Proceedings of the Edinburgh Mathematical Society (1989) 32, 165–168 ©

BOOK REVIEWS

WHITELAW, T. A., *Introduction to abstract algebra* (Blackie, 2nd edition, 1988) pp. viii + 200, 0 216 92259 3, Paper £10.95.

The most obvious difference between this new edition and the first edition (1978: reviewed in Vol. 22 (1979), p. 68) is the insertion of a chapter on the symmetric group. This is certainly a good idea, and the new chapter has all the virtues (and all the delicious schoolmasterly quirks, as in "Then, as the student should verify in detail,...") of the rest of the book. Throughout, the exercises are a particularly strong feature, being varied, interesting and far from trivial. Those who learn and those who teach abstract algebra at this level have reason to be grateful to Dr. Whitelaw for disseminating his skill and experience in this area beyond the walls of Glasgow University.

J. M. HOWIE

WHITE, NEIL (ed.), *Combinatorial geometries* (Encyclopaedia of Mathematics and its applications, Vol. 29, Cambridge University Press, 1987), xii + 212 pp., 0 521 33339 3, £25.

This is the second of a three-volume series intended to cover matroids and combinatorial geometries. Like the first volume, it consists of a collection of expositions by various experts, edited to ensure uniformity of presentation throughout.

The first volume was described as a "primer in the basic axioms and constructions of matroids". Historically, matroids arose as a generalization of linear dependence, and later they were seen to be closely related to geometric lattices and combinatorial geometries. The result is a subject which can be approached from varying starting points using different sets of axioms. The approaches and axiom systems are described in the first volume. The present second volume begins with three chapters on coordinatization, or the representation of matroids as vector space matroids, including the particular cases of binary matroids (those representable over GF(2), discussed by J. C. Fournier), and unimodular matroids (those representable over every field, discussed by Neil White, the editor). Then Richard Brualdi deals with connections with matching theory. Matching matroids arise from matchings in graphs, while transversal matroids arise from matchings in bipartite graphs; remarkably these two classes of matroids are identical. There then follows a chapter by R. Cordovil and B. Lindström on simplicial matroids.

Attempts to generalise graph theory to matroid theory have produced analogues of chromatic polynomials, called characteristic polynomials. These are studied by T. Zaslavsky, along with some invariants of matroids such as the Tutte–Grothendieck invariant. Martin Aigner then studies geometric lattices, on which a rank function can be defined, and their Whitney numbers. For some lattices these numbers are unimodal, in fact log-concave.

The final chapter, written by Ulrich Faigle, is the longest. It deals with matroids in combinatorial optimization; this is the aspect of matroids which has been of widespread interest in recent years. The greedy algorithm works precisely in the matroid setting. Other optimization problems with constraints presented by integer-valued submodular functions (the discrete analogue of convex functions; the rank function of a matroid is submodular) can be considered from the point of view of integral matroids, which are collections of vectors in integral polyhedral matroids. The author's aim is to show how matroids are not only abstractable from optimization problems but are an essential tool in their analysis. There is a vast wealth of information in this chapter, which is possibly the most valuable of the whole book.

This volume can be highly recommended for inclusion in every mathematics library. It is well produced, with exercises and references at the end of each chapter.

IAN ANDERSON

MULDOWNEY, P. *A general theory of integration in function spaces, including Wiener and Feynman integrals* (Pitman Research Notes in Mathematics Series **153**, Longman Scientific and Technical, Harlow 1987), pp. 115, 0 582 99465 9, paper, £12.

In this monograph the theory of the Henstock integral is expanded, and applied to the path integrals named after Wiener and Feynman.

The Henstock integral is a generalization of the Riemann integral combining simplicity with considerable power. Recall that a function $f(x)$ defined for $0 \leq x \leq 1$ is said to be Riemann-integrable with integral c if for all sufficiently fine partitions

$$0 = a_0 < a_1 < \cdots < a_n = 1$$

and for all attached points $a_{i-1} \leq x_i \leq a_i$ the finite sum $\sum f(x_i)(a_i - a_{i-1})$ approximates c. Here the fineness of the partition is measured by the mesh $\delta = \sup\{a_i - a_{i-1}\}$. Henstock's generalization consists in allowing the mesh to depend on the point x_i, so that in the course of the $\varepsilon - \delta$ argument formalizing the above the fineness condition becomes

$$a_i - a_{i-1} \leq \delta(x_i)$$

for some positive function $\delta(x)$.

Henstock's procedure applies as well to integrals of infinite range, and agrees with the Lebesgue integral when that applies. Moreover it applies to some integrands of varying sign which are *not* absolutely integrable. This is indeed its major strength, although for this reason it clearly cannot be related to an underlying measure theory and so its various convergence theorems require more careful statement than those of the Lebesgue integral.

To fulfil the intention of the monograph it is necessary to apply the Henstock procedure to domains which are product spaces (indeed, of uncountably many factors). It will come as no surprise to measure-theorists that the simplicity of the original definition becomes less apparent; product spaces are the source of several technical difficulties for more orthodox integration theories also. The technicalities do however appear to differ in an interesting way. For example if the space of continuous functions is viewed as a subset of $\mathbb{R}^{[0,1]}$ then its indicator function is immediately Wiener-integrable if the Henstock procedure is employed.

What is lost and what is gained by using the Henstock integral?

Considerations of simplicity apart, we know *a priori* little or nothing can be gained in the theory of integration of nonnegative functions. Recall the summary of Solovay (1970): "the existence of a non-Lebesgue measurable set cannot be proved in Zermelo–Fraenkel set theory if use of the axiom of choice is disallowed." This confirms the empirical experience of analysts and probabilists, that Lebesgue integration theory is sufficient for the demands of absolute integration. The Henstock integral encompasses the Lebesgue theory for nonnegative functions; whether by strict inclusion is not clear from the monograph but any gain in this direction must clearly be of some sophistication in the style of mathematical logic. For practitioners, the simplicity of Henstock's definition must be matched against the enormous expressive power of the measure theory underlying Lebesgue's integral.

In the case of non-absolute integration Henstock's procedure provides for example an unambiguous value for Fresnel's integral $\int_0^\infty \exp(iy^2)\,dy$, which of course does not exist in Lebesgue's sense. The attraction of Henstock's theory is that it provides an automatic procedure for evaluating such integrals, together with a reasonable quantity of limit theorems and in