EDUCATION

REFORMING THE HIGH SCHOOL MATHEMATICS CURRICULUM

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The editor has asked me to describe recent developments in Ontario resulting from the initiative of the Ontario Teachers' Federation in setting up a Mathematics Commission. But he went farther, he gave me carte blanche to use as much space as I wanted to expatiate on all my pet ideas!

It is often claimed, and seems to be widely admitted, that, in the past, the educational system in Ontario was one of the soundest of large public systems in North America. As far as mathematics is concerned there is some objective evidence that this is still the case. Every student in the "general" course in our high schools takes one mathematics course (usually five periods per week) in Grades 9, 10, 11 and 12. In Grade 13, nearly everyone takes at least one course and a very large percentage take three courses. It is normal in Ontario for a freshman entering engineering or an honours course in mathematics or physics to have seven high school mathematics courses behind him. This contrasts vividly with the desperate statistics emanating from the U.S.A. in recent years which suggest that it is unusual for the American high school student to take three courses and unheard of for him to take four, unless he is attending an outstanding private school. Last year, in the MAA contest for high school students, the median mark obtained by Ontario students was twice that for the students of New York State excluding Metropolitan New York.

However, there are disquietening signs:

(i) In the MAA contest we were completely outclassed by Metropolitan New York and New Jersey. The team marks for the top ten percent of their schools ranged from 206 to 400, (450 was the maximum possible) whereas ours ranged from 127 to 242.

(ii) Whereas twenty or thirty years ago there were many able mathematicians teaching high school the proportion and quality of teachers with type A specialist certificate in mathematics seems to be dropping steadily.

(iii) It is generally admitted that good students in many other countries (France and Scotland may be chosen as non-political examples) are two years ahead of Ontario students of

comparable ability at age 17.

(iv) Many of the mathematically-gifted students, such as those in the mathematics, physics and chemistry honours courses at Toronto, complain that the high school course was dull and uninteresting, and that it lacked any real challenge for them. It has been my observation that for too many this has resulted in very poor work habits.

(v) The majority of teachers responsible for initiating students into algebra and geometry in Grades 9 and 10 have specialist qualification in English or physical education. They choose mathematics as a second subject because it is "easy" to teach! Many of these teachers are uninspired and uninspiring slaves to a text-book.

(vi) The text-books which enjoy greatest popularity for their "teachability" are divided into inconsequential snippets. One successful group of authors considered but rejected the idea of including a university professor in their group since such an association might stigmatize them as "high-brow" and adversely affect their sales!

(vii) The terrifying prospect of the Last Judgment in the form of the provincial examinations at the end of Grade 13 hangs over the whole school system with stultifying power. The main criterion of "success" of a school is the performance of its students at Grade 13. Probably, Hilda Neatby would consider this to be a better criterion than the performance of the school football team; possibly so. However the examiner (who is usually a university professor) "dare not be too original because he might confuse the pupils who have not been prepared for the unexpected" (F. MacKinnon, The Politics of Education). A question which tests anything other than rote learning is met with howls of anguish from teachers, students and parents.

(viii) The scope of mathematical research has been increasing exponentially over many years. The ideas in most of our mathematics courses in high schools have changed since 1900 only by being watered down. Perhaps it is time to rethink the whole content of the high school curriculum. There have been a number of important and interesting efforts along these lines in the U.S.A. in the last few years, notably the work of the Commission on Mathematics of the College Entrance Examination Board (CEEB), of the School Mathematics Study Group centred in Yale University, and the experiment at Ball State Teachers' College, Indiana.

The Ontario Mathematics Commission. It was against this background that, on the insistence of Prof. D.T. Faught of Assumption University, Windsor, and Mr. Howard Mulligan, a teacher of Central Technical School in Toronto, the Ontario Teachers' Federation called together a group, in February 1959, to discuss the question of whether or not some changes in the mathematics curriculum in Ontario were desirable and, if so, to take steps to effect them. The core of the group consisted

of high school teachers chosen by the OTF and university mathematicians, representing all but two of the Ontario universities, appointed by their mathematics departments. There was unanimity that changes were desirable. A proposal for the revision of algebra from Grades 9 through 12 was produced by a sub-committee. This served as the basic document for discussion at a meeting of the full group held at Lakefield Preparatory School in August, 1959. There, thirty university professors and high school teachers spent six days exchanging opinions and finally produced a preliminary outline for a total reorganization of the high school mathematics curriculum. Everyone seemed to enjoy the experience of working together. We were all astonished at the remarkable measure of unanimity achieved.

The main result accomplished is summarized in the following extract from the minutes of the Lakefield meeting.

"First Draft, August 1959, for Revision of Mathematics Curriculum in the Secondary Schools of Ontario.

Preamble. The proposed curriculum is an attempt to realize these convictions which were held, almost unanimously, by the members of the Lakefield Workshop:

(1) The present Ontario curriculum is essentially sound so that its central core should be incorporated in any new curriculum.

(2) The material of the present curriculum could be more easily understood and assimilated if the rigid barriers between algebra, geometry and trigonometry were broken down.

(3) The present material in Grades 9 and 10 is not very challenging or stimulating to the average student, and certainly not at all for the gifted student. We do not propose any radical change in content for these grades, but we do believe that the material can be made mathematically more significant by a change of approach along the lines suggested in the Report of the Commission of the CEEB. In particular much greater care should be taken, than is at present the case, to explicitly set forth the set in which various relations are valid.

(4) The present examinations in Grade 13 are unsatisfactory because they try to serve two contradictory purposes (a) the provision of a necessary credit for students with no intention of pursuing mathematics after they leave high school; (b) the testing of an important minority of students as to their fitness to pursue advanced studies in engineering, the physical or mathematical sciences. The result is parrot-like regurgita-

tion by the poorer students and the dulling of interest and enthusiasm in the mathematically gifted.

To implement these ideas we propose (a) that coordinate geometry be gradually introduced with algebra from Grade 9; (b) that less emphasis be placed on the solution of triangles by logarithms and on the detailed discussion of the properties of the ellipse, parabola and hyperbola; (c) that in Grade 11 geometry, considerably fewer results should appear as stated theorems, proofs of which are to be memorized, and that several theorems should be proved both synthetically and analytically, in order to bring out the particular values of the two approaches; (d) that in Grade 13 there should be one course, Mathematics A, designed for students for whom this would be a terminal course and in which the examination would not probe too deeply, and two additional courses, Mathematics B and Mathematics C, designed for future mathematical practitioners in which the examiners would test for mathematical power and maturity.

Mathematics 9

1. Operations with simple algebraic expressions (9,I).

2. Positive and negative rational numbers (9,II).

3. Linear equations and inequalities in one variable (9,III).

4. Polynomial expressions and factoring (9,VI).

5. Rational algebraic expressions (9,VII).

6. Informal geometry including simple areas (10,I).

Remarks. The numbers in brackets refer to pages 36-47 of the Report of the Commission of the CEEB.

The above differs very little from the current syllabus. However, it is intended that the approach to this material be brought up-to-date by emphasis on the structure of the number system, by the introduction of problems which draw attention to the importance of the domain of definition of the variables, and the use of simple set notation.

The present commercial arithmetic should continue to be available as an option, but for the general student the material of the present arithmetic syllabus should be integrated as problems during the course of the above development and not treated separately.

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It is the belief of the Commission that the proposed new approach to algebra will be more meaningful and therefore easier for all students.

Here, as in higher grades, the approach to geometry should emphasize the intuitive elements, abstraction as simplification, the game aspect, its applicability to practical problems, an aesthetic aspect, and logical pattern. It is most desirable that before Grade 9 the student should be familiar with the ruler, compasses and protractor; methods of simple construction; terminology; some simple properties of geometric figures. Informal discussion of similarity should start in Grade 9 if not before.

Mathematics 10

1. Descriptive statistics (including the ideas of median, mean, and quartile) (9, X).

2. Deductive reasoning (9, VIII; 10, II).

3. Synthetic geometry through Pythagoras' theorem; results on similar triangles needed for proof of equation of a straight line; simple results in three-dimensional geometry which can be stated in terms of parallel and orthogonal planes and the perpendicular line to a plane (10, III, VI).

4. Two dimensional coordinate geometry (10, IV).

5. Linear equations and inequalities in two variables (9, V).

6. Variation (9, IV).

Remarks. It must be recalled that many students leave high school at the end of Grade 10. The new topic (1) would possibly be of more value to such students than any other item proposed.

It must be admitted that Euclidean geometry is too complex to be exactly presented as a deductive system in high school. Rather than attempting to prove the obvious (an enterprise which commands the interest only of students of great mathematical maturity) emphasis should be laid on the development of significant self-contained sequences of results, which provide interesting examples of deductive reasoning achieving striking non-obvious conclusions, and which begin with assumptions intuitively acceptable to the students.

Mathematics 11

1. The number system once more, to real numbers defined as infinite decimals. Practice in arithmetic, using root tables. Radicals. Solution of quadratic equations.

2. A selection of the more important of the theorems of the present Grade 11 geometry, including the circle and similar triangles, treated synthetically.

3. Coordinate geometry used to give alternative solutions to some of the results obtained in Mathematics 10 and in (2).

4. The volume of a sphere, cone, prism; construction of models of regular polyhedra; great circles on spheres.

5. Trigonometrical functions defined for general angle. Their evaluation with the aid of tables. Solution of right-angled triangles with applications.

Remarks. Perhaps it would be possible to treat coordinate geometry as a deductive system once the real numbers have been invented or granted. However, it is probably wisest to defer discussion of deductive systems until Grade 13 or beyond.

Mathematics 12

1. Indices. Four-place logarithms.

2. Sine and cosine laws. Solution of general triangle. Simple trigonometrical equations.

3. Complex numbers.

4. The quadratic function.

5. Simple properties of conics deduced from graphs of their equations.

6. Compound relations,

7. Linear equations in three variables.

8. Mathematical induction applied to finite A.P. and G.P.

9. Two-dimensional vectors: associative and commutative laws for addition, multiplication by scalars, resolution.

Remarks. The possibilities which arise in the solution of a system of linear equations are most easily characterized by the ranks of the matrix and the augmented matrix of the

system. With the help of second order determinants these ideas could be easily introduced in Grade 10. If this were done they could be extended to the third order case here.

If there proves to be too much material in the above course, items 8 and 9 could be omitted."

It was clear at Lakefield that, if any significant change is to be effected, new text-books would have to be produced - three for Grade 13 and four for the lower grades. Lakefield had engendered such a sense of confidence that the moment had come for effective action that it proved quite easy to recruit a team of six high school teachers and six university professors who are now well launched on the project of writing the necessary texts. It is proposed that each text should appear in the first instance in a tentative form and be subject to one or two years trial. The book for Grade 9 will be ready in August for experimental use this fall. An attempt will be made to get thirty or forty teachers, approved by the Department of Education, to undertake to use the new text and to make regular critical reports on its success. These teachers will be expected to attend a course during the summer in which the text will be studied and will probably participate in briefing and reporting sessions during the school year. From the experience of these teachers, which will be carefully followed by the Inspectors, the Department of Education and the Mathematics Commission will be able to evaluate the appropriateness of the material discussed in the text for groups of students of varying ability.

The first three chapters of the Grade 9 text, which I have seen in draft, strike a most satisfactory balance between practice in manipulative skills and presentation of mathematical ideas. The authors, well-known and experienced teachers, are aware that, for this age-level, pages of solid text are worse than useless, so they introduce many and interesting problems, and, at the same time, set forth ideas in simple but precise language. It is their hope that the pupils will actually read the text!

In addition to progress with the Grade 9 text, work is going forward at other levels so that by the first of August fairly extensive Outlines, of thirty to forty pages each, should be ready for Grades 10, 11 and 12. It was apparent soon after the Lakefield meeting that the description of the proposed curriculum given in the above extract from the Minutes was much too brief to convey any clear impression as to what is really intended. On the basis of these more detailed Outlines and the first draft of the Grade 9 text, it will be possible for the Commission to review the whole proposal at a meeting

planned for August 20 - 22, 1960.

If we move ahead according to our rather ambitious schedule, a new course would be introduced in Grade 9, throughout the province in 1962 and in successive grades annually thereafter. Since much of what is currently done in Grade 13 would come down into the lower years there will need to be a radical change in the mathematics content for this grade. For administrative convenience, it has been proposed that three courses should continue to be offered in Grade 13. The nature and content of these courses is still under debate. One suggestion, which I personally support, is that one course, which would be required of anyone who takes mathematics in Grade 13, would seek to get across to students an understanding of the nature of mathematics and reasons for its significance in the modern world. This would be a terminal course for students who do not intend to pursue mathematics further. At the same time it would open up to the intending mathematical practitioner some of the logical and cultural roots of his subject. The two other courses would be more traditional, and would provide substantial preparation for the student intending to pursue mathematics, physics or engineering. One course might treat vectors, three-dimensional geometry, the general second degree equation, and the other, elementary functions, analytical trigonometry and, possibly, introductory calculus. The debate as to whether or not calculus should be taught in high school will permit the release of enormous quantities of irrational prejudice! Since I taught calculus to myself at the age of fourteen and it is not uncommon in Europe to teach it to fifteen or sixteen year olds, my own inclination is to try it in high school here when an adequate textbook is available.

Problems and Dangers. It seems to me that the phrase "modern mathematics" has caused unnecessary confusion and alarm in much of the current debate. It has become a slogan with which to impress "educators". To assert in angry tones that nothing is being taught in our high schools which was not known in 1750, could easily lead to the conclusion that we should not teach anything which was discovered before 1900. Retarded Grade 9 students should be immediately set to work on Russell and Whitehead's Principia Mathematica! Many teachers, fearing that the proponents of "modern mathematics" are advocating such absurdities (and some of them do seem to advocate absurdities!), react purely negatively in defense of the status quo. A young mathematics teacher recently sought the advice of an inspector in Northern Ontario as to how he could acquaint himself with the new ideas in mathematics. He was told not to worry since no significant changes in the curriculum in Ontario were desirable or, even if desirable, likely to be introduced!

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However, the proponents of change, even if they talk about "modern mathematics" may have ideas which are worth considering.

It has been the experience of those of us actively involved in the Ontario Mathematics Commission that the teachers and professors have much to learn from each other and are quite willing to do so. This realization is the most important step forward which has been achieved in Ontario, since hitherto there has been a great gulf fixed between the university and high school departments of mathematics. Whatever the reasons for this gulf, it is clearly in the interest of the country and of university departments of mathematics that the mathematical preparation of incoming university students be greatly improved. The key to this lies with (a) teacher training, (b) curriculum and (c) text-books.

University professors must take greater responsibility at each of these points than they have hitherto. In doing so, they will be both welcomed and opposed. They will be welcomed because the majority of teachers respect the university and are eager that their students be given the best education possible. They will be opposed because it is a basic tendency of the natural man to want to be left undisturbed, especially if he is the author or publisher of a text-book which is bringing in a regular, even if modest, profit.

With respect to teacher-training, the universities should be offering in summer and regular sessions more courses which bear more immediately upon the content of the high school curriculum than does the staple calculus sequence. Most honours course programmes are designed to produce research mathematicians of the calibre of C.F. Gauss. In fact few Gausses have been produced in Canada recently, and a rather small percentage of honours course graduates end up as research mathematicians at all. We need to think seriously about the proper training for a high school teacher and ensure that such training is available. If the changes proposed by the Ontario Mathematics Commission are to be brought into effect in Ontario within our lifetime, much time and effort on the part of university professors will be needed in summer and extension courses.

Changing curriculum is necessarily a slow and uncertain business. It is usually impossible to find any convincing objective reasons why a particular topic should or should not be included. Such decisions should be made only by persons with mature competence in the particular field concerned. In mathematics, this is especially important at a few critical points even at an elementary level. Even if you do not use the words, you cannot proceed very far in mathematics without such ideas as set, number, numeral, domain of definition of a

variable or a function, symbol. Is it important that when these ideas first appear they should do so in a form which will be consistent with the use to be made of them at the most advanced levels? Often for the innocent pupil a correct definition is as easy to grasp as is the insufficient or imprecise definition to which the teacher is wedded by years of familiarity.

Since, in many schools, the initiation of students into algebra and geometry is, for the most part, in the hands of specialists in physical education or English, the text-book is of decisive importance. The majority of teachers seem to be slaves to the text, and for most students in Grades 9, 10 and 11 the text-book is the only source of mathematical enlightenment available. My ideal text-book would be easily read by the student, mathematically accurate, proceed by easy stages so that even the dullest could follow it, yet contain stimulating enrichment material for the good student, and be foolproof in the sense that even a bad teacher could not go too far astray if he used it.

Further Reading. In order to capitalize on the current fad, every publisher has to have a book on "modern mathematics"; so there is ample choice. The following material contains more good ideas than we are likely to incorporate into the Ontario school system for many years. Naturally, I do not find all of it to be relevant or perfect.

1. Report and Appendices of the Mathematics Commission of the CEEB, obtainable for $2.00 from the Education Testing Service, Box 592, Princeton, N.J.

2. Publications of the School Mathematics Study Group, Drawer 2502A Yale Station, New Haven, Conn. Ask to be put on their mailing list for a free news letter which describes the material they are gradually producing for Grades 7 to 13.

3. Modern Mathematics, by Aiken and Besseman, McGraw Hill. A very attractive booklet illustrating the use of set notation in algebra and coordinate geometry. Suitable for Grades 9 and 10.

4. The New Mathematics, by I. Adler, John Day, New York, distributed in Canada by Longmans Green and Co. There is a set of problems published separately. This gives a very readable account of the number system up to the level which I advocate as essential in the high school.

5. Elementary Mathematics, by K.O. May, Wiley. This covers the whole of high school mathematics and reaches the level of maturity which I regard as desirable in a

freshman about to enter engineering or to major in mathematics.

Anyone wishing to keep abreast of developments in Ontario may write to the Ontario Mathematics Commission, 34 Prince Arthur Ave., Toronto 5, Ont., and ask to be put on the mailing list, in the event that the Commission issues announcements or news letters.

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