

that  $f(1/10)$  is transcendental. The text rounds up nicely with a chapter on Lambert's irrationality proofs, which involve a generalisation of simple continued fractions.

There is no shortage of number theory books, and many of them include some of the topics being mentioned here. However, as a book devoted entirely to the subject of irrationality and transcendence at this level, I only know of [1], from which I first learned about such matters, and the more recent [2]. All three books are excellent but, perhaps surprisingly, neither [1] nor [2] is mentioned in the otherwise adequate bibliography.

### References

1. Ivan Niven, *Irrational Numbers*, Carus Mathematical Monographs, Series Number 11, The Mathematical Association of America (1956).
2. Edward B. Burger and Robert Tubbs, *Making Transcendence Transparent: An intuitive approach to classical transcendental number theory*, Springer (2004).

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**Theory of infinite sequences and series** by Ludmila Bourchtein and Andrei Bourchtein, pp 377, £54.99 (paper), ISBN 978-3-03079-430-9, Springer Verlag (2022)

The notion of a limit of sequence is fundamental in analysis and with it the related and perhaps more important notion of a sum of an infinite series. This textbook is divided in five chapters: the first two are devoted to sequences and series of numbers, followed by two chapters on sequences and series of functions and the last, fifth chapter is on power series.

The first chapter on sequences of numbers contains the standard material found in any analysis textbook. Much attention is paid to calculation of limits and to this end the Stolz-Cesaro theorems, which are discrete versions of L'Hôpital's rules, are used for evaluation of indeterminate forms  $\frac{0}{0}$  and  $\frac{\infty}{\infty}$ . There is no mention of the important notion of accumulation points of sequences nor of recurrent sequences like the famous Fibonacci sequence. We encounter only explicitly defined sequences throughout. It is strange that a sequence is denoted simply by  $a_n$ , which is the same as the general term, not using the common notation  $(a_n)$ .

After some introductory examples of convergent and divergent series of numbers, Chapter 2 deals mainly with various tests for convergence of series. First the familiar D'Alembert's test (ratio test) and Cauchy's test (root test) are treated, followed by the integral, comparison and Cauchy condensation tests. As a nice application of the last-named, the authors give the convergence of the  $p$ -series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  for  $p > 1$ , which in most textbooks is treated with the integral test. When the first two tests fail, there are more delicate ones to be tried, like the Raabe and the Jamet test. These are shown to be by-products of a more general Kummer and Cauchy chain of tests. Other topics discussed are absolute and conditional convergence, the Cauchy product and the astonishing Riemann theorem for the existence of rearrangement of conditionally convergent series to any previously chosen value.

In Chapter 3, after a brief discussion of pointwise convergence, the fundamental concept of uniform convergence of sequences of functions is thoroughly discussed. Under uniform convergence the limit function  $f$  inherits the nice properties of the functions  $f_n$  such as boundedness, continuity, differentiability and integrability. For pointwise convergence this is not the case and the authors demonstrate this with plenty of examples and counterexamples. This chapter is preparatory for the next,

which is focused on uniform convergence of series of functions. After explaining the relationship between absolute and uniform convergence of such series, the authors prove Dirichlet's and Abel's tests for uniform convergence, which are analogous to the tests for series from Chapter 2. As in the previous chapters, there are ample examples with some of them used several times in order to present different ways of showing that a series is or isn't uniformly convergent. Sufficient conditions for uniform convergence of series, such as the Weierstrass  $M$ -Test and Dini's theorem for convergence of series of continuous functions on a compact set, are included. Karl Weierstrass (1815-1897) is a towering figure in the theory of series and the authors present in nicely fitted compartments of the book two of his celebrated results: the approximation theorem which states that continuous function on a closed interval can be uniformly approximated by a sequence of polynomials, and his example of an everywhere continuous and nowhere differentiable function.

The final chapter is the largest and is devoted to power series and Taylor series. For all elementary functions the corresponding Taylor series are calculated. Common applications of Taylor series included are numerical approximations, calculations of limits (no L'Hôpital's rule as usual) and solution of ordinary differential equations.

Two main problems are involved when dealing with number series: to determine whether the series converges, and, if it does, to what sum. The main criticism one can point at the authors is that this second aspect of the theory of infinite series is entirely suppressed in favour of the first. We don't find in the book the exact value even of the most celebrated Euler series  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ . This so-called Basel problem, with solution  $\pi^2/6$  found by Euler (1707-1783) in 1734, was a great motivation in the development of mathematics. Throughout the text there are other brief historical remarks which make the reading more enjoyable.

There is no shortage of exercises and problems, proposed at the end of each chapter, with detailed solutions provided as an electronic supplement of the book. Among the exercises there are some classic results but unfortunately they are not named, so the student misses out valuable information. For instance, the convergence of the important sequence  $1 + \frac{1}{2} + \dots + \frac{1}{n} - \ln n$  is relegated to an exercise, without any mention that its limit is the well-known Euler-Mascheroni constant. Nevertheless, university students working through the exercises will gain solid grounding in that part of analysis concerned with convergence of sequences and series. This book is primarily to be recommended to them.

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**A student's guide to Laplace transforms** by Daniel Fleisch, pp. 218, £17.99 (paper), ISBN 978-1-00909-629-4, Cambridge University Press (2022)

This book aims to give students an intuitive understanding of Laplace transforms and their physical meaning, and I think that in this it succeeds very well.

It makes no claims to cover advanced abstract theory, and the level of mathematical sophistication expected of the reader is not high, but within these limits the material is covered with appropriate rigour. The language is informal and accessible but accurate, and it reads well. The emphasis is on practicalities, but complications are not overlooked. The explanations are extremely clear and detailed