

CORRESPONDENCE.

ON CERTAIN MEANS FURNISHED BY THE CENSUS OF 1851 FOR EXTENDING THE APPLICATION OF THE PRINCIPLE OF ASSURANCE TO THE SOCIAL CONDITION.

To the Editor of the Assurance Magazine.

SIR,—In the Sixteenth Annual Report of the Registrar-General of Births, Deaths, and Marriages, published on the 7th of the present month, it is shown that during the year ending the 31st December, 1853, there were 313,756 males and 298,635 females born, being 21 males to every 20 female children born—a difference of five per cent. in favour of female births. Observations by Corboux, and previous reports of the Registrar-General, tend to show that the relation which subsists between the numbers of male and female births is constant. The total number of births, therefore, exclusive of those still-born, for the year 1853, was 612,391. The survivors of these will, of course, be 27 years of age in the year 1879.

Until very recently, our predictions respecting such a number of human beings, in reference to the state in which they will actually be found at any particular future age of life, would have been limited to the single assertion that 354,758 of them would attain the age of 27, already mentioned (Carlisle Mortality). In addition to this, we could have determined how many of each sex this number consisted of, and have shown that it was made up of 169,158 males and 185,600 females. Beyond this we could scarcely have extended our predictions.

The age of 27 has been arbitrarily selected, for the sake of illustration; but for ages greater or less than 27 we could have assigned the numbers living at such ages, and also have distinguished the sexes with the same degree of accuracy. The correct determination of a series of values of this character, corresponding to each age of life, has long been regarded of paramount importance, as these values constitute the elements entering into all life contingency calculations.

There are, however, other inquiries, of scarcely less interest and practical utility, relating to the *social condition* of the population of this country, which day by day press themselves upon our notice; and it may fairly be presumed that much of the talent and industry which have been bestowed upon the leading inquiry just referred to will be brought to bear upon questions relating to many great problems connected with the civil condition of the people. In the year 1851, the most important of all considerations relating to the social state was made one of the heads of inquiry in taking the census; and tabular results, constructed by means of the returns, and published in 1854, show, for the eleven great divisions of the kingdom, the conjugal condition of the population, for quinquennial periods, commencing at the age of fifteen.

Portions of these tables will be quoted in a subsequent part of this article. Many social questions have already been solved by the results so arrived at; and the numerical relations which have been thus ascertained, with great care and consummate skill, enable us now to assert much *more* respecting the after-lifetime, and also the status, of the large numbers that are constantly being supplied to the population from time to time. The deductions which have been derived from the census returns are entitled to confidence, inasmuch as the observations have been made upon very large numbers, and every possible pains and caution employed to rectify those returns which were supposed to be of a mendacious character.

Errors in all cases are sure to arise in so vast an enumeration as the census estimates; but these are as likely to be in one direction as another, and, by a sort of compensation, do not materially affect final conclusions. I shall confine my remarks to the additional tabular matter relating to the *conjugal condition*, and endeavour to show in what way the real knowledge thus supplied to us may be made available for the purpose of extending the principle and practice of assurance, to provide in a pecuniary sense for various necessities connected with the social state which, from their very nature, may with singular propriety and advantage be brought under their operation. At all events, if the practice of assurance is to undergo any amplification, I believe it will be in this direction. We now possess reliable and veracious data, forming a very good index of the conjugal condition, to enable us to complete our calculations, and are independent of the dangerous practice (heretofore resorted to in some cases with the view of effecting this object) of proceeding upon a rough guess or a shrewd conjecture, false data or no data at all.

The following paragraph, which I shall quote at length, from the Registrar-General's Report, points out also in what various ways the facts elicited by the late census may be made subservient for the purposes of assurance, and applied to extend its operations.

“Without entering into any further or profounder analysis, it is sufficiently evident that the returns open a new field of philosophical inquiry into a subject which has hitherto been treated lightly; and the fortune-teller may yet share the glory or the shame of the astrologists and the alchemists, whose success was the evidence of undiscovered truth, as well as of their bold rapacity and mankind's credulity. The passions and affections of men are governed by laws as certain as those of the heavenly bodies; but it is not true, as the phenomena are complicated, that the acts of particular individuals can always be predicted, and in discarding this notion we get rid of the vulgar error: but it is true that the acts of numbers of individuals can be predicted with sufficient certainty for practical purposes; for the marriage returns, and these in conjunction with the life table, furnish the means of calculating the chances that a man or woman, young or old, and unmarried, will marry before, in, or after a given year of age—of calculating the *probability* of remaining a spinster or a bachelor, or of being in the married state, at any given age—the probability of bearing children, or of being a widower or a widow; and these calculations will serve not merely to gratify idle curiosity, but to guide the course of men's lives, to regulate the population, to make provisions for children who marry as well as for those who do not marry, and to direct the establishment and conduct of social institutions which may mitigate the calamities of premature death.” Following out the suggestions here given, by aid of the tables relating to the conjugal condition we are enabled to distribute the survivors in the year 1879 of the males and females born in the year 1852, nearly as they will actually be found to be *socially* related when the former period shall have arrived. In a similar manner may the numbers surviving at any earlier or later years than the date selected be distributed according to sex and conjugal condition.

For the sake of illustration, using the tables of mortality that apply to the sexes separately, the males will be distributed as follows:—

Bachelors.	Husbands.	Widowers.
74,667	94,491	1,929

The females will be ranged under each status thus:—

Spinsters.	Wives.	Widows.
73,797	108,361	3,442

If, therefore, out of a given number of births (for the purposes of this article I shall select female births) we are enabled to assign the numbers respectively belonging to each of the classes—spinsters, wives, and widows—at the particular age of 27 or any later or earlier age, it is evident that with the life table we possess the requisite elements for calculation, so as to determine the premiums payable at each particular age to cover the risk of having to pay a specified amount in the event of the person by whom or on whose behalf such premium was paid existing in a specified social status (suppose the first of the three just mentioned) when an assigned epoch shall have arrived. The problem is this: What sum, as a single payment, shall be paid by an individual A, of age x years next birthday, so that, after the expiration of n years, A being of a certain status (α) when that period has been completed, the sum of £ p shall then be paid? The solution is simple enough.

Let l_x denote the number living of age x according to the tables (the value of x having proper limits), $x+n=a$ constant, α_{x+n} β_{x+n} γ_{x+n} be respectively the numbers surviving out of l_x individuals of each status (α), (β), and (γ), then the single payment will be $\frac{p\alpha_{x+n}r^n}{l_x}$. The payments may be spread over a limited number of years—the annual premium being the same as that of a temporary annuity for that time payable at the beginning of each year, and of equivalent value to the single payment just determined. By means of columns N and D of Griffith Davies, and the expression $\frac{D_{m+n}}{N_{m-1} - N_{m+n-1}}$, the annual premium may be ascertained very readily.

If the premiums are returnable provided an event (β) takes place during the period of t years, then the single payment already shown must be augmented by the sum of the present values of all the probable claims that may arise for a return of premiums during the period within which the event (β) may occur. If $\pi_{m,t}$ denote the probability that the event (β) will occur in any year within the period t , then $\sum r^t \pi_{m,t}$ for all values of t from 1 to t , must be added to the preceding value, $\frac{\alpha_{x+n}r^n}{l_x}$. This present or single payment, so increased, may in like manner be spread over a definite number of years.

By similar means, with the necessary elements the single and annual premiums may be calculated to assure a sum payable at a fixed period in the event of an individual on whose behalf the premiums have been paid being, on the arrival of such period, either blind, deaf and dumb, paralytic or lunatic, or subject to any other infirmity; the data employed being, of course, properly authenticated. Some of the affections just named are not characterized, however, by that uniformity and constancy of occurrence which distinguish the operations of the human will, as manifested in the act of marriage, and in other respects among very large numbers. Lunacy, for instance, appears to have increased of late years, from accelerating causes not very clearly understood, the regular and gradual increase of the population being alone insufficient to account for the alarming increase that has latterly taken place in the number of persons in a state of *amentia*.

With increasing means of calculation at our disposal, it is obvious that the practice and application of assurance, hitherto circumscribed, can be materially enlarged, and adapted to many contingencies and casualties to which mankind are subject, but which as yet have not been brought within their sphere of operation. In fact, to restrict the uses of assurance to three or four contingencies to which life and property are liable, is opposed to the very spirit and function which belong to it. Risks of all kinds, and not those only of a particular class, may in the course of time be made matters of precise calculation, as reliable data are obtained from time to time; and the calamities which fall upon individuals may be greatly mitigated in their effects, if not wholly removed.

Much has yet to be accomplished, although so much has already been done, in ameliorating the condition of the people, and lessening the anxieties for the present and the future which are felt more or less by all classes of the community; nor is there any room for doubt that the benefits which will accrue from a more enlarged application of the principles of assurance will also be increased *pari passu* as it is made to embrace a greater variety of objects. We have seen that means are at hand for extending the sphere of assurance to some conditions of life; it may also be desirable to point out in a particular instance the necessity for such an extension. The female population of this kingdom from the age of 25 and upwards is greatly in excess of the males of the same age and upwards; and it is an undoubted fact that immense numbers of women reach the middle age of life, and are left unmarried—first in consequence of the redundancy just alluded to, and secondly in consequence of the large numbers of the other sex who, either from habits of prudence or other causes, pass through life in a state of celibacy.

We have already seen that the number of females born is less than that of the males; and it is highly probable, amongst very large numbers in each particular country or climate, the relation which one bears to the other is always constant at birth, from which time the numbers of each sex converge year by year to a ratio of absolute equality: the age at which the numbers are balanced probably coincides with the average age of puberty. From this period the numbers begin to diverge year by year, the excess now being invariably in favour of the female population, and to a large extent. The causes, if we stop for a moment to consider them, appear nearly obvious. The life of the male is more precarious: upon man devolve the weightier cares and duties of life; he is less kept in check by the rules of society, and can with less damage to his position addict himself to vicious practices; he is more exposed to the casualties of life and the effects of climate, not to mention the consequences of protracted wars: all these are inimical to life, and tend to shorten it. We shall presently see, by reference to the population tables, the disparity in the numbers of the sexes. It will serve the present purpose to select the age-period from 25 to 30. These tables record 808,705 males of ages between 25 and under 30, and 903,733 females of various ages between the same two periods, thereby showing a redundant female population of that quinquennial age-period of nearly 100,000. If we consider the disparity in the numbers of each sex as actually existing in some of the principal towns of England and Scotland, it will in a great many instances appear even more startling. The following results, selected from the population tables, will place the subject in a very clear point of view. The ages are between 25 and 30:—

NAME OF TOWN OR CITY.	POPULATION.		NAME OF COUNTY.	POPULATION.	
	Males.	Females.		Males.	Females.
London	101,886	124,746	Surrey	7,494	8,362
Bath	1,556	3,145	Sussex	12,174	13,786
Bedford	369	504	Essex	12,377	13,205
Brighton	2,554	3,874	Suffolk	11,623	13,044
Bristol	5,072	7,153	Norfolk	15,339	17,963
Bury St. Edmunds . .	437	594	Wiltshire	8,126	9,278
Coventry	1,466	1,653	Devonshire	19,501	24,168
Exeter	1,032	1,593	Somersetshire	14,447	18,581
Halifax	1,476	1,666	Gloucestershire	14,630	18,586
King's Lynn	795	942	Cheshire	16,820	18,941
Lancaster	634	749	Lancashire	88,424	97,409
Macclesfield	1,553	1,879	Dumfriesshire	2,566	3,055
Worcester	955	1,344	Renfrewshire	5,970	7,455
Yarmouth	1,015	1,436	Lanarkshire	23,439	25,987
Carnarvon	289	393	Edinburghshire	10,322	13,339
Aberdeen	2,299	3,537	Fifeshire	5,067	6,298
Dumfries	362	543	Forfarshire	6,462	8,924
Edinburgh & Leith . .	7,772	10,426	Aberdeenshire	7,158	8,886
Glasgow	14,675	17,618	Invernesshire	2,990	4,069
Inverness	318	478	Orkneys & Shetland . .	1,393	2,629
Kilmarnock	688	765	Jersey	2,232	2,890
Paisley	1,137	1,437			
Perth	936	1,065			
Dundee	2,868	4,162			
Norwich	2,582	3,406			
Plymouth & Devonport	3,798	4,674			
Shrewsbury	751	951			

A slight examination of the preceding table will show that in every place and county named the number of females greatly preponderates, and that in several places the number of women is double that of the men. In Bath, for example, there are 3,145 women of ages between 25 and 30, to 1,556 men of the same age-period. In a few cases, the aggregation of females in particular towns may arise from manufacturing operations in which women are employed; but these are extremely rare. Whatever may be the causes of this disparity which prevails throughout the country, and which is even greater at more advanced periods of life, the fact cannot be controverted that there were 680,938 women above the age of 30 (and consequently beyond the age most fruitful in marriages) who were returned as unmarried in 1851, and above the age of 25 there were 1,049,329 returned as unmarried. By the time the next census is taken, these numbers will have become so unusually augmented, especially if the present war is to continue for any length of time, as to excite some apprehension.* There can be little doubt that those districts in which the great mass of the population is for the most part poor are the most favourable to marriage, and that localities chiefly inhabited by the professional and middle classes are characterized by the greatest redundancy of unmarried women. For these classes especially the system of assurance may be made of great service; and sufficient sums may be assured to them at an inconsiderable outlay, if commenced in the earlier stages of life, by which the penury that almost invariably accompanies their declining years may be replaced by comparative comfort and ease.

* In the *Times* of the 2nd August it is stated, from the returns of the Registrar-General, that the marriages during the quarter ending March, 1853, were 35,014; in the quarter ending March, 1855, the number returned is only 29,131.

The following table, arranged from the census, shows the proportions of unmarried, married and widowed women in every thousand at each of fourteen periods of life; and on account of its great practical utility, I venture to give it *in extenso*.

SPINSTERS.				
AGES.	GREAT BRITAIN.	ENGLAND AND WALES.	SCOTLAND.	LONDON.
All ages.	603·47	597·93	637·15	
15 to 20	975·20	974·55	978·73	969·4
20 to 25	696·73	687·29	748·40	696·6
25 to 30	407·53	397·61	465·72	435·3
30 to 35	258·06	248·55	316·29	282·9
35 to 40	188·71	178·89	248·78	213·4
40 to 45	156·10	144·66	223·49	174·8
45 to 50	137·27	125·69	208·84	163·1
50 to 55	132·39	118·84	206·82	151·5
55 to 60	126·36	113·57	204·70	155·1
60 to 65	130·62	116·31	210·78	155·9
65 to 70	123·55	109·84	204·51	154·8
70 to 75	127·23	112·36	210·63	155·7
75 to 80	123·54	108·98	208·43	152·3
80 and upwards }	126·24	108·80	208·16	153·6
WIVES.				
All ages.	322·42	329·71	279·06	
15 to 20	24·50	25·16	20·94	30·2
20 to 25	298·31	307·75	246·78	297·3
25 to 30	573·32	583·84	511·63	543·2
30 to 35	700·55	711·45	634·36	669·0
35 to 40	746·27	758·43	673·03	705·2
40 to 45	740·26	755·81	649·70	692·1
45 to 50	722·89	739·20	624·00	653·7
50 to 55	668·08	688·07	559·60	580·0
55 to 60	619·08	638·89	499·48	509·9
60 to 65	515·97	536·22	403·70	390·3
65 to 70	432·05	448·39	336·34	301·8
70 to 75	323·33	336·63	250·30	205·5
75 to 80	233·31	243·23	175·94	129·0
80 and upwards }	120·71	127·80	88·11	77·5
WIDOWS.				
All ages.	74·11	72·37	83·78	
15 to 20	0·30	0·29	0·34	·4
20 to 25	4·96	4·97	4·82	6·1
25 to 30	19·14	18·55	22·65	21·5
30 to 35	41·39	40·00	49·35	48·1
35 to 40	65·01	62·67	78·19	81·4
40 to 45	103·64	99·53	126·82	133·1
45 to 50	139·84	135·11	167·15	183·2
50 to 55	199·52	193·09	233·59	268·5
55 to 60	254·56	247·54	295·82	335·1
60 to 65	353·41	347·48	385·53	453·8
65 to 70	444·39	441·77	459·15	543·5
70 to 75	549·45	551·02	539·07	638·8
75 to 80	643·16	647·79	615·64	718·7
80 and upwards }	753·05	763·39	703·73	768·9

The values for intermediate years in each quinquennial period may be interpolated, and the relative numbers of individuals belonging to the three conditions ascertained for each year of life.

The probability also may be determined that a female of a given age will marry prior to the attainment of a stated age, as also the probability that she will marry in some particular year, and consequently the probability that these events will *not* take place becomes known. Of course these values apply to the general population.

There is also ample evidence to show that marriages are by far most frequent among the lower classes; and that, if the population could be resolved into three great classes with any degree of accuracy, and the upper, middle and lower classes distinguished from each other by well defined lines of separation, the numbers unmarried in the middle periods of life belonging to the first two classes would constitute a much larger fraction of the whole than the unmarried portion of the lower classes compared with the whole number of them.

According to the returns of births, deaths and marriages for the year 1852, in which 158,782 marriages took place, the enormous number of 70,772 cases occurred in which each woman signed the marriage register *with her mark!* It is also shown in the census report, that in the counties of Stafford and Durham the number of women in every hundred of the age-period from 20 to 40, who are married, is greater in these two manufacturing districts than in any other part of the empire.

These two facts alone serve to show that the poor enter into the married state much more readily than the members of the upper and middle classes; and we may fairly conclude, that the greater proportion of the female population who are unmarried, in the middle period of life, consists to a great extent of the professional and upper classes of society. Whatever may be the case, it is certain that, if every man in Great Britain were to marry before attaining the age of 30, a very large number of women would necessarily be left unmarried. Attempts have been made from time to time to remedy in some degree the social evils resulting from the numerical inequality in the respective number of the sexes of a marriageable age in certain localities, such as Bath, Edinburgh, and other large towns; but in consequence of the want of trustworthy data, furnished by competent authority, these attempts have hitherto not been carried out.

In a work published by Mr. Bridges and entitled *The Prudent Man's Almanac for 1852*, the author has thrown out suggestions for the extension of the benefits of assurance: he states, "that as an alternative of emigration, and other social movements to obviate the compulsory celibacy of an immense proportion of the female population, an Association might be established to promote what might be called marriage insurance. The object and *modus operandi* would be, in consideration of a small yearly premium to guarantee a sum of money to any female remaining unmarried at a given age." An anonymous writer, in a pamphlet published in Edinburgh in the year 1851, and entitled *Poor Scotch Old Maids*, after relating many stories and quoting instances of the extreme indigence endured by unmarried women of good families in Scotland, gives a series of tables showing the sums which might be assured to unmarried women, and payable to them on attaining ages ranging from 50 to 55 (the time being contingent), for small annual payments, spread over a limited number of years; but (for want of veracious data, such as that which was given for the first time by the late census, and published in 1854) throws entire

discredit upon his calculations by stating that “the accuracy of the results ascertained and shown by the tables are to a considerable extent necessarily hypothetical, and are not of course guaranteed to the contributors.” This admission of itself was sufficient to condemn the whole plan; for no set of tables could be safely adopted in the construction of which hypothetical data had been employed. There are, however, suggestions in the pamphlet referred to which I have found useful, notwithstanding the impracticability and complicated nature of the plan, which fixes no precise time for the liquidation of individual claims, but regulates such period according to the age of the youngest member on the books.

In concluding this article, I may be permitted to observe, that the habit of generalizing too hastily and drawing inferences from numbers too small to admit of fair averages has often been productive of error. The tables, however, which have been constructed from the census returns, are entirely free from objection on this ground, while the conclusions which have been arrived at could scarcely have been misinterpreted. Doubtless even more precise information will be acquired relating to the conjugal condition by means of the next decennial returns; and the conjugal aspect of society, as it applies to the upper, middle, and lower ranks of society (if these distinctions of rank or any other for a similar purpose can be well defined), will be thoroughly ascertained.

GEORGE SCOTT.

40, King William Street, London Bridge,
August 17th, 1855.

ON THE MEANS OF APPROXIMATING TO THE RATE OF INTEREST YIELDED BY CERTAIN INVESTMENTS, &c.

To the Editor of the Assurance Magazine.

SIR,—Although the following problem is one which must frequently present itself to persons who have occasion to estimate the comparative advantages of different investments, &c., I am not aware that a convenient mode of solving it has ever been pointed out.

Problem.—A bond securing an advance of £1 at ρ per annum is purchased for £1 + p exactly n years before the principal becomes repayable, and immediately after a year’s interest has been received by the seller. What is the rate of interest obtained by the purchaser?

Let r = the rate of interest sought, then $1 + p = \frac{1 - (1 + r)^{-n}}{r} \cdot \rho + (1 + r)^{-n}$, or $p = \frac{1 - (1 + r)^{-n}}{r} (\rho - r)$.

Again, let $\phi r = \frac{1 - (1 + r)^{-n}}{r} (\rho - r) - p$ [1],

or $\phi r = 0$; and let $r = r_1 + h$ where r_1 is an assumed near value of r and h the difference between it and the correct value. Expanding the left-hand member of the equation $\phi(r_1 + h) = 0$, we have $\phi r_1 + \phi' r_1 h + \dots = 0$, or $h = -\frac{\phi r_1}{\phi' r_1}$ (nearly, when h is small); that is, an approximate value of h may be found by substituting the assumed value r_1 for r in the right-hand member of Equation [1], dividing the expression so obtained by its differential coefficient, and changing the sign of the result. Performing these operations, we shall find