THE q-LOG P DISTRIBUTION OF CLASSICAL ALGOLS*

EDWIN BUDDING

Carter Observatory, Wellington, New Zealand

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Abstract. Data on a large sample ($\sim$400) of candidates for the classical Algol-type eclipsing binary configurations have been collected. The mass-ratio q has been calculated on the assumption of their semi-detached nature, and then compared with corresponding period for some subsets of the sample. The tendency to slight overluminosity (compared with corresponding MS stars) of the primaries in classical Algols is confirmed.

The role of the loss of angular momentum in classical Algol evolution is considered, together with a method for determining representative parameters to characterize such effects using the sample data. The possible existence of a class of post-contact classical Algols is noted.

1. Catalogue of Classical Algol Candidate Stars

A list of some 400 candidate objects has been compiled to provide the basis for a catalogue of stars, which have been suspected at some time or other, for more or less well established reasons, as falling into that category of semi-detached binary exemplified by β Per (Algol). One broad purpose of this undertaking is to provide a more extensive range of material which might be useful in providing statistical tests on some theories of binary evolution, rather in the way considered by Giuricin and Mardirossian (1981a), the receipt from whom of published and unpublished work the author is glad to acknowledge.

Also of basic usefulness to the present work has been the catalogue of Brancewicz and Dworak (1980, hereafter referred to as BD) whose uniform procedure, even if carrying through systematic effects associated with uncertainties attaching to some adopted relations, provides a potentially important basis for statistical comparisons.

A good proportion of the candidate systems (311) were previously compiled by the present author in a rather preliminary survey of their properties (Budding, 1981). These systems were taken from the listing of those eclipsing binaries described as EA in the well-known *General Catalogue* of Kukarkin *et al.* (1969), and the designation EA2 was applied to the binaries in question to distinguish them from another group – EA1 – showing 'Algol' type light curves in the traditional sense, but being made up of pairs of essentially unevolved stars. The EA2 designation implies the classical semi-detached evolved star containing configuration as typified by Algol itself.

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Algol data

	Name	Period	M_1	Spectrum	q_{BD}	r_{L_2}	q_{SD}	Δm	r_1	SD status	Remarks
1	TT AND	$2^d.7652$	3.25	A0:+F7:III	0.60	74	0.25	2.5v	0.17	0.7	
			1.8				0.22				
2	TW AND	$4^d.1228$	2.11	A8+K2	0.20	96	0.20	2.3v	0.14	0.9	Well documented case.
3	UU AND	$1^d.4863$	1.84	F5+K5IV:	0.25	·148	0.40	3.0v	0.32	0.7	Lowish mass.
4	WW AND	$23^d.2853$	2.92	A5+(F6IV)	0.30	37	0.05	1.1v	0.08	0.5	Solution errors if *s/d*. Perhaps EA1 solution possible.
			3.2				0.4				
5	XZ AND	$1^d.3573$	3.26	A0+G8:III:	0.69	91	0.51	3.0p	0.30	0.9	
6	BO AND	$5^d.7973$	5.42	B8	0.50	80	0.24	1.0p	0.19	0.7	
7	CO AND	$1^d.8277$	1.45	F8	0.69	86	0.42	1.0p	0.15	0.7	EA1 solution possible? If *s/d* must be one of coolest.
8	CP AND	$3^d.6089$	2.14	A7	0.62	68	0.16	1.5p	0.17	0.7	
9	DW APS	$2^d.3130$	2.93	A0	0.71	86	0.42	1.3p	0.18	0.7	
			1.3				0.23				
10	RY AQR	$1^d.9666$	3.98	A3+G8:IV:	0.30	83	0.18	1.3p	0.20	0.9	Popper's mass seems low; his q seems high.
11	CX AQR	$0^d.5560$	1.47	F2p+(G5)	0.70	106	0.95	1.1p	0.23	0.3	EA1 solution likely.
12	CZ AQR	$0^d.8628$	2.96	A5p	0.50	100	0.52	1.5p	0.30	0.7	Early stage of *s/d* phase?
13	XZ AQL	$2^d.1392$	2.73	A2	0.48	67	0.13	1.3p	0.23	0.5	Perhaps occultation possibility should betried.
14	YZ AQL	$4^d.6723$	3.13	A3	0.82	87	0.50	3.7v	0.15	0.7	Solution errors.
15	FK AQL	$2^d.6509$	6.29	B9+G5III:	0.35	119	0.75	2.4p	0.20	0.7	
			1.48				0.26oc				
16	KO AQL	$2^d.8640$	2.90	A0V+(F8IV)	0.20	63	0.05tr	1.0p	0.20	0.9	Transit solution probably wrong.
17	KP AQL	$3^d.3675$	2.68	A3V	0.87	33	0.02?	0.8p	0.12	0.3	
18	QS AQL	$2^d.5133$	6.82	B5V+(G8IV)	0.17	161	>1	0.15p	0.17	0.3	Solution ambiguous. Eclipse depths shallow and secondary depth ~ one third that of primary!
19	QY AQL	$7^d.2296$	2.69	F0+(K3IV)	0.28	83	0.17	3.2p	0.19	0.9	

Fig. 1. First page of provisional form of catalogue of classical Algol (EA2) binaries.

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The first page of a provisional form of the catalogue is presented as Figure 1. A few remarks may be made about this. The name and period of each binary is straightforward enough, though a good many of the candidates show period variations affecting higher decimal digits of the period than those included. Each entry contains (where possible) the mass of the primary star as given by BD, (to two significant decimal figures). Sometimes additional estimates of this mass are included for comparison. These generally well-known examples will be remarked on subsequently. The spectra can be traced to the sources listed by BD. Their mass ratio q_{BD} is also listed. Later, reasons will be given why it is thought that there has been a tendency to systematically overestimate this quantity (at least for EA2 systems). Also of interest is the quantity tabulated by BD as RL2 – since it provides a clue to the likelihood of a semi-detached binary.

The quantity q_{SD} corresponds to the mass-ratio which would be implied by the quoted relative radius of the subgiant component, were this to be in contact with its surrounding Roche lobe. The value comes from an interpolation on the tabulated data of Kopal (1959) (rather than any use of formulae). Also supplied is the depth in magnitudes of the primary eclipse. This usually comes directly from Kukarkin *et al.* (1969), who supply the suffix *p*, *v*, etc., depending on the wavelength of observation. The key point here is the fact that EA binaries with deep primary (and shallow secondary) minima are very likely to conform to the EA2 type characteristics in other respects (i.e., to be semi-detached). The relative radius of the primary star r_1 is useful since, taken together with the other items, it can allow calculation of an overall picture of the salient physical quantities characterizing the binary system.

The quantity described as SD status is meant to indicate the likelihood of a genuine classical evolved Algol binary based on the information available. It takes the 5 odd values from 0.1 up to 0.9 in steps of 0.2.

The weight 0.9 refers to well-known cases, whose semi-detached state has usually been attested by a number of authorities. It is this group for which additional comparison data is often presented. 0.7 attaches to those binaries which show similar properties to the 0.9 examples, but have received relatively little attention. 0.5 corresponds to candidates which might possibly be EA2 systems, but for which an alternative EA1 or other configuration is about equally likely. 0.3 is given to those candidates for which a semi-detached configuration may just be possible but the supporting evidence seems poor, or indeed an alternative arrangement seems more likely. 0.1 is given to those cases which have somehow appeared as candidates in an initial search, but further examination has made appear very unlikely to be EA2 binaries. An additional column contains a few brief words on possible peculiarities of the system in question. The author would be glad to receive early comments on the style of, or possible improvements which could be made to, this catalogue.

Figure 2 shows the plot of q_{BD} against q_{SD} from which the general excess of q_{BD} values can easily be ascertained. An explanation of this is offered as follows: the

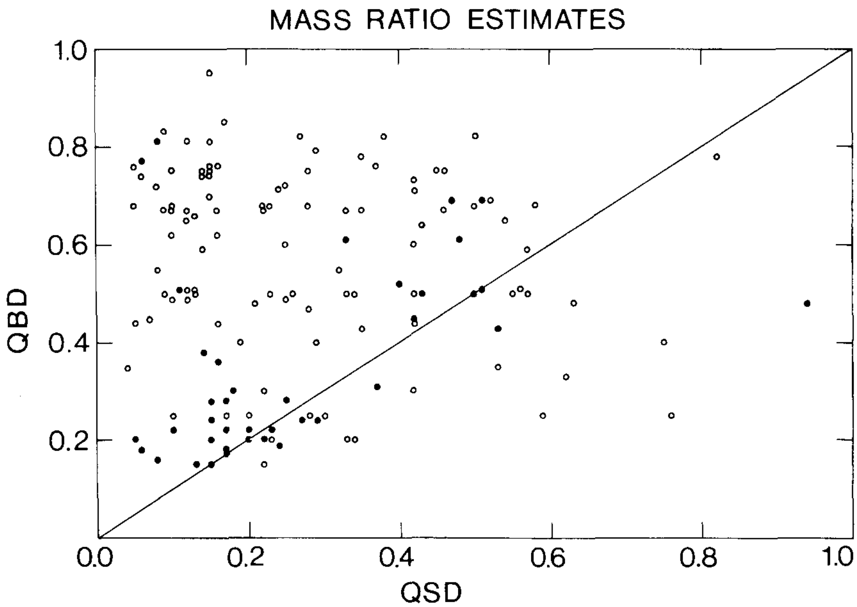

Fig. 2. Plot of q_{BD} against q_{SD} for 147 EA2 candidate binaries with primaries in the mass range $2 < M_{1(BD)} < 4\,M_\odot$. The trend of $q_{BD} > q_{SD}$ is evident even for the s.d. status 0.9 systems (full circles). Open circles denote s.d. status 0.7 systems.

five basic relations put together in BD – essentially the same as those considered, for example, by Hall (1974), and including a mass-luminosity relation which might be applicable, but only for the primary component in the case of EA2 systems – refer to eight separate quantities, some of which can be regarded as known. The final reduced expression, obtained from elimination between these five relations, can be cast in the form

$$\log M_1 = f(\log P, \log r_1, \log T_1, \log(1+q), \alpha_i), \tag{1.1}$$

where M_1 is the primary mass, P the period, r_1 the primary's radius in units of the binary mean separation, T_1 its surface temperature, q the mass ratio, and α_i are some known constants.

The binaries considered all have known primary spectral type from which it is assumed T_1 can be derived. P is, of course, known to high accuracy, while r_1 obtains from the solution of the light curves. We are left with M_1 and q, the former turning out to have a relatively small dependence on the assumed value of the latter. Subgiant components in semi-detached binaries are known to show luminosity excesses over the normal mass: luminosity law which, in some cases may be very considerable (see, e.g., Giannone and Gianuzzi, 1972). BD do not make clear what their procedure would be in the case of semi-detached suspects, though in those well-known cases where secondary masses have been more confidently specified in source material (status 0.9 systems) the general agreement between q_{SD} and q_{BD} is better.

If we concentrate just on the primary, however, and keep in mind the iterative improvement solution method of BD which starts with a q value of 0.75 (quite higher than average for a semi-detached system); the adopted Main-Sequence mass luminosity relation would tend to impart a Main-Sequence character to this primary star. A reasonable form for (1.1) can be shown to be

$$\log(1+q) = 4.76 \log M_1 - 5.90 \log T_1 - 1.95 \log P - 2.95 \log r_1 + 20.24, \tag{1.2}$$

where T_1, P, and r_1 are taken to be known for a particular case. If a value of q satisfying the above equation is perturbed downwards in value, a corresponding reduction of M_1 is clearly entailed – i.e., if M_1 were already in keeping with its derived R_1 and T_1 as a Main-Sequence star – a lower value would imply the star to be somewhat overluminous for its mass. This is indeed what has been found to be the case for certain well studies semi-detached binaries (Hall, 1974). The implication is that a more generally self consistent solution is found when the q-values are reduced from those of BD to those of the trend of q_{SD}, on the basis that the primaries in these systems tend, possibly as a result of processes connected with the binary evolution, to be rather over-luminous for their mass, compared with normal Main Sequence stars.

2. Existence of Trends in the q-log P Diagram

In pursuing the aforementioned aim of providing material to test binary evolution theory the distribution of the systems in the $q : \log P$ plane was examined. These two quantities can be readily related to various schemes of binary evolution, and, for genuine EA2 stars with known light curve solutions (i.e., known r_2) q_{SD} $(= q(r_2))$, as well as P, is directly accessible. This distribution has been considered with such purposes in mind by such authors as Svechnikov (1969), Ziółkowski (1976), and Giuricin and Mardirossian (1981a).

At first data was taken from the compilation of Giuricin and Mardirossian (1981a) for those 0.9 status systems for which a primary mass of less than $6\,M_{\odot}$ had been calculated. The distribution of 76 such relatively low-mass classical Algol candidates in the $q : \log P$ plane is shown in Figure 3. A negative correlation of q with $\log P$ can be eye-judged from this diagram ($r = -0.45$), and though the scatter in the plane is quite wide it can be observed that there are relatively few points with very small q and period, or relatively large q ($\gtrsim 0.5$) and large period.

The existence of this general trend is in broad agreement with the Roche-lobe overflow (RLOF) theories of binary evolution, and even on fairly general grounds we might expect that if the contact component does tend to lose mass then lower mass ratios will tend to go with longer orbital periods. Further insight into Figure 3 may thus be afforded by considering the form of how q would vary with $\log P$ according to some of the discussed evolutionary schemes.

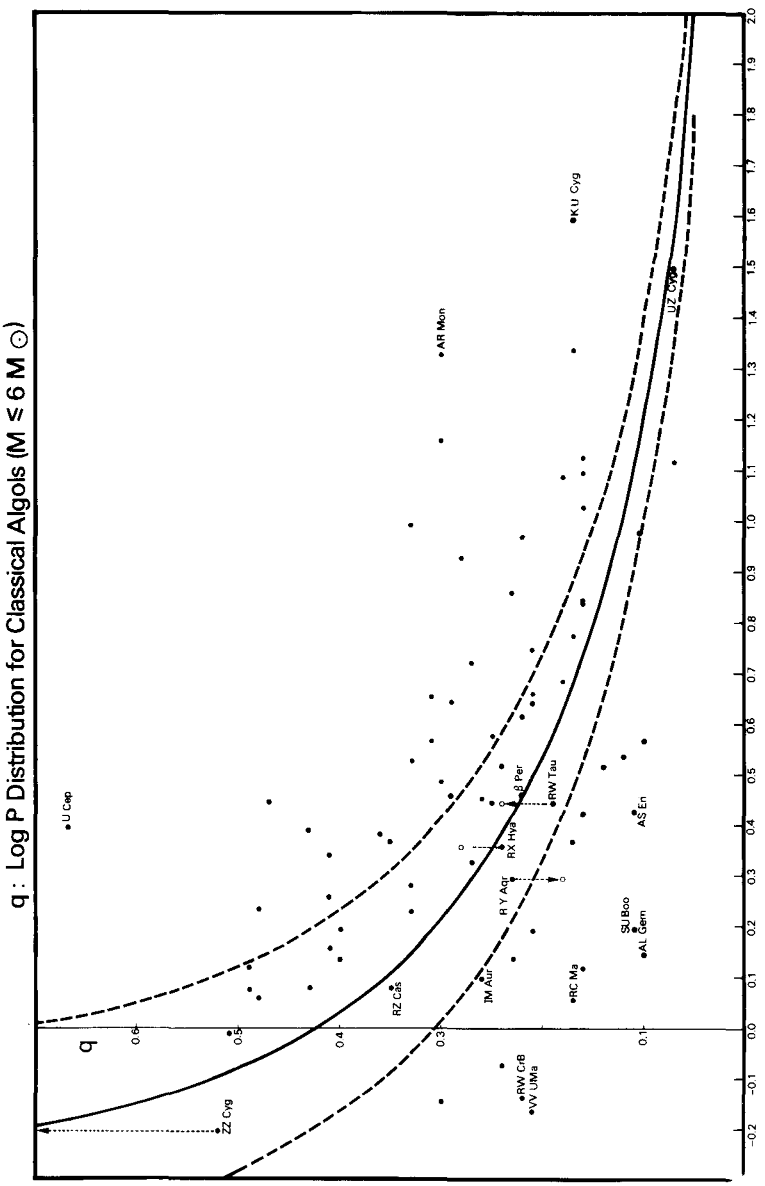

Fig. 3. Distribution of a selection of low mass EA2 systems in the q: log P plane, taken from the compilation of Giuricin and Mardirossian (1981a).

Perhaps the simplest case to consider is the 'conservative' trend (which corresponds to the curves actually shown in Figure 3) which is given by

$$\log P = \log P_0 + 6 \log(1+q) - 3 \log q - \log 64, \tag{2.1}$$

where P_0 corresponds to the minimum period, obtaining when $q = 1$. The curve shown as a continuous line corresponds to $P_0 = 0.587$ days, which might be appropriate for a conservative RLOF process for a pair of stars of total (constant) mass M about $5\,M_{\odot}$, which would just be in contact as an equal mass pair. P_0 (contact) is actually specified by

$$P_0\,(\text{contact}) = 0.082\alpha^{3/2}\,(M/2)^{(3n-1)/2}, \tag{2.2}$$

where n is a mass : radius relationship index, which might be expected to be ~ 0.7 for the stars in question; while α expresses the ratio of separation of the centres of equal mass to the nominal ('volumetric') radii of the corresponding critical lobe surfaces (typically, $\alpha \sim 2.6$).

From (2.2) we may deduce that any appropriate abscissal shifts of the continuous curve shown in Figure 3, in dependence on the expected range of total mass of the 76 systems should be relatively slight (or order 0.1 either way). Hence, while the majority ($\sim\frac{2}{3}$) of the systems considered lying to the right of the continuous curve would not, on this evidence, be in conflict with a conservative RLOF scheme, a substantial number of systems exist which, if projected backwards along a conservative path in the $q : \log P$ plane, would, at some point, no longer be able to satisfy the initial premise of being a semi-detached system. This result is not new. The likelihood of non-conservative evolutionary trends in the properties of classical Algol systems has been considered by a number of authors, at least since the work of Paczyński and Ziółkowski (1967), and the compilation of Svechnikov (1969), if not before. However, the accumulation of systems on the right of the conservative limit in Figure 3 is notable. The distribution of these EA2 systems over a range of possible values of $\log P_0$ is shown as a histogram in Figure 4. In what follows, features and possible interpretations of such diagrams as Figures 3 and 4 will be considered.

3. Possible Interpretations and Checks

Apart from the majority of systems clustering somewhat above the continuous curve in Figure 3, there exists a group of systems rather more spread out, below and somewhat to the left of the curve. Some care may be required to establish facts in the case of such systems. For example, KO Aql would have appeared among this group if earlier solutions for q had been adhered to. Such solutions were calculated on the premise that the primary eclipse is caused by a transit of the smaller star across the disk of the larger one. It was shown, essentially already by Russell (1912), that occultation primary solutions producing a quite similar form of light curve can generally be found as an alternative to transit primary solutions (though

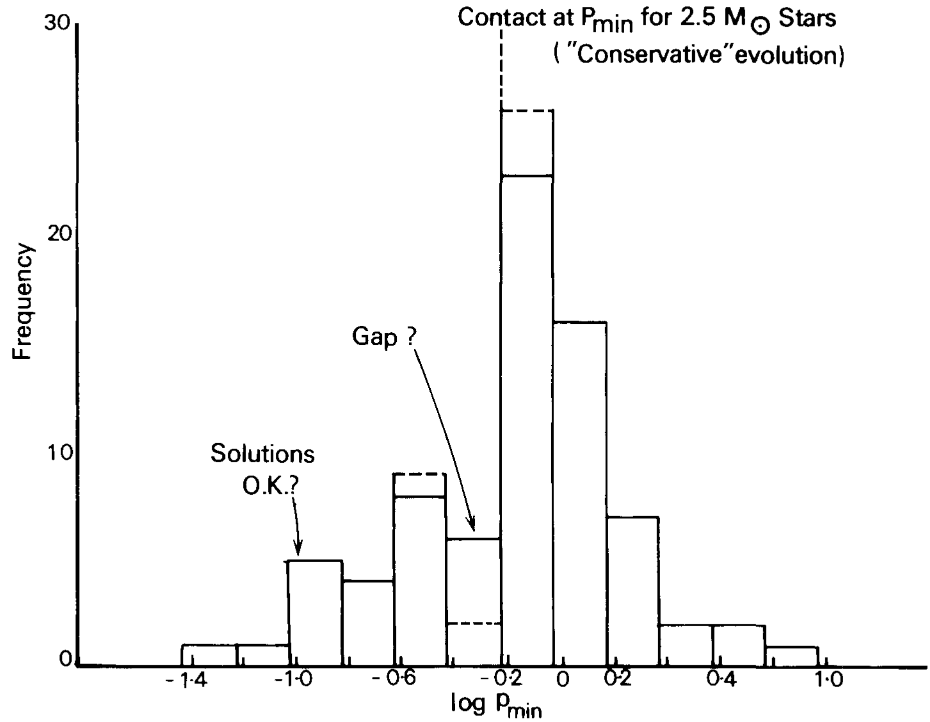

Fig. 4. Numbers of Algols falling into successive intervals of 0.2 in $\log P_{\min}$ (days) on the assumption of 'conservative' evolution (Equation (2.1)). A sizeable number of systems (to the left of the contact limit) violate an initial premise of the conservative RLOF mechanism since their Roche lobes, at minimum period, are not large enough to contain both stars. Some of these systems may result from faulty photometric solutions producing too low q-values. On the other hand, the suggestion of a 'gap' may indicate a qualitative difference in the evolution progress for such systems.

the reverse is not always the case). The more recent discussions of KO Aql (see, e.g., Blanco and Cristaldi, 1974; Giuricin and Mardirossian, 1981b) have shown, in a reasonable way, that the occultation alternative provides a much more satisfactory explanation of the evidence. Perhaps such an explanation might fit some members of the low q, $\log P$ group – transit primary hypotheses account for some 46 % of the solutions in this group, though only 30 % of the entire set are solved in this way. An occultation alternative for a given model, since it raises the secondary relative radius, will clearly increase the corresponding value of q, perhaps to allow a point to move into the majority domain.

This point can be carried a little further by considering what radii of primaries might be reasonably expected for the mass of systems involved, and following through the implications on the relative dimensions of components. In this way, it would be very difficult to expect any real EA2 system, of any normally encountered mass ratio (i.e., $q \lesssim 0.6$), of total mass several $M_{\odot}$ with a period less than one day, for example, to have a secondary star actually larger than the primary. Similarly, cases like

AS Eri or RV Oph, for which occultation primary solutions have already been advanced, presumably cannot allow larger q values, despite their relatively short periods. Systems which might, perhaps, be profitably re-examined to check the assumed eclipse type, however, include SU Boo, XX Cep, IM Aur, UX Her, V338 Her, and AL Gem.

Prominent among those stars well to the lower right of Figure 3 is the well-known system R CMa, once regarded as the prototype for a subgroup of peculiar overluminous Algol systems (Kopal, 1959). The particular problems posed by the original members of the 'R CMa group' appear to have been largely disposed of with careful reobservation and analysis (Sahade and Ringuelet, 1970; Okazaki, 1977; Hall and Neff, 1979). In the case of R CMa itself, its overluminosity problem was made less acute by reducing the adopted mass ratio; however, let us note here that this operation would accentuate its evolution problem in the sense previously mentioned. The careful consideration which has been given to R CMa, as well as some other members of the low $q:\log P$ group, such as AS Eri (cf. Popper, 1973; Refsdal *et al.*, 1974), do allow us to have confidence that at least some of these points really do lie below the main congregation.

Keeping in mind the matter of confident positioning of points in the $q:\log P$ plane, Figure 4, nevertheless, directs attention to the possibility of some definite gap between the main aggregation and the low $q:\log P$ group. This possibility is enhanced by the fact that of the 6 binaries in the range $0.372 < P_0 < 0.587$, 3 (RY Aqr, RX Hya, and RW Tau) are re-positioned outside this region by the q-values of the catalogue, while the case of ZZ Cyg as a *bona fide* EA2 system is not so well established (Hall and Cannon, 1974). The possible existence of a gap in the $q:\log P$ distribution at the minimum period-contact limit, raises intriguing notions about the possible role of a common envelope in providing angular momentum loss in binary evolution. Derivation of specific information on possible mass or angular momentum loss was perhaps the immediate stimulus to the compilation of the catalogue, so that further testing of any particular features of the distribution might be checked from a larger data-base of systems confined within a smaller range of masses.

Let us also note in this context, however, the third body in the R CMa-system (cf. Radhakrishnan *et al.*, 1983). Total angular momentum, in the evolution of some Algol systems, might not be so much lost as redistributed, if there happened to be some third orbit in which it could be deposited.

The distribution of 147 points corresponding to BD primary masses in the range 2 to 4 $M_{\odot}$ is shown in Figure 5, together with some possible interactive evolution tracks generalized to mass and angular momentum loss by the f and g parameters, which have become widely referred to (see, e.g., Paczyński and Ziółkowski, 1967; Thomas, 1977), and corresponding histograms to Figure 4, can be drawn up for a range of values of f and g. Some examples are shown in Figure 6.

There appears to have been some confusion in the literature over the precise meaning of f and g. Here we shall adopt f to mean the fraction of matter lost by the loser which is subsequently lost entirely from the system, (with the implication that

$1-f$ is the fraction transferred to the gainer). The orbital angular momentum of the system J $(= (PG^2/2\pi M)^{1/3} M_1 M_2)$ is taken to decrease with a power law dependence on the total mass $(J/J_0) = (M/M_0)^g$. The formulae (5) and (6) of Giuricin and Mardirossian (1981a), attributed to Vanbeveren *et al.* (1979), are then straightforward to derive, and are general enough to encompass a range of possibilities.

In what follows, it will be convenient to write p in place of $\log P$. An interpretation may be put on the observed frequencies h_i in histograms like those of Figure 6, by referring to the observed density of points in the p, q-plane, $\sigma \equiv \sigma(p, q; \chi(p_0), f_1, g_1, \alpha_j)$, where p and q are regarded as the basic independent variables. $\chi(p_0)$ represents a range of initial condition, for given mass, in terms of a distribution of the logarithm of the minimum period p_0, and f and g are the parameters already referred to. The additional quantities α_j are meant to refer to other matters influencing the observed density distribution, or possible selection effects; we shall neglect the role of such quantities in the vicinity of the highest density of points. An estimate of the particular values f_1 and g_1 which best characterize the observed distribution might be empirically arrived at in the following way.

Curves $\sigma \equiv$ const. represent contours of constant density in the p, q-plane. One such curve $\sigma = \sigma_{max}$, say, refers to an elongated region of minimum diameter $\sim \Delta p_0$, wherein the point density attains a maximum. Now the curve ϕ_{ev} $(p, q; f, g) =$ const. (p_0)

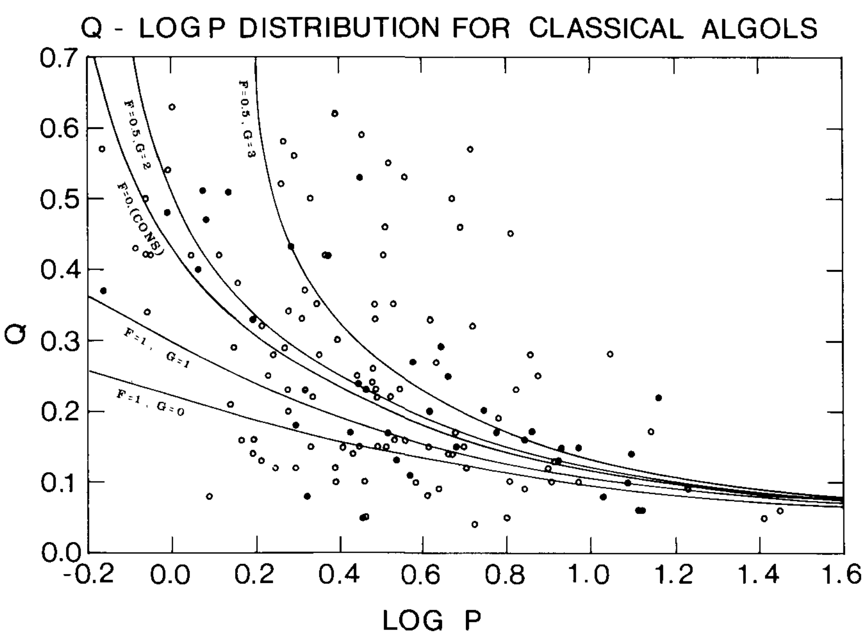

Fig. 5. Similar to Figure 3, but with the data corresponding to the 147 system from the catalogue, referred to also in Figure 2.

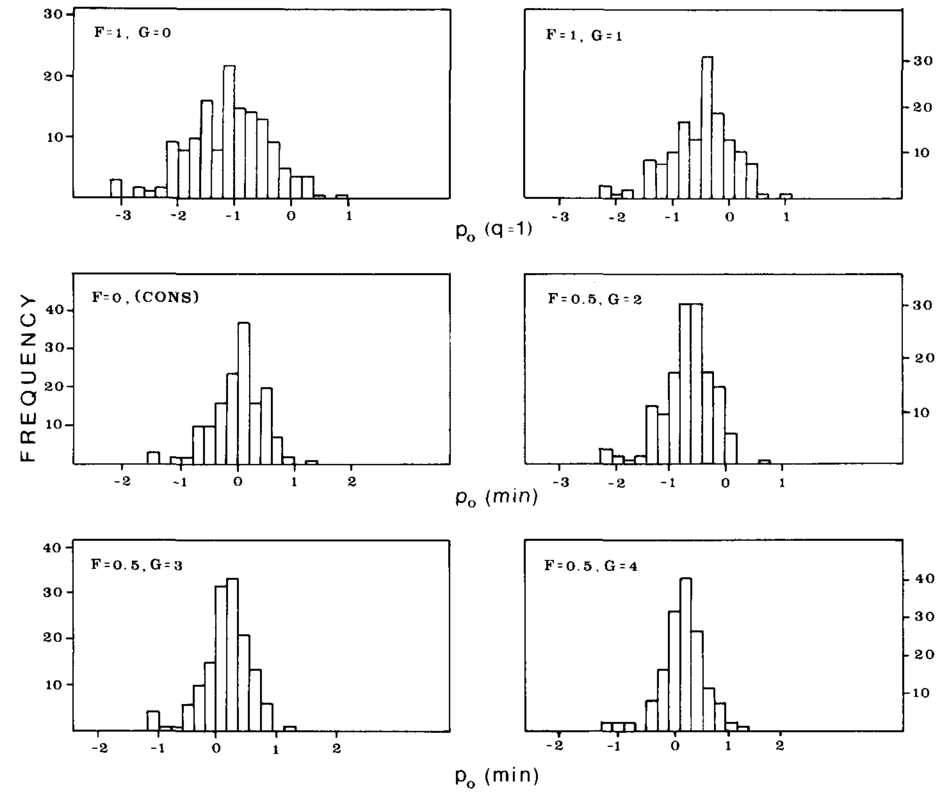

Fig. 6. Similar to Figure 4, but with the inclusion of non-conservative evolution trends such as those shown in Figure 5. The low g histograms clearly involve unacceptably low initial periods for the protomorph systems, as well as indicating non-parallelism between the densest region of the $p-q$ plane and the evolution tracks. The matching ameliorates with higher g and lower f. Thus the $f = 0.5$, $g = 4$ combination clearly aligns well with the density maximum as well as allowing the great majority of evolution tracks to be free from the 'over-contact' problem of Figure 4. With all considered combinations of f and g, however, there is a persistent knot of low p, q-systems whose explanation must remain puzzling.

represents a trial evolution track dependent on the parameters f and g, and specified for given p_0. In the most straightforward approach, we attribute the observed density distribution, for given mass, in the p, q-plane, only to different values of p_0 as expressed by the function $\chi(p_0)$. The existence of some maximum density region $\sigma = \sigma_{\max}$ would imply, neglecting any observational selection effects, some largest value in the frequency $\chi(p_0)$ of EA2 binaries, at, say $P_{0\max}$. The number of points $h_i(f, g)$ between $\phi_{\mathrm{ev}} = c(p_0 - (\Delta p_0)/2)$ and $\phi_{\mathrm{ev}} = c(p_0 + (\Delta p_0)/2)$, will be maximal when $\phi_{\mathrm{ev}} = c(p_0)$ coincides with the major axis of the elongated region $\sigma = \sigma_{\max}$. This condition occurs when $f = f_1, g = g_1$ (and $p_0 = p_{0\max}$); i.e., maximizing the histogram maximum $h_{i\max}$ in terms of f and g is equivalent to determining their best representative values, and will also lead to an estimate for $p_{0\max}$ (which might be compared to the 'contact' p_0, for example).

The conservative case considered previously proves to be something of a discriminant in this context. Thus $\log P$ specified by (2.1), $\equiv p_c$ say, we can now compare with the more generalized formula of (for example) Vanbeveren *et al.* (1979). We will then have

$$p - p_c = (3g-5)\log \frac{1+q}{1+q(1-f)}.$$

If f is ~ 0 the difference in ϕ curves is not too sensitive to g. For $f > 0$, if $g > \frac{5}{3}$, ϕ_{ev} will lie above $\phi_{\mathrm{ev}}(\mathrm{cons}) = c$, and below if $g < \frac{5}{3}$. A tendency of the data to congregate roughly parallel to the $\phi_{\mathrm{ev}}(\mathrm{cons})$ curve might be interpreted as evidencing relatively little systemic mass loss in general, however the two parameters may not be so clearly resolved in this way – i.e. the effect of higher f may be simulated, to some extent, by higher g.

In view of this, the numerous uncertainties surrounding this rather simplistic approach and inherent noise in the histogram distributions which is not negligible, it would be dangerous to place too much weight on any of its results so far; though indications are that f is not likely to be so far from zero for many EA2 systems, while $g > \frac{5}{3}$, and indeed a value of g significantly larger than $\frac{5}{3}$ may be considered appropriate*. A value for $f \sim 1$ with $g < \frac{5}{3}$ leads to unacceptably low p_0 for a large faction of the binaries. This is in general agreement with the conclusion of other authors, such as Giuricin and Mardirossian (1081a), and Popov (1970), on low-mass Algols.

A few more general remarks may be made about the enlarged sample distribution shown in Figure 5. Again there are a number of cases of possible incorrect eclipse type assumption, and, even if the basic type of eclipse is right, the accuracy of the solutions for r_2 may be doubtful, particularly in the case of the status 0.7 systems. Also, the gap of Figures 3 and 4 seems to have disappeared on Figures 5 and 6. It is still, perhaps, too preliminary to rule out a possibly qualitative change in the evolution pattern which may occur with over-contact at p_0, though we have found no clear corroboration of this possibility by taking into account a larger number of classical Algols whose primaries fall into the mass range $3 \pm 1\ M_{\odot}$.

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* A 'good' pair of values, in the previously considered optimization sense, was found to be $f = 0.5$, $g = 4$ (see Figure 6).

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