Funktionentheorie, by A. Hurwitz and R. Courant, with an appendix by H. Röhrl. 4th edition. Springer Verlag, Berlin-Göttingen-Heidelberg-New York, 1964. xiii + 706 pages. Price DM. 49.00.

The first edition of this well-known treatise appeared in 1922 and contained the lectures of Hurwitz on complex analysis, strictly in the spirit of Weierstrass, i.e. analytic functions were defined and built up by power-series. A second part dealing with the theory of elliptic functions - both in the manner of Jacobi and of Weierstrass with their sometimes conflicting notations - was then added as an application of these methods. In 1925 Courant added a third part on geometric function theory where the whole material was built up anew with the existence of the complex derivative as a starting point. This latter part was enlarged in the 3rd edition of 1929 and has served admirably as an introduction to analytic functions for many mathematicians. Now in the 4th edition of 1964 a fourth part written by H. Röhrl has been added. The first two parts were left unchanged - probably due to a sentimental reluctance or out of respect for the founder of the enterprise - although the book has now acquired a rather unwieldy size, and in my opinion the first part could have been omitted without damage to the further development. At present there is probably nobody left who would want to introduce analytic functions by such an awkward and inelegant way as power series. Moreover the theory starts again on p. 257; the equivalence of the two definitions is proved on p. 304 and the theory is then carried much further on geometrical grounds. Some trace of justification for keeping the first part could perhaps be found in the theory of functions of several complex variables - which, however, is not treated in this book.

Contrary to the first two parts, the third part has been modernized here and there: Dirichlet's principle is now proved under the weak assumption that the considered families are of class C'', and this in turn assures the existence of harmonic functions for given singularities under the same assumptions. Abelian integrals on compact Riemann surfaces are dealt with more fully. A proof for the identity of the topological and analytic definition of the genus has been added, as well as Carlemann's generalisation of the maximum principle, and more detailed investigations of the boundary values of holomorphic functions. The references to the literature on topological concepts are up to date and the terms are in line with modern usage - although it is unsatisfactory that many formal definitions of fundamental topological concepts are scattered throughout the book, sometimes in footnotes and after previous implicit use. This also leads to a lot of repetition: Homology, although used for Abelian integrals p. 504, is formally introduced in a footnote on p. 632; triangulation is partially treated pp. 501-504 and partially in a footnote pp. 596-597; Riemann surfaces are more or less intuitively defined on pp. 383-391, but the underlying concepts of topological space and differentiable manifold appear only in the new appendix p. 551; relative compactness is relegated to a footnote p. 560; covering surfaces and covering transformations appear twice on p. 525 and p. 684, although homotopy of curves and fundamental groups are introduced early on p. 270 (though only for planar domains). Contour integrals are not defined over chains (as e.g. in Ahlfors), and generality of the integral theorems is lost by omitting Artin's winding number. Astonishingly enough Bieberbach's rotation theorem p. 424 is not presented in its "modern" form due to Golusin and Basilewitsch (dating back to 1936).

In the introduction to the new appendix, differential forms are sketchily introduced and no treatment of the general Grassmann algebra is given. The first two sections of part IV contain a lucid presentation of the conformal mapping on the boundary of a domain with a fairly complete account of the concepts of "ends" and "prime-ends" following Caratheodory. Then the conformal automorphisms of Riemann surfaces are treated; Greenberg's famous result of 1960 that every countable group is isomorphic to the group of automorphisms of a suitable Riemann surface is mentioned without proof. In the same section discontinuous groups of conformal transformations are introduced and their study is continued in the next section on "Fuchsian groups". As a preparation for the definition of quasiconformal maps, the concept of modulus of quadrangles and annuli is introduced in section 5. Sections 6 and 7 present at first (p. 614) a topological definition of quasi-conformal maps following a paper by Ahlfors (1953); this is then applied to differentiable maps in order to obtain the original classical definition of Grötzsch (1928): the connection between the two definitions seems not quite clear - at least not to me. Section 7 ("extremal quasiconformal maps") culminates in the definition of the Teichmüller distance between two surfaces of genus one which vanishes if and only if they can be conformally mapped onto each other. The generalisations to higher genus by Ahlfors and Bers - based also on the conjectures or "semi-proofs" of Teichmüller - are mentioned without proofs (on p. 538 the problem of the moduli - though only for "schlichtartig" surfaces - had been taken up).

The second chapter of the appendix is concerned with meromorphic functions on compact Riemann surfaces, with the classical formulae of Riemann-Roch, and with Cauchy's integral formula. A modern and very natural touch is then provided by the consideration of holomorphic vector space bundles and their cross-sections in whose development Röhrl himself - together with Grauert, Behnke, Stein, Remmert etc. - played an outstanding part. As applications of this approach we obtain generalisations of the Mittag-Leffler theorem (theorem of H. Florack), of Weierstrass' product theorem, the realisation of abstract non-compact Riemann surfaces as covering surfaces of the finite complex plane. In the last section we return to automorphic functions (already introduced p. 443) and gain some new insight by means of principal bundles.

Although modern articles in scientific journals, up to about 1962, are liberally quoted, there is a somewhat surprising lack of references to other important textbooks; e.g. uniformisation is discussed cursorily on p. 523, but the standard book of Nevanlinna is not mentioned. Nor does one find references to the well-known textbooks of Ahlfors, Sario, Hille, Lehner, Thron. Pfluger and Behnke-Sommer, however, are quoted several times.

Complex analysis has assumed such huge dimensions that a textbook can never be complete. A personal impression of mine is that the theory of univalent functions - with coefficient estimates along Jenkins' line - and some rudiments of Schiffer's variational methods should have found at least a tiny refuge in such a large survey.

As for the style, the book is written for the greater part in the carefully thought-out way of the fathers and grandfathers of complex analysis; some exceptions occur in the new appendix where modern English corruptions mar the beauty of the German presentation (e.g. "geliftete Kurve" on p. 685!). However I would be at a loss to find a better translation with the proper technical connotation.

The index of the former editions has not always been rewritten conforming to the new arrangement of the text; e.g. on p. 364 one looks in vain for non-Euclidean motions as announced by the index.

Summing up: The new version of Hurwitz-Courant-Röhrl will continue to be a standard textbook of the first class. One cannot escape the impression, however, that the wealth of material seems to have overwhelmed the authors to such an extent that the organisation has suffered sometimes. Naturally, in this age of gentle educational approach, the aforementioned repetitions can also be interpreted as highly sophisticated pedagogical devices enabling the student to digest piecemeal the more complicated aspects of the theory!

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Mathematical Theory of Optics, by R. K. Luneburg. University of California Press, Berkeley, 1964. xxx + 448 pages. $12.50.

The volume under review is a corrected version of a famous set of lectures delivered by the late Rudolf Luneburg at Brown University in 1944. Mimeographed copies were prepared, but had a very limited distribution as they were not generally available to the scientific public. It is no exaggeration to state that these notes have had a profound impact upon the progress of classical optics in spite of their inaccessibility, and for this reason their publication in book form is a major event. Luneburg was that rara avis, the productive scientist who did not publish his results! As a manifestation of this attitude, one finds that the volume has an intensely personal flavor reminiscent of Lord Rayleigh's

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