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A STATISTICAL STUDY OF FLARES AS SELF-ORGANIZED CRITICAL PHENOMENA

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ABSTRACT The statistical property of flares is examined in terms of the number of flares versus its magnitude, i.e. the "Importance" reported in the IAU *Quarterly Bulletin of Solar Activity* for 1961-1983. It is shown that the number of flare and its magnitude follow a power law relationship indicative of the self-organized critical phenomenon, regardless of the phase of the solar cycle.

ANALYSIS AND RESULTS

It is well known that (1) the solar atmospheric magnetic field is subject to a continuous agitation of the photospheric and sub-photospheric convective activities, such as granulation, supergranulation (Nakagawa and Priest 1973), proper motions of sunspots (Tanaka and Nakagawa 1973), as well as the large scale differential rotation (Nakagawa and Levine 1974), and as the conseqence magnetic energy is accumulated in the atmospheric field as deviations from the potential field (Nakagawa et al. 1971, 1973; Nakagawa and Raadu 1972), also that (2) the energy released in flares can be accounted for through the change of magnetic energy content of the atmospheric magnetic field before and after a flare (Tanaka and Nakagawa 1973).

This situation resembles the dynamical mechanism leading to the onset of earthquakes in which the sliding motion of earth crust along faults results in the accumulation of strain energy in the crust, and subsequent release of this accumulated energy is manifested as the earthquake (Bak and Tang 1989). The above scenario can be interpreted as the accumulation of magnetic energy in the solar atmosphere by relative motions of unipolar magnetic regions and sunspots across the magnetic neutral line, including the subsequent energy release as flare.

Bak and his associates (Bak et al. 1988) introduced the term "self-organized criticality" to describe the instability of such a complex large number of interacting system and noted that (1) the system evolves toward a self-organized critical state in which a small perturbation leads to a coherent large scale instability, and that (2) the self-organized critical phenomenon is characterized by a succession of instabilities of various

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scales such that a power law relation holds between the number of event and the scale, i.e., the energy released in the events.

In order to examine such a statistical characteristic of self-organized criticality for flares, the flares listed in the IAU *Quarterly Bulletin on Solar Activity* for 1961-1983 are used. The data are divided into three groups of the period 1961-1965, 1966-1975 and 1976-1983, in consideration of the difference in the flare "Importance" classification and the apparent improvements of observation noted by a marked increase of number of small(S) flare reported after 1976. The flares are classified by their "Importance" presupposing that the "Importance" represents a logarithmic scale of energy released in flares. After the least-squares fitting, the results of analysis yield the following exponential relation between the number n(M) and the flare magnitude denoted by "Importance" M

$$n(M) = 1.90 \times 10^5 \exp[-2.09(M+1)] \quad (1)$$

where S flare is denoted as M=0.

Next the validity of this exponential relationship is examined for the period of the solar minimum (1973-1976) and the maxima (1967-1970, 1979-1981). For the solar minimum, we obtain

$$n(M) = 1.98 \times 10^4 \exp[-2.23(M+1)] \quad (2)$$

and for the solar maxima

$$n(M) = 3.05 \times 10^5 \exp[-2.45(M+1)] \quad (3)$$

These results are shown in Figures 1 and 2.

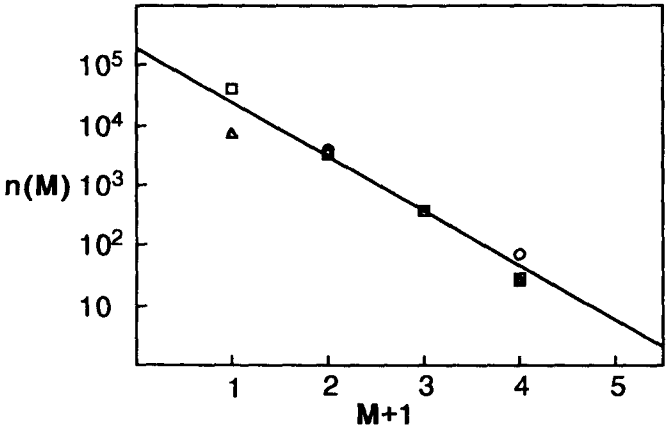

Fig. 1. The number of flares n(M) and flare magnitude in terms of "Importance" M for 1961-1965 (circles), 1966-1975 (triangles) and 1976-1983 (squares).

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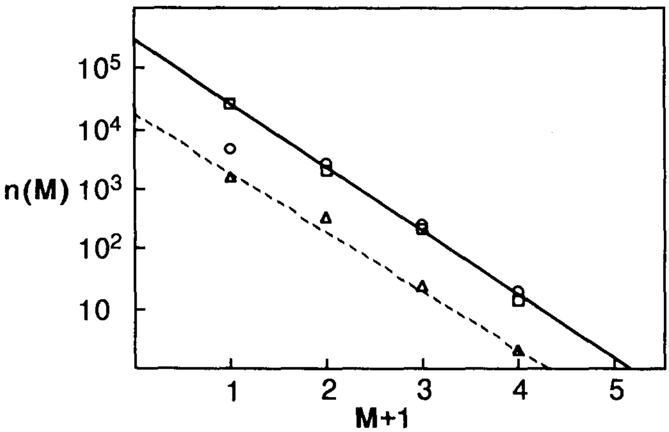

Fig. 2. The number of flares n(M) and flare "Importance" M for the solar minimum 1973-1976 (triangles) and the solar maxima 1967-1970 (circles) and 1979-1981 (squares).

These figures clearly show the exponential dependence of number of flares on flare "Importance" as the general rule. Therefore, it is expected that a power law relationship between the number of flares f(E) and the energy E released in flares results, if the flare "Importance" M is to be represented in terms of exponential function of energy. Assigning 10^{29} ergs for S flares, 10^{30} ergs for "1" flares, 10^{31} ergs for "2" flares and 10^{32} ergs for "3" flares, the power relations corresponding to Equations (1)-(3) become

$$f(E) \propto E^{-0.91}, \quad f(E) \propto E^{-0.97}, \quad f(E) \propto E^{-1.07} \qquad (4)$$

The results obtained in Equation (4) imply that energy released in flares is closely proportional to the amount of energy accumulated between successive flares, as the exact proportionality is the exponent of -1.00. The present results, therefore, suggest that an approximately constant rate of energy supply between flares in all phases of solar activity. It should be noted, however, that the total amount of energy released by flares varies in phase with the solar activity since the number of flares is clearly proportional to the number of sunspots.

DISCUSSION AND REMARKS

The result of present study is in agreement with that reported by Tang et al. (1984), if we interprete the flare "Importance" in terms of the area (or size) of magnetic activity. These authors showed that (1) an exponential relation holds between the number and size of magnetic activity (i.e., number of region de-

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creases exponentially with area size) and (2) the average area of active region varies in phase with the solar cycle, implying that the smaller amount of energy is released in flares during the solar minimum.

Apart from such a study, the evidnce of flare itself as the self-organized critical phenomenon has been given by Dennis (1985) from the analysis of over 6000 solar hard X-ray bursts recorded during SMM (Solar Maximum Mission). He obtained a power law relationship between the rate of flares N(P) and the peak counting rate P as

$$N(P) = 110\ P^{-1.8} \tag{5}$$

resembling results of Equation (4).

Finally, the results of present study can be summarized as follows:

(1) the dynamical mechanism leading to the onset of flares is clearly the self-organized criticality.

(2) the roughly inverse power law relationship between the number of flares f(E) and the energy released in flare E suggests that the amount of energy releazed in flares is supplied with an approximately constant rate between successive flares independent of the solar activity.

(3) the amount of energy released in flares, nevertheless, varies in phase with the solar activity, as the number of flares is proportional to the number of sunspots.

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