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The estimation of continuous-time systems using discrete data

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The problem treated is the estimation of the general continuous-time system

$$(1) \qquad y(t) = B\int_R \Gamma(\tau;\ \theta)z(t-\tau)d\tau + x(t)\ ,\quad t \in R\ ,$$

and several interesting special cases, when the dependent and independent vector variables $y(t)$ and $z(t)$ are observed only at the discrete, equally-spaced time-points $t = 1, \ldots, N$. The vector $x(t)$ is a residual and θ and B are respectively a vector and matrix of parameters to be estimated, with $\Gamma(\tau;\ \theta)$ a possible nonlinear function of θ. Clearly (1) cannot be directly estimated when only discrete data are available, so we consider the approximation

$$(2) \qquad w_y(s) = B\tilde{\Gamma}(\lambda_s;\ \theta)w_z(s) + w_x(s)\ ,$$

where

$$w_x(s) = (2\pi N)^{-\frac{1}{2}} \sum_{t=1}^{N} x(t)\exp(it\lambda_s)\ ,\quad \tilde{\Gamma}(\lambda;\ \theta) = \int_R \Gamma(t;\ \theta)\exp(-it\lambda)dt\ ,$$

$\lambda_s = 2\pi s/N$, $|s| < \frac{1}{2}N$, with $w_y(s), w_z(s)$ defined like $w_x(s)$. Then the estimates $\hat{\theta}, \hat{B}$ are the values that minimize Q, a weighted sum, over a symmetric subset of frequencies $\mathcal{B} \subset (-\pi, \pi)$, of squared deviations from (2). For θ and B to be identified it is usually necessary to assume that the continuous-time process $z(t)$ has no spectral mass over the frequencies $\lambda + 2\pi j$, $\lambda \in \mathcal{B}$, $j = \pm 1, \pm 2, \ldots$. Then if $\mathcal{B}$ can be

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chosen so that this is true and if additional, fairly general, conditions hold (principally if all the processes are strictly stationary and ergodic), $\hat{\theta}$ and $\hat{B}$ converge almost surely to θ and B respectively as $N \to \infty$, and $N^{\frac{1}{2}}(\hat{\theta}-\theta, \hat{B}-B)$ has a limiting multivariate normal distribution. Moreover if a particular weighting is used in Q, the estimates are asymptotically efficient.

The special cases of (1) that are considered are vector regressions and (the solutions of) autoregressions involving a finite number of unknown lags, for which Γ is taken to be a linear combination of Dirac delta functions, and (the solutions of) systems of linear differential equations and difference-differential equations. For each of these systems the stability, solution, identification, estimation and asymptotic theory are discussed. The methods are illustrated by estimating four simple scalar models of these types, using economic and oceanographic data.