1})(Taylor and Manchester, 1977,1981; Gullahorn and Rankin, 1978; Lyne, 1981) their acceleration must be due to their formation process (Shklovsky, 1970; Iben, 1972; Buchler, 1973) or their relativistic nature (Harrison and Tademaru, 1977). The radio pulsars with measured run-away velocities belong to the maximum part of the velocity distribution as they are easiest measurable. Hanson (1979) in a statistical study estimated the mean of the run-away velocity distribution to be about 100 km.s^{-1}.

We will assume here that the observed run-away velocities of radio pulsars originating from single stars are due to the asymmetry of the supernova explosion which created them and to be 75 - 150 km.s^{-1}

For collapsed stars formed in binaries the run-away velocities depend on the initial binary conditions. In a binary system mass exchange will cause the originally most massive star (primary) to become the less massive one. As the evolution of this primary remains faster than the evolution of the now more massive secondary, it will as first become a supernova while its companion is still a main sequence star (van den Heuvel and Heise, 1972). Hence during the first supernova explosion in a binary less than half of the total mass will leave the system, which cannot enhance the disruption of the remaining system. According to two-dimensional hydrodynamical calculations by Fryxell and Arnett (1981) the momentum imparted by the supernova shell to a companion, of polytropic structure with polytropic index n=3, decreases from 80% to 35% of the momentum incident on the geometric cross section of the star as the kinetic energy of the incident matter increases from 20% to 60% of the binding energy of the companion star. They find a fairly good agreement between their hydrodynamical results and the analytic model of Wheeler, Lecar and McKee (1975) for the evaluation of the mass loss due

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to stripping and ablation and the reduction of the effective cross section in case that the ratio ψ of the momentum of the impacting matter to the momentum which the undisturbed companion would have if it moved with outer escape velocity (as defined below) ranges from:

$$\psi = 6.10^{-3} - 4.10^{-2} \tag{5.1}$$

As the effects of the impact may become important only for larger values of ψ we make the following extrapolation.

The mass loss from the companion due to stripping and ablation is assumed to be given by the results for polytropic models of Wheeler et al. (1975) which can be fitted by a simple function of $\mathrm{Log}_{10}\ \psi$ as:

$$M_2^f = M_2^o \cdot \left[1 - \left\{\frac{\mathrm{Log}\ \psi + 3}{4}\right\}^c\right] \quad , c = \begin{cases} 4.3 \text{ if } \psi > .01 \\ 3.3 \text{ if } \psi \leq .01 \end{cases} \tag{5.2}$$

where:

$$\psi = F_{in} \cdot \left[\frac{V_{SN\ shell} - 1}{v_{es}}\right] \tag{5.3}$$

with:

$$F_{in} = \frac{(M_1^o - M_1^f)}{4.M_2^o} \cdot \left[\frac{R_2^o}{r}\right]^2 \tag{5.4}$$

and

$$v_{es} = \left\{2.G.\frac{M_2^o}{R_2^o}\right\}^{1/2} \tag{5.5}$$

F_{in} denotes the ratio of the mass of the supernova shell which interacts with the companion to the total mass of this companion; v_{es} stands for the magnitude of the escape velocity in the outer part of the companion; $V_{SN\ shell}$ denotes the ejection velocity of the supernova shell. Observations give values between $0.5-3.10^4$ km.s^{-1} (Shklovsky, 1968; Schatzman, 1965; Zwicky, 1965; Minkowski, 1969).

We assume here that $V_{SN\ shell} = 10^4$ km.s^{-1}.
R_2^o represents the pre-supernova radius of the companion star, which is still on the main sequence.

We assume here that R_2^o equals two times the radius on the zero age main sequence given by Plavec (1968) in solar units:

$$R_2^o = 2\ 10^{(0.63 \log M_2^o - 0.08)} \tag{5.6}$$

As to the impacted momentum we assume it equals 30% of the incident momentum, hence

$$v_{im} = 0.3\ F_{in} \cdot V_{SN\ shell} \tag{5.7}$$

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This way the effects of the interaction of the supernova shell with the companion star will be overestimated if they play a role in the faith of the post-supernova binary ($\psi > .1$).

6. THE OUTCOME OF THE FIRST SUPERNOVA EXPLOSION IN MASSIVE BINARIES

6.1. The data

Out of the best known data on Wolf-Rayet + OB binaries we selected our pre-supernova systems by assuming a value for the inclination i of the orbital plane. Except for V444 Cyg. the only system with an accurately measured value of i = 78°4 (Münch 1950) For the other systems the choice of the inclination was inferred from the eclipse condition taking into account the radius of the Wolf-Rayet envelope the spectral type of the OB star and the Roche radii of both components. The value retained yields the best mass-spectral type relation for the OB star and a minimum mass of 5 $M_\odot$ for the Wolf-Rayet star. For the two systems with highly eccentric orbits, i.e. γ^2Vel ($e^o \simeq 0.40$) and HD 90657 ($e^o = 0.42$) this condition was tested at periastron.

Table II . The pre-supernova data.

	System (★ eclipses)	Spectral type	P^o (days)	$M_{WR}.\sin^3 i$ $M_{OB}.\sin^3 i$ $(M_\odot)$	q^o	i	M_{WR} M_{OB}	v_c^o (km.s^{-1})
a	CX Cep	WN5 O	2.12	5.3 12.	2.3	55°	9.6 22.	524
b	HD 193576= V444 Cyg ★	WN6 O6I	4.21	9.5 24.	2.4	78°4	11. 26.	439
c	HD 94546	WN4 O	4.90	8. 23.	2.9	60°	12. 35.	452
d	HD 90657	WN5 O6	6.42 e^o=.42	7.6 17.	2.2	50°	17. 37.	432
e	HD 211853= GP Cep ★	WN6 O6I	6.69	7.6 20.	2.6	75°	8.5 22.	353
f	HD 152270	WC7 O5-8	8.89	1.8 4.9	2.7	30°	9.3 26.	337
g	HD 186943	WN4 O9V	9.55	9. 18.	2.0	60°	9.4 19.	306
h	HD 168206= CV Cer (★)	WC8 O8-9	29.7	11. 22.	2.0	75°	12. 25.	229
i	HD 68273= γ^2 Vel	WC8 O9I	78.5 e^o=.4	17. 32.	1.9	70°	20. 39.	194
j	HD 190918	WN4 O9I	85.0	0.20 0.77	3.8	20°	5.9 22.	147

(a) Massey and Conti (1980)
(b) Münch (1950)
(c) Niemela (1980)
(d) Niemela (1976)
(e) Hiltner (1945) and Massey (1981)
(f) Seggewiss (1974)
(g) Massey (1981)
(h) Massey and Niemela (1980)
(i) Niemela and Sahade (1980)
(j) Wilson (1949)

The assumption is made that these data, as given in Table II, are comparable with the pre-supernova binary parameters at the time the Wolf-Rayet star becomes a supernova

6.2. The results

Table III shows the results of a supernova event leaving a collapsed star of mass $M_p = 1.5\ M_\odot$ for an assumed value of $V_{SN\ shell} = 10^4$ km.s^{-1} and a random orientated kick velocity $\bar{v}_k$ of 75 km.s^{-1} (case a) and 150 km.s^{-1} (case b), respectively. In case of an eccentric orbit the results for the supernova explosion occurring at periastron and at apastron are given. In the first three columns the pre-supernova parameters are shown. The initial mass of the pre-supernova star M_{WR} and its companion M^o_{OB} are given in the first column in solar units, in the second column the initial period P^o, expressed in days, and the initial eccentricity e^o, if non zero, is shown. Column three gives the barycentric velocities at the instant before the explosion of both components v^o_{WR} and v^o_{OB}. If $e^o \neq 0$ the upper and lower case correspond to an explosion occurring at periastron and at apastron, respectively. Column four gives the mass loss parameter α and if $e^o \neq 0$ the survival parameter β^2 is also given. The effects of the impact are shown in the subsequent columns. The fifth column shows the magnitude of the impact velocity v_{im} and the value of the momentum parameter ψ and column six the final mass of the companion M^f_{OB}. In column seven the run-away velocity of the bound system $v_{b\ sym}$ or of the disrupted components $v^\infty_{p\ sym}$ and $v^\infty_{OB\ sym}$ are given for the symmetric supernova explosion ($\|\bar{v}_k\| = 0$). In case of an asymmetric explosion the survival probability P and the extreme values of the run-away velocity of the bound system v_b, of the disrupted companion star v^∞_{OB}, and of the single collapsed star v^∞_p are shown respectively in columns eight, nine and ten for case a ($\|\bar{v}_k\| = 75$ km.s^{-1}) and in colums eleven, twelve and thirteen for case b ($\|\bar{v}_k\| = 150$ km.s^{-1}). In case of an eccentric initial orbit the value of the mean survival probability $\langle P \rangle$ is also shown in columns seven, eight and eleven, respectively.

6.3. The survival probability

A massive close binary revolving in a nearly circular initial orbit survives the symmetric supernova explosion of its most evolved and hence less massive component. If the initial revolution is eccentric the system is disrupted in case the symmetric explosion occurs at the less probable positions around periastron in the initial orbit. The mean survival probability over one revolution for the short periodic system (d) is $\langle P_{sym} \rangle = .80$, due to the influence of the impact especially around periastron; whereas for the long periodic system (i) : $\langle P_{sym} \rangle = .92$.

In case the supernova explosion is asymmetric the survival condition also depends on the ratio of the asymmetric kick velocity to the initial circular velocity. As v^o_c decreases for increasing values of a^o (equation 2.8), the influence of the asymmetric kick velocity will increase with P^o.
Originally circular systems with an initial orbital period up to about

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Table III. The survival probability and the run-away velocities.

								case a: $\|\bar{v}_k\|$=75 km.s^{-1}				case b: $\|\bar{v}_k\|$=150 km.s^{-1}			
	M_{WR}	P^o (e^o)	v^o_{WR} v^o_{OB}	α (β^2)	v_{im} ψ	M^f_2	v_b sym/or [v^∞_P sym, v^∞_{OB} sym] ($<P_{sym}>$)	P ($<P>$)	v_b	v^∞_{OB}	v^∞_P	P ($<P>$)	v_p	v^∞_{OB}	v^∞_P
a	9.6	2.12	365	.66	78.	19.3	141	.98	135	136	142	.68	134	135	116
	22.		159		.28				147	137	151		152	145	263
b	11.	4.21	309	.70	34.	24.4	110	1.	105			.70	105	106	79
	26.		131		.12				115				119	113	221
c	12.	4.90	337	.74	29.	33.3	100	1.	96			.77	95	96	81
	35.		115		.09				103				106	100	205
d	17.	6.46	463	.64	60.	32.8	221	.20	200	200	180	.31	199	201	143
	37.	(.42)	213	(1.1)	.26		204		202	208	295		205	212	385
							(.80)	(.80)				(.71)			
			189	.70	13.	36.2	77	1.	74			1.	71		
			87	(.49)	.04				80				83		
e	8.5	6.69	255	.75	15	21.4	77	1.	71			.68	72	72	44
	22.		98		.05				82				86	81	202
f	9.3	8.89	248	.77	11.	25.5	71	1.	66			.68	67	67	39
	26.		89		.04				75				79	75	199
g	9.4	9.55	205	.71	11.	18.6	79	.86	75	75	66	.59	77	75	25
	19.		101		.04				85	78	100		90	87	204
h	12.	29.7	155	.72	3.	24.8	61	.75	59	59	40	.50	62	58	1
	25.		74		.01				66	63	98		70	68	201
i	20.	78.5	195	.68	3.	38.6	88	.42	89	89	46	.36	91	89	5
	39.	(.4)	100	(1.1)	.01		90		92	94	152		95	96	246
							(.92)	(.69)				(.43)			
			84	.69	1.	39.0	38	.89	36	36	27	.48	39	35	20
			43	(4.9)	0.				41	37	55		44	41	162
j	5.9	85.0	116	.84	0.	22.0	22	.71	20	19	9	.41	25	18	22
	22.		31		0.				26	24	94		31	28	202

one week stay together for case a and have a survival probability of $.7 \leq P \leq .8$ for case b. Except the shortest system (a) which has a relatively small value for α = .66 and is disrupted for a kick directed along $\bar{v}^o + \bar{v}_{im}$ of minimum magnitude $v_{k\ min}$ = 78 km.s^{-1} if v_{im} = 0, with P = 1 (case a) and P = .71 (case b), respectively. Whereas here v_{im} = 78 km.s^{-1} so that $v_{k\ min}$ = 73 km.s^{-1}, with P = .98 (case a) and P = .68 (case b), respectively. Hence the impact lowers the survival probability here with a few percent.

The short periodic system (d) with an initially eccentric orbit is disrupted for some directions of the asymmetric kick velocity if the explosion occurs near periastron in the initial orbit, with P_{per} = .20 (case a) and P_{per} = .31 (case b), respectively; but stays bound for an explosion occurring around apastron, so the mean survival probability over one orbit is as high as $<P>$ = .80 (case a), i.e. equal to the symmetric value, and $<P>$ = .71 (case b), respectively. This last value lies in the range of the survival probabilities of the circular short periodic systems.

For originally circular systems having an initial orbital period of the order of months the survival probability lies in the range $.7 \leq P \leq .9$ (case a) and $.4 \leq P \leq .6$ (case b), respectively. The long periodic system (i) with an initially eccentric orbit has a survival probability around periastron as low as $P_{per} = .42$ (case a) and $P_{per} = .36$ (case b), respectively and around apastron up to $P_{ap} = .89$ (case a) and $P_{ap} = .48$ (case b), respectively; with a mean survival probability of $\langle P \rangle = .69$ (case a) and $\langle P \rangle = .43$ (case b), respectively, both in the range of the corresponding survival probabilities for long periodic circular systems.

6.4. The run-away velocities

The magnitudes of the run-away velocities of the OB stars with a collapsed companion $\bar{v}_b$ and of the disrupted OB stars $\bar{v}_{OB}^{\infty}$ are comparable. They are slightly smaller than the barycentric velocity $\bar{v}_{OB}^{o}$ of the OB star at the instant of the supernova explosion and their asymmetric values enclose the symmetric one : $\bar{v}_{b\ sym}$ or $\bar{v}^{\infty}_{OB\ sym}$. According to the pre-supernova momentum relation with respect to the initial center of gravity C_g^o (eq. 2.7), these run-away velocities are inversely proportional to the pre-supernova mass ratio q^o. They decrease for increasing values of the initial orbital period. The possible values of $\bar{v}_b$ are slightly larger than those of $\bar{v}_{OB}^{\infty}$ (except for an explosion occurring near periastron in an elliptic initial orbit where the opposite is true) and their maximum values are most probable (cf. figure 4).

Originally circular systems with an orbital period up to one week create high velocity OB stars (single or with a collapsed companion) with run-away velocities between 75-150 $km.s^{-1}$. If the initial orbit is eccentric, as in the short periodic system (d), a supernova explosion occurring at the less probable positions near periastron in the initial orbit releases run-away OB stars faster than 200 $km.s^{-1}$.
For originally long periodic circular systems the post-supernova system, consisting of an OB star and a collapsed companion, and the disrupted single OB stars have run-away velocities in the range 60-70 $km.s^{-1}$ if the initial orbital period is about one month and smaller than 25 $km.s^{-1}$ if the initial orbital period is longer than a few months. If the initial orbit is eccentric and the explosion occurs near periastron somewhat faster run-away OB stars are released.

In case a nearly circular system is disrupted a single pulsar is formed with a run-away velocity up to $v_p^{\infty} \leq 100\ km.s^{-1}$ (case a) and $v_p^{\infty} \leq 200\ km.s^{-1}$ (case b), respectively; independent of the initial orbital period. Few of them have a negligible run-away velocity (cf. figure 4). Except system (a) where $140\ km.s^{-1} \leq v_p^{\infty} \leq 150\ km.s^{-1}$ (case a) and $115\ km.s^{-1} \leq v_p^{\infty} \leq 265\ km.s^{-1}$ (case b), respectively due to the relative large mass loss and the high value of the impact velocity.
In case the supernova explosion disrupts an initially eccentric revolving system the run-away velocity of the collapsed star is comparable with the value for circular systems. Only in the less probable cases the supernova explosion occurs near periastron in the initial orbit, much higher

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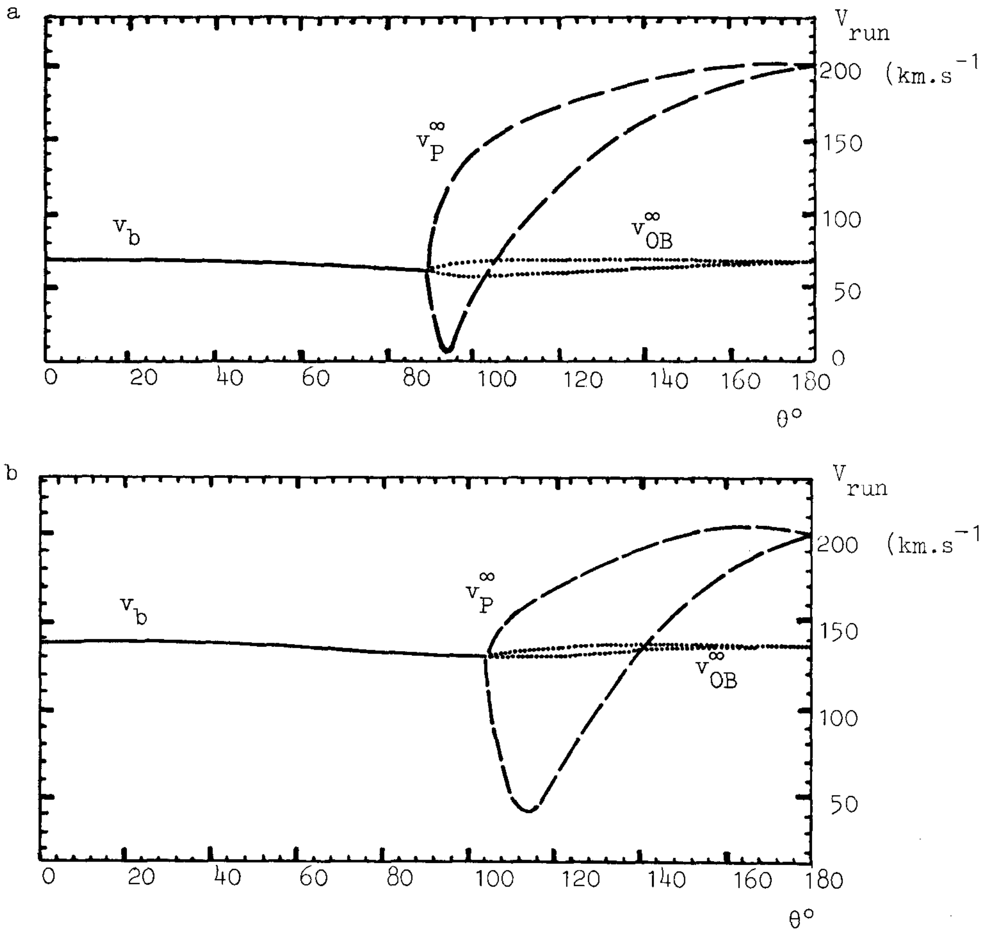

Figure 4. The run-away velocity range as a function of the angle θ between the asymmetric kick velocity $\bar{v}_k$ and the vector $\bar{v}^o + \bar{v}_{im}$. The outcome for a case (b) supernova is shown for the system (h) in part (a) and for the system (d) with $E^o = \pi/2$ in part (b).

velocity pulsars are created. For example the symmetric explosion occurring near periastron in the short periodic system (d) releases a run-away pulsar with v_p^∞ sym = 220 km.s^{-1}; for an asymmetric explosion near periastron : 180 km.s^{-1} $\leq v_p^\infty \leq$ 295 km.s^{-1} (case a) and 140 km.s^{-1} $\leq v_p^\infty \leq$ 385 km.s^{-1} (case b), respectively. For an explosion near periastron in the long periodic system (i) : v_p^∞ sym = 90 km.s^{-1}, $v_p^\infty \leq$ 150 km.s^{-1} (case a) and $v_p^\infty \leq$ 245 km.s^{-1} (case b), respectively.

7. DISCUSSION AND CONCLUDING REMARKS

In view of the above we can conclude that the main effect of the impact of the ejected supernova shell on the companion star is the induced mass loss out of the system. Its magnitude is roughly inversely proportional to the square of the separation at the time of the supernova explosion. But, even for the shortest initial orbital periods considered or near periastron in eccentric orbits, the influence of the impact on the survival probability and the run-away velocities is marginal. Though $\bar{v}_{im}$ enters the equation (3.4) of $\bar{v}_g$ multiplied with q^f = 15-26.

The asymmetric kick velocity $\bar{v}_K$, the collapsed star is supposed to receive due to the asymmetry of the supernova explosion, lowers the survival probability of the remaining system and its importance is inversely proportional to the initial circular velocity v_c^o; i.e. increases for increasing values of the initial orbital period. The run-away velocity of the disrupted collapsed stars depends only on the magnitude of the asymmetric kick velocity. Whereas its influence on the run-away velocities of the remaining systems and of the disrupted OB stars is found to be marginal.

An initially eccentric orbit is mostly disrupted if the explosion occurs in the less probable positions near periastron. As most of the short periodic systems are expected to be circularized by tidal forces, OB stars with run-away velocities larger than 160 $km.s^{-1}$ should be very rare. Initially eccentric systems with long orbital periods may be quite common. The range of their run-away velocities is somewhat larger than in the circular case, but their mean survival probability equals the circular value.

If the supernova explosion is symmetric all the run-away OB stars originating from initially circular systems will have a collapsed companion. Single run-away OB stars are formed only in case the supernova explosion occurs at the less probable positions near periastron in an initially eccentric orbit. In case of an asymmetric supernova explosion however the mean survival probability and the run-away velocity of the OB star (be it single or with a collapsed companion) are related. They both decrease for increasing values of the initial orbital period.
Hence most of the high velocity OB stars ($\geq$ 60 $km.s^{-1}$) have a collapsed companion. They originate from systems, with an initial orbital period up to one month, which have a large survival probability. Whereas systems with an initial orbital period of a few months create low velocity OB stars and about half of them are disrupted. Single run-away OB stars are expected to be less numerous and most of them have small run-away velocities.

In a bound system the collapsed star can show up as an X-ray pulsar, in the very rare case a suitable mode of mass transfer is available (cf. Davidson and Ostriker, 1973; Illarionov and Sunyaev, 1975; Savonije, 1979,1978). From observations of massive X-ray binaries we know that most of them have a high z-latitude, indicating that it are run-away objects; the observed orbital periods are up to about one month and they revolve in nearly circular orbits. The theoretically expected number of massive X-ray binaries in the galaxy (van den Heuvel, 1974) is in agreement with the observed number in case their survival probability is high. This supports our results that a supernova explosion in a massive close binary, with an initial orbital period up to a few months, creates run-away OB stars most of which have a collapsed companion. Hence an asymmetric supernova explosion in a massive close binary with initial period up to one year cannot lower the survival probability by orders of magnitude (cf. De Cuyper, De Grève, de Loore and van den Heuvel, 1976).

If the collapsed star is not directly observable, most of the bound systems will be regarded as single stars. This is due to the small barycentric velocity of the much heavier OB star in its final orbit, to the marginal effects the collapsed star has on its photometric light variations (unless the side towards the pulsar companion is heated by electromagnetic radiation from the pulsar beams) and to the observational limits on these quantities for OB stars.

In spite of the above, Stone (1979, 1981a,b) derived a mass-velocity relation for O-stars from a theoretical evolution model. Starting from a "standard" zero-age-main-sequence WR+OB binary with fixed initial orbital period and mass ratio, equal to their so called "most probable" value, only the initial total mass of the system was used as a free parameter. The system was assumed to revolve in a circular orbit and the period change due to mass and momentum loss and exchange was calculated neglecting the rotational energy and angular momentum of both components. The observational evidence given for the supposed increase of the run-away velocity of O-stars with their mass is weak. It depends on only three high run-away velocities of stars heavier than 60 $M_\odot$, which may be due to selection effects. Whereas the given run-away velocities of stars smaller than 60 $M_\odot$ are uncorrelated.

Our results show no evidence for a correlation between the mass of the remaining companion and its run-away velocity, in agreement with the observed run-away OB-star velocities (Cruz-González et al., 1974; Stone, 1979).

If the system is disrupted a single pulsar is released with maximum run-away velocity merely depending on the magnitude of the asymmetric kick velocity. We started by deriving this magnitude from observations of single radio pulsars. The run-away velocities calculated for pulsars escaping from massive binaries are spread around the magnitude of the asymmetric kick velocity used. But even for $\|\bar{v}_k\|$ = 150 km.s^{-1} they are not larger than 200 km.s^{-1} for initially circular systems and about 400 km.s^{-1} for supernovae occurring near periastron in initially short periodic eccentric orbits. The highest run-away velocities observed

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($\lesssim$ 650 km.s^{-1}) are probably the outcome of the second supernova explosion in a massive close binary which survived the supernova explosion of its less massive companion. Here the effect of the second supernova depends critically on the outcome of the instable spiral-in of the collapsed star into the envelope of its more massive companion. Hence the supposed asymmetric supernova explosions reproduce the observed run-away velocities of pulsars.

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