P. E. BOYNTON
Department of Astronomy, University of Washington
Seattle, Washington 98195

Small angular scale fluctuations in the temperature of the relict radiation may provide crucial information regarding the evolution of large scale structure in the universe. Some aspects of the interface between theory and observation in this developing study are considered.

I. Introduction

A central, unresolved issue in modern cosmology is how to understand the large scale structuring process which has transformed a relatively homogeneous early universe (on scales of galaxies, clusters, and super-clusters) into the highly inhomogeneous world of the current epoch. Although several alternatives have been pursued (see for example Ozernoi 1974, Press and Schechter 1974), the classical gravitational instability picture whose linearized theory was first discussed by Lifschitz (1946) is generally regarded as the most appealing framework for the discussion of such a process (cf. Weinberg 1972).

This picture provides for the growth of structure from incipient density perturbations in the early universe (but does not address the question of their origin). More specifically, within the context of the Standard Big Bang cosmology, the growth of these "seed" perturbations is restricted in time to the era following hydrogen recombination, and initially (i.e. in the linear regime) growth proceeds only as a power law in time, and not exponentially. Such painfully slow development might be viewed as a difficulty in using this process to explain the striking density contrast currently found on large scales.

But I prefer to argue the converse. Based on a Friedmann cosmology (specified by H_o and Ω_o) the locally observed distribution of matter ($z < 1$) implies a distinct evolutionary history of structure, and in particular specifies the amplitude of density perturbations which must have been present at the time of recombination. The elements of this argument may be summarized as follows:

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(1) From studies of the spatial correlations of galaxies, (e.g. Davis et al. 1977) one finds that density perturbations with masses larger than $3 \times 10^{14}\ \Omega_o h^{-1} M_\odot$ are just now entering the non-linear regime, $(\delta\rho/\rho)_{z=0} \simeq 1$.

(2) Density perturbations are free to grow from the time the Jeans mass falls below the perturbation mass. This transition takes place during recombination as a consequence of the decoupling of matter and radiation.

(3) In an open universe ($\Omega_o < 1$), growth in the linear regime continues as $\delta\rho/\rho \propto (1+z)^{-1}$ until the universe approaches "free" expansion; that is, until $\Omega(z)$ begins to depart from unity, which occurs nominally at $1 + z_f \simeq \Omega_o^{-1}$. Growth is thereby limited to the interval $z_f < z < z_{r(ecombination)}$, and the resulting growth factor $(1+z_r)/(1+z_f)$ ranges from 10^2 to 10^3 for $0.1 < \Omega_o < 1.0$. Thus, for those mass scales just now approaching non-linear condensation, the fractional density fluctuation back at z_r must have been in the range 10^{-2} to 10^{-3}, depending on Ω_o.

(4) Because of the tight coupling between matter and radiation up to recombination, density perturbations of this amplitude and angular scale ($\sim$ minutes of arc) might produce *observable* fine scale temperature anisotropy in the cosmic microwave background radiation.

Consequently, a natural by-product of this evolutionary scenario is a "fossil record" of the texture of the early universe which may be available to us for direct inspection. Such a record was necessarily imprinted on the radiation field as the universe became transparent (in the *last-scattering* process). So, by definition, the fractional anisotropies, $\delta T/T$, produced in this manner are the signatures of the most distant, highest redshift objects observable; hence the apparent hyperbole expressed in the title of this contribution.

If we could understand in detail the coupling between $\delta\rho$ and δT, this series of four concepts in the evolutionary scenario taken together would provide a test of the gravitational instability hypothesis through measuring or even through placing upper limits on temperature anisotropy for angular scales corresponding to progenitors of structure which clearly does exist at the current epoch. The motivation for fine-scale anisotropy observations is therefore clear.

II. Status of Observations

With one exception, little has changed since IAU Symposium #79 and there seems no need to repeat the summary of observational efforts I

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presented at that time, especially in view of the recent, comprehensive discussion of fine-scale observations by Partridge (1979a). Moreover, the announcement by Partridge (1979b) of a 95% confidence upper limit of roughly 10^{-4} for an rms fractional temperature fluctuation observed on an angular scale of $\sim$10 arc minutes, continues a well-established trend: that each improvement in technique fails to provide a detection of anisotropy, and instead places still more stringent upper limits on the amplitude of "primordial" temperature fluctuations.

In the past, these descending observational limits may have been paralleled by informal downward revisions in the expected fluctuation amplitude based on gravitational instability theory. In any case, an actual conflict between theory and observation on this topic has yet to occur. Motivated at least partly by the resilient ingenuity of theoreticians as a group, Harry van der Laan (1977) asked publicly at Tallinn if a *firm* lower limit to expected temperature anisotropy could be established. There are several considerations which could provide lower bound estimates. An estimate of minimal anisotropy present at recombination $(\Delta T/T)_{min}$ in the context of gravitational instability theory can be made, and one approach is discussed in the following section. But there is no lower bound to *observable* fluctuations $(\Delta T/T)_{obs}$ because of the unsettled issue of the history of the optical depth of the Universe following initial recombination. Reionization of the intergalactic medium may have occurred during the collapse of an early generation of high density contrast systems. The consequent increase in optical depth to Thomson scattering subsequent to z_r could erase all traces of information about structure present at recombination. In this situation, the lower bound $(\Delta T/T)_{min}$ provides a critical goal in planning the sensitivity of future observations. We may yet discover small angular scale temperature anisotropies at some level between $(\Delta T/T)_{min}$ and the current observational upper bound, and if successful we will have clearly learned something. On the other hand, if $(\Delta T/T)_{obs}$ is forced below $(\Delta T/T)_{min}$ the situation is ambiguous: either the universe is opaque back to recombination, or gravitational instability theory is incorrect. Aside from questions of practicality, there seems little motivation for improving observational sensitivity to search for fluctuations below $(\Delta T/T)_{min}$.

III. Defining the Critical Measurement

Van der Laan's question, rephrased in the spirit of that viewpoint developed in the Introduction, becomes: what is the smallest value of $(\delta T/T)_{z_r}$ consistent with the currently perceived value of $(\delta\rho/\rho)_{z=0}$ on interesting angular scales and which are also accessible to measurement? The question itself provides a prescription for formulating a reply: characterize the current mass spectrum of inhomogeneities, extrapolate that distribution back to the recombination era, define a *minimal* coupling to the radiation field thereby determining a lower bound for anisotropy consistent with gravitational instability. The details of this procedure have been spelled out by Davis and Boynton (1979); a selective review of that work follows.

What constitutes minimal coupling? An arbitrary perturbation in the pre-recombination matter-radiation fluid can be decomposed into familiar isothermal and adiabatic components, or alternatively considered in terms of matter and radiation perturbations. In the case of an adiabatic disturbance, matter and radiation are strongly correlated, $\delta T/T = 1/3\ \delta\rho/\rho$, prior to the onset of recombination. Although an isothermal disturbance may be thought of as a pure matter perturbation before passing through the horizon, subsequent gravitational coupling gives rise to a "spontaneous" radiation component. Therefore we have chosen to define *minimal* coupling through a disturbance which is artificially constrained as a *pure matter perturbation* throughout the recombination process. The only coupling to the radiation field is through Thomson scattering, and anisotropy is produced solely through large scale mass motion via the familiar Doppler effect, $\delta T/T \propto (v/c) \cos\phi$, where the scattering element is traveling at speed v at an angle ϕ relative to the direction to the observer. Even this minimal coupling mechanism is fairly effective because of the necessarily simultaneous reduction in optical depth, τ, and Jeans mass brought about by recombination. Consequently, photons are last scattered in material just developing a turbulent velocity field through the condensation process. To carry out our prescription, this velocity field must be related to the "seed" density perturbation spectrum. Such a relation follows directly from mass conservation, $\partial/\partial t(\delta\rho/\rho) = -\nabla\cdot v$, yielding fourier amplitudes $v_k \propto \delta_k\ 1/k$. Then the temperature fluctuation is given by

$$\delta T/T = \int (v/c) \cos\phi\ e^{-\tau(z)} (d\tau/dz) dz, \qquad (1)$$

where $e^{-\tau(z)} d\tau/dz$ defines a shell of last scatter with mean redshift $z_r \simeq 1100$, and width Δz of ~ 150 (Zel'dovich and Sunyaev 1972).

It is customary to examine the power spectrum of fluctuations, and in this case

$$|a_k|^2 \equiv |(\delta T/T)_k|^2 \propto (|\delta_k|^2/k^2) e^{-k^2 \cos^2\phi\ \sigma_x^2}$$

where σ_x is the coordinate width of the last scattering shell. This spectral density clearly exhibits the primary features of the minimal coupling process. First, the "velocity derived" temperature perturbations exhibit a considerably steeper spectrum (by a factor of k^{-2}) for small k compared to the "density derived" case: that is, compared to the adiabatic radiation component fluctuations for which $|a_k|^2 \propto |\delta_k|^2$.

This steepening implies a correspondingly broader autocovariance function, hence a more extended angular correlation for the minimal coupling perturbations. Second, the exponential cut-off at small scales arises directly from the finite width of the last scattering shell. For perturbations with linear size smaller than σ_x, the fluctuation is reduced by averaging over the several scattering elements which may contribute along the sight line. Third, not so obvious is the remarkable coincidence that the smallest mass scale which remains unaffected by this exponential cut-off is also the minimum mass which still lies in the linear growth regime at the present epoch[1]. This fact enables a trivial extrapolation (both normalization and spectral shape) of the current density spectrum back to z_r.

Although the power spectrum of temperature fluctuations adequately illustrates these points, it does not provide a convenient interface between theory and observation. The customary beam-switching observing technique involves the measurement of an ensemble of *temperature differences* between adjacent "patches" of sky. The "patch" angular size is determined by the width of the antenna beam pattern, θ_B, and the separation of patch pairs is just the antenna beam throw, $\Delta\theta$. These temperature differences are then employed to compute a mean-square measure of temperature fluctuation, $(\Delta T/T)^2_{obs}$. Calculating a comparable measure from a theoretical power spectrum $|a_k|_T^2$ involves computing the fourier transform of this single-difference sampling function including the effects of beam shape. The square of this transform, $|f_k|^2$, defines a power density filter whose product with the power density spectrum, when integrated, yields the desired mean square temperature difference:

$$\int |a_k|_T^2 \cdot |f_k|^2 \, d^3k = \left(\frac{\Delta T}{T}\right)_T^2 \qquad (2)$$

An equivalent, yet computationally simpler and more transparent procedure follows from recognizing the variance given by this equation as an autocovariance $C(\Delta\theta)$ evaluated at zero lag:

$$\int e^{ik\cdot\Delta\theta} |a_k|_T^2 \cdot |f_k|^2 \, d^3k = C(\Delta\theta=0) = (\delta T/T)^2 \qquad (3)$$

By application of the convolution theorem, the fourier transform of the product $|a_k|_T^2$ can be expressed as the convolution of the corresponding autocovariance functions: $C = C_T * C_{sampling}$

$$\text{where} \quad C_T = \int e^{ik\cdot\theta} |a_k|_T^2 \, d^3k \quad \text{and} \quad C_{samp} = \int e^{ik\cdot\theta} |f_k|^2 \, d^3k, \qquad (4)$$

$$\text{and} \quad \left(\frac{\Delta T}{T}\right)_T^2 = C(0) = \int 2\pi\theta \, C_{samp}(\theta) \cdot C_T(\theta) d\theta \qquad (5)$$

For the minimal coupling model, we characterize the density fluctuation power spectrum as a power law $|\delta_k|^2 = |\delta_o|^2 k^n$. The resulting theoretical autocovariance function C_T is parametrized by this power law index and Ω_o. A typical example $(n = -0.8, \ \Omega_o = 1.0)$ is shown in figure 1. Also depicted is an arbitrarily scaled covariance $C_m(\theta)$ describing the matter distribution, or alternatively the radiation component covariance for the adiabatic case prior to decoupling. Note the striking contrast between C_T and C_m. For the minimal coupling model, we define $(\Delta T/T)^2_{min} \equiv (\Delta T/T)^2_T$.

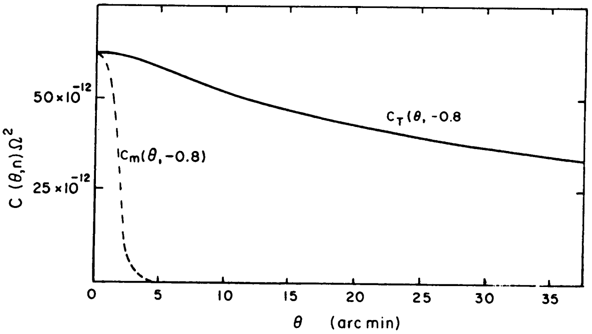

Figure 1. Theoretical autocovariance for minimal coupling model.

Comparison between this or any theory and a particular observing program can be carried out as follows: the observation provides a value of $(\Delta T/T)^2_{obs}$ corresponding to a particular sampling function $f(\theta)$ defined by the beam shape and switching amplitude. A one-dimensional representation of a typical sampling function (the customary Gaussian approximation to an antenna response function) is shown in figure 2a. According to eq. 5, the autocovariance of this same sampling function (fig. 2b) times $2\pi\theta$, when multiplied (at zero lag) by the theoretical autocovariance of temperature fluctuations (fig. 2c) and integrated over θ yields a value $(\Delta T/T)^2_{min}$ expected from the theory which is then

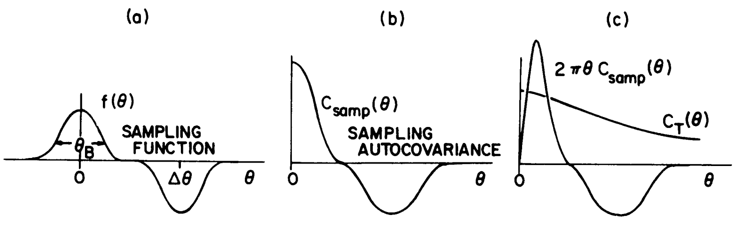

Figure 2. One dimensional representation of sampling function (see note 2).

directly comparable to $(\Delta T/T)^2_{obs}$. Concise comparison between any theory and observation is therefore possible if the theoretical covariance function for sky fluctuations and the observational sky sampling function are both made available.

The integral implied by figure 2c is particularly simple for $\theta_B \ll \Delta\theta$ and $\theta_B < \theta_{corr}$ where θ_{corr} is the correlation angle characteristic of $C_T(\theta)$. In that case, the sampling has a δ-function character which yields $(\Delta T/T)^2_{min} = 2(C_T(0) - C_T(\Delta\theta))$. This result is equivalent to that computed from a simple two-point difference spanning an angle $\Delta\theta$, $(\Delta T/T)^2_{min} = \mathrm{var}\,(\delta T/T(\theta) - \delta T/T(\theta + \theta\Delta))$, but from elementary error propagation considerations $\mathrm{var}(\delta T/T(\theta) - \delta T/T(\theta + \Delta\theta)) = \mathrm{var}(\delta T/T) + \mathrm{var}(\delta T/T) - 2\,\mathrm{cov}(\delta T/T(\theta), \delta T/T(\theta + \Delta\theta)) = 2C_T(0) - 2C_T(\Delta\theta)$

No such simple expression is possible in dealing directly with the power spectrum representation.

For the large θ_{corr} encountered in the minimal coupling case, typical millimeter wave beam switching geometries roughly satisfy the point sampling approximation introduced above. The resulting $(\Delta T/T)_{min}$ calculated for the sky autocovariance function of figure 1 and also for functions appropriate to $n = 0.0$ and $+0.8$ are shown in figure 3 for $\Omega_0 = 1.0$. From this figure, it is clear that $(\Delta T/T)_{min}$ is insensitive (unfortunately) to the power law index n; therefore, choosing $n = 0$ as representative allows the display of $(\Delta T/T)_{min}$ for various values of Ω_0 shown in figure 4.

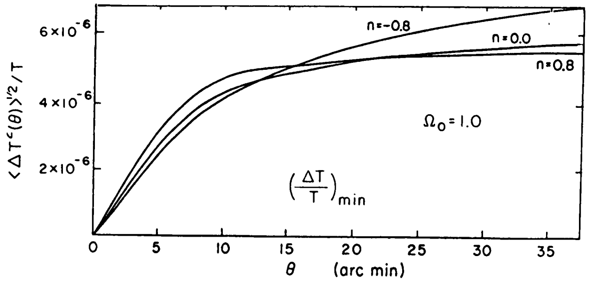

Figure 3. Angular dependence of differentially measured temperature fluctuations.

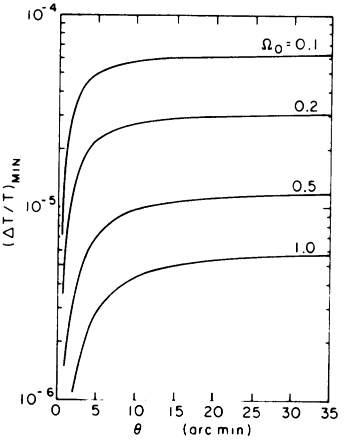

Figure 4. $(\Delta T/T)_{min}$ for various values of Ω_0.

Being based on minimal coupling, these curves provide lower limit values for $(\Delta T/T)_{obs}$. In a sense no definitive measurement is implied if Ω_0 is not predetermined. If upper limits on $(\Delta T/T)_{obs}$ continue to be reduced, larger values of Ω_0 must be accepted if the gravitation instability hypothesis is to be retained. In this sense, from figure 4, Partridge's upper limit implies $\Omega_0 > 0.05$. Although Ω_0 is still poorly determined, current observations suggest $\Omega_0 < 0.3$, thus $(\Delta T/T)_{min} \sim 2 \times 10^{-5}$ might be considered a tentative critical value in designing future observing programs and instrumentation.

As already mentioned, it is always possible to reduce $(\Delta T/T)_{obs}$ to arbitrarily low values through postulating sufficiently early reionization of the intergalactic medium following initial recombination. Although information about fluctuations present at z_r is lost in that case, the same process of scattering in the presence of large scale mass motion produces temperature fluctuations at an epoch for which the optical depth to Thomson scattering again becomes small. However, scattering at lower z yields structure on larger angular scales, degrees rather than minutes of arc (Davis 1979).

As emphasized by Rees (1979), if seed perturbations are purely adiabatic, reionization is less likely. Perturbations on mass scales which might collapse and release gravitational energy at early epochs are strongly damped during recombination in this case. At the same time, the expected temperature fluctuations at recombination are much larger and consequently existing upper limits on $(\Delta T/T)_{obs}$ constrain Ω_o to much larger values. Recent calculations of $(\Delta T/T)_T$ by Silk and Wilson (1979) for initially adiabatic perturbations, when compared to Partridge's 95% upper limit, require $\Omega_o > 1$ to avoid rejecting the gravitational instability hypothesis. Given the current uneasiness about such large values of Ω_o, this comparison of theory and experiment provides a fairly compelling argument against the presence of purely *adiabatic* disturbances in the early universe.

It would be interesting to be forced to contemplate our non-existence by observing sufficiently small upper limits for the amplitude of fine scale anisotropy. But because reionization can always be invoked to explain the absence of fluctuations observed from the current epoch, this irony is easily avoided. Until it is possible to infer independently the z-dependence of the optical depth of the universe for $z < 1000$ (Basko and Polnarev 1979), the utility of the minimal coupling limits presented in figure 4 is primarily as that gauge of experimental effort requested by van der Laan. That is, until observational limits are forced below $\sim 10^{-5}$ there remains the possibility of discovering the texture of the early universe, studying conditions in the shell of last-scattering, and confirming a clear view back to the initial decoupling of matter and radiation. However, if no cosmological anisotropy is detected on small scales even at this level, our sense of *horizon* may have to shrink a little.

FOOTNOTES

1. This smallest mass is also well above the "Silk damping" mass, and therefore the "velocity derived" component of temperature fluctuations is also correct for initially adiabatic perturbations.

2. The simple integral of the product of the two functions shown in figure 2c is a good approximation to $(\Delta T/T)^2$ for $\theta_B < \Delta\theta$. Note in figure 2c that the positive portion of the sampling autocovariance is multiplied by $2\pi\theta$ whereas the negative portion is not, this

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choice is appropriate to an approximate one-dimensional representation of the two-dimensional convolution implicit in equation (5). We stress this one-dimensional representation only because it provides a simple visualization of this linkage between theory and observation.

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DISCUSSION

Turner: What influence on your calculations and conclusions would changes in the assumed value of the amplitude and large-scale cut-off or break in the present-day matter covariance function have? Both of these quantities are significantly uncertain.

Boynton: Uncertainty in the amplitude of the matter covariance function (on mass scales which would be expected to contribute to observable temperature fluctuations) translates directly to the same fractional uncertainty in the estimated mean square temperature fluctuation level. However, these estimates are rather insensitive to the shape of the matter covariance function in the restricted sense that they do not depend strongly on the local power law index (cf. Fig. 3).

Gaskell: Why are you choosing to constrain the adiabatic fluctuations rather than, say, that Ω_0 could be greater than unity?

Boynton: Primarily because the conventional wisdom regarding the value of Ω_0 allows one to pose this rather interesting constraint. Heresy would appeal to me only if I had something new to bring to bear on the closure issue, and I don't.

Birkinshaw: As a by-product of the microwave background observations described later in this volume, we have a limit to the value of $\Delta T/T$ on blank sky for a beam-throw of 15 arcmin; this limit is $\sim 2 \times 10^{-4}$.

Rees: If there is an isothermal component to the initial inhomogeneities, the first generation of gravitationally-bound systems (of masses 10^7 to 10^9 $M_\odot$) will condense out immediately after recombination. The ensuing heat input may well be sufficient to reionize a fraction of the remaining diffuse matter; one only needs $n_e/n \gtrsim 10^{-3}\ \Omega^{-1/2}$ in order to smear out and delay the "last scattering epoch" and thereby reduce the Boynton-Davis fluctuations still further. (On the other hand, your exclusion of a "pure adiabatic" model for the fluctuations is on firmer ground, because in this model no bound systems form until clusters of the Silk mass turn around at $z \lesssim 10$, which is an epoch too recent to produce an opaque intercluster medium.)

One further point: if there is early production of grains or molecules, the "last scattering" of the microwave background may be due not to free electrons, but to some wavelength-dependent opacity. This means that the last scattering surface must be located at a wavelength-dependent redshift. There must then be no detailed correlations between the $\Delta T/T$ observed at two different wavelengths from the same piece of sky, contrary to the expectations of the standard model.

Boynton: I certainly agree with you about the possible impact of reionization on this question, but the point about excluding adiabatic fluctuations has been made by Silk and Wilson, not me. If wavelength-dependent opacity can be identified, it will make both the observing and interpretation tasks more difficult; but on the plus side, it might be exploited to provide a three-dimensional view of the pattern of early inhomogeneities. The angular scale of observed fluctuations will still be a crude index to the last-scattering epoch.

Silk: Wilson and I have also predicted that the minimal level of small-scale anisotropy expected from primordial isothermal density fluctuations is of the order 10^{-5} if there is no appreciable rescattering after decoupling. However, I believe that rescattering is probably inevitable for the following reason: in the hierarchical clustering model of galaxy and cluster formation, one begins with primordial isothermal fluctuations that systematically merge into deeper and deeper potential wells. These fluctuations must remain at least partly gaseous until clustering has developed. One can calculate the release of gravitational binding energy required in this model, and it must result in substantial amounts of ionizing radiation at very early epochs ($z > 30$ or more). The intervening medium will therefore be reionized at an early epoch if an isothermal fluctuation spectrum is the principal source of inhomogeneity in the early universe.

G. Burbidge: If no fluctuations can be found we have no direct evidence at all that galaxies are formed at early epochs through gravitational instability. That is, the current view of galaxy formation would be a purely theoretical concept.

Boynton: Lamentably so, but I think, rather, that galaxy formation would *remain* a purely theoretical concept.

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