ROTATING STELLAR ATMOSPHERES AND THE ENERGY DISTRIBUTION OF ALTAIR

PETER R. JORDAHL

The University of Texas at Austin, Austin, Tex., U.S.A.

Abstract. Models of rotating stellar atmospheres have been constructed for masses appropriate to middle and late A stars, and some of the photometric properties of these models are discussed. A specific model for the rapidly rotating star Altair has been constructed matching the observed radius, luminosity, and projected rotational velocity. It is found that the energy distribution predicted for the model agrees well with that observed for Altair, after the latter has been corrected for line blanketing.

This investigation was begun in the hope of comparing predictions from rotating model atmospheres and observations of a relatively rapidly rotating bright star, and thus, to some degree, testing the validity of the theory. The star Altair (α Aquilae) was chosen for the following reasons: (1) the spectral type (A7) indicates that most of its flux is emitted in the accessible regions of the spectrum. (2) Collins (1965) has suggested that the observable effects of rotation on the continua of A stars might be larger than for the B stars which he studied, due in part to (1). (3) Altair has a $V_e \sin i$ of 250 km/sec (Slettebak, 1966) determined by the standard method; Stoeckly (1968) has carefully studied line profiles in Altair photoelectrically and has derived a similar value based on a more complete model for rotation effects. (4) Altair has an excellent parallax ($\pi = 0.194 \pm 0.004$ (Jenkins, 1963)) and an interferometric angular diameter determination (Hanbury Brown *et al.*, 1967); thus we can attempt to match the total luminosity and radius of the star with our model.

The major disadvantage of the use of Altair as a test case is that it is a field star, and therefore (1) we cannot independently determine its approximate age by the cluster turnoff method, and (2) we cannot say anything about its composition from similar but sharp-lined stars which are members of the same group.

An attempt was made to associate Altair with one of the stellar groups isolated and identified by Eggen (1962). In the $U-V$ plane Altair cannot be identified with any of these groups, although it is much closer to the Hyades group than to any other, and is significantly distant from the sun-Sirius group. Therefore, we might *assume* (for lack of any better information) that the composition of Altair is similar to that of the Hyades, and that therefore blanketing corrections to the energy distribution of Altair (which of course cannot be measured directly for a star with a large $V_e \sin i$) may be obtained from the work of Oke and Conti (1966) on the Hyades.

Following Collins (1965), we assume uniform rotation, a Roche model for the potential, and von Zeipel's theorem for the relation between local gravity and local radiative flux. The polar radius and total energy generation are assumed to vary with angular velocity according to the prescription given by Faulkner *et al.* (1968) for a 2.0 $M_\odot$ interior model (computed with uniform rotation). (This variation is much less than that suggested earlier by Roxburgh *et al.* (1965); the newer results make a fit

A. Slettebak (ed.), Stellar Rotation, 116–121.

between model and observations possible over a much smaller range in the latter). The stellar interior models used to obtain the non-rotating luminosity and radius for a given mass, age and chemical composition were from the evolutionary sequences of Kelsall (unpublished; see Kelsall and Strömgren, 1966) which were computed for a range of composition and mass, and which incorporated modern opacities, including the bound-bound contribution.

The composition selected for the interior models was $X=0.67$, $Z=0.03$, these values agreeing well with those determined by Kelsall and Strömgren to give the best fit between the zero-age Kelsall models and the observations of the zero-age main sequence. The evolutionary sequences are given for a sufficient range of composition and age that a mass-luminosity-radius (MLR) relation can be obtained for the above composition for ages of 0, 1, and 2×10^8 years.

Thus, for a given age, mass, angular velocity (here expressed in terms of a ratio $v=\omega/\omega_c$, where ω_c is the 'critical' or 'break-up' angular velocity), and inclination i, we may construct a rotating model which gives the shape, size, and variation of gravity (and hence temperature) over the surface.

To obtain the radiation distribution from the model, the following procedure was employed: A grid of 21 model atmospheres was generated for $6500\,\mathrm{K}\leqslant T_e\leqslant 9500\,\mathrm{K}$, $3.5\leqslant \log g\leqslant 4.5$, using a program kindly supplied by Robert Kurucz of Harvard-SAO. The atmospheres were constructed with $X=0.7$ and normal solar metal abundances and incorporated all the usual sources of continuous opacity; also included were the absorption from neutral Mg and Si (of great importance in the rocket ultraviolet) and an approximation to the blended wings of the Balmer lines, as suggested by Strom and Avrett (1964; their model 5). This latter makes the theoretical energy distributions much more realistic in the region between Hδ and the Balmer limit.

For each atmosphere, emergent intensities were calculated for 62 frequencies and for $1.0\geqslant\mu\geqslant 0.05$, and these data were incorporated into the rotation program. For a specific rotating model (age, M, v, i given) the gravity g (and hence the effective temperature T_e) and μ are determined for a set of points (θ,φ) on the surface. For each point (θ,φ) and each frequency ν, the intensity $I_\nu(0,\mu[\theta,\varphi];T_e[\theta],\log g[\theta])$ in the direction of the observer is obtained from the model atmosphere intensities by quadratic interpolation in T_e, $\log g$, and μ. The monochromatic flux F_ν per unit solid angle in the direction of the observer is then obtained by a numerical quadrature over the surface of the model.

If these values of F_ν are integrated over all frequencies and multiplied by 4π steradians, we obtain the total luminosity L which would be *inferred* for the model if it were observed *at the given inclination.* Note that for rapidly rotating models this value can vary by a factor of 2 or more with inclination, while of course the true total luminosity of the model remains constant.

Energy distributions were calculated as above for models of $M=1.7$, 1.8, and 1.9 $M_\odot$, $i=0°$ and 90°, and a range of $v\,(=\omega/\omega_c)$ from 0. to 0.99. These were then subjected to another program which computes the parameters of the Strömgren 4-color (*uvby*) photometric system (based on a preprint from Matsushima, 1969). Some of the results

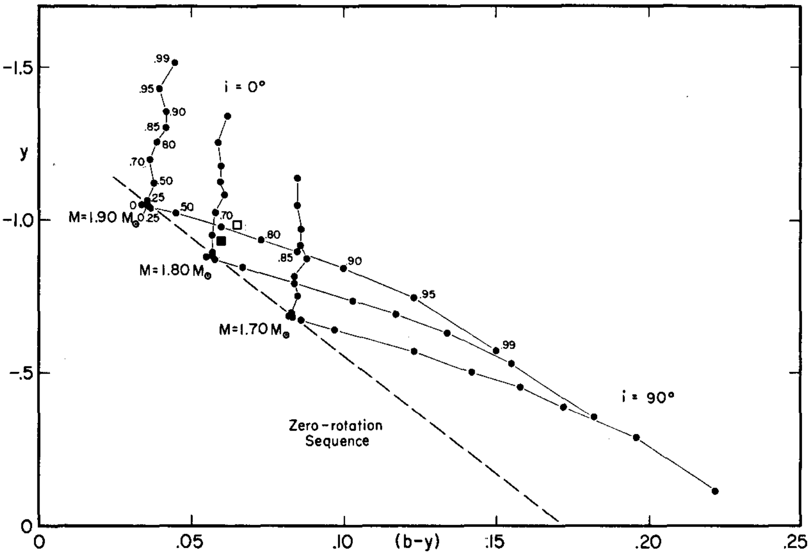

Fig. 1. Pole-on ($i = 0°$) and equator-on ($i = 90°$) sequences of zero-age rotating models in the [y, $(b - y)$] color-magnitude diagram. Points for $M = 1.90\ M_\odot$ are labeled by the ratio of angular velocity to critical angular velocity. The squares show the position of non-rotating models for ages 1×10^8 years (filled) and 2×10^8 years (open) for $M = 1.80\ M_\odot$. (Note: magnitude y has an arbitrary zero point).

are shown in Figure 1, which demonstrates the variation in magnitude (in the y filter; *arbitrary* zero point) and color $(b - y)$ with the parameters mentioned. The results for $1.9\ M_\odot$ are labeled by the value of v. Also shown in Figure 1 are the positions of non-rotating models for $1.8\ M_\odot$ and ages of 1 and 2×10^8 years (filled and open squares, respectively). The important thing to notice is that for the model with $v = 0.85$ (corresponding to $V_e = 260$ km/sec, about the largest observed in this mass range) the change in color (equator-on sequence) or magnitude (pole-on sequence) from the non-rotating model is only about half the maximum change predicted (for $v = 0.99$). Hence, although these theoretical models predict a rather large effect for 'near-break-up' stars, we shall probably not find such large effects. Taking the opposite view, *if* such large photometric effects are found and definitely attributed to rotation, then there is a greater impetus for incorporating a more complex model of rotation.

Another parameter which can be derived from the tilted, distorted rotating model is the average apparent radius $\bar{R}$. This is an average of the 'radii' of the object along one quadrant of the projected disk. These radii were not weighted by limb darkening and gravity darkening in taking the average. (However, the values of angular diameter taken from Hanbury Brown *et al.* incorporated *their* approximate correction for the effects of *limb* darkening.)

We now have three parameters with which we may compare models and stars: $V_e \sin i$, $\bar{R}$ and L. $\bar{R}$ and L, however, may be determined with reasonable accuracy only for the nearest stars, since both depend directly on distance; in addition, the angular

https://doi.org/10.1017/S0252921100027111 Published online by Cambridge University Press

diameter has been measured for only a few stars. Altair satisfies both requirements. Therefore a program has been written which performs a differential corrector procedure on the variables M, v, and i to fit the observed $V_e \sin i$, $\bar{R}$, and L, starting with a reasonable first guess for the variables. The derivatives required for the differential corrector procedure were calculated separately for each iteration, thus requiring four complete rotating models per iteration. For the age-zero Kelsall MLR relation the values (for a good fit) of these variables are:

$V_e \sin i = 250$ km/sec $\quad M = 1.80\, M_\odot$
$\bar{R} = 1.65\, R_\odot \quad v = 0.85$
$L = 11.0\, L_\odot \quad i = 73\overset{\circ}{.}5.$

No fit was possible for ages of 1.0 and 2.0×10^8 years, because the non-rotating model radius was already too large for these ages.

Observations of the energy distribution of Altair were made with the Ebert scanner on the 36″ telescope at McDonald Observatory. Those covering 3200 Å to 7000 Å

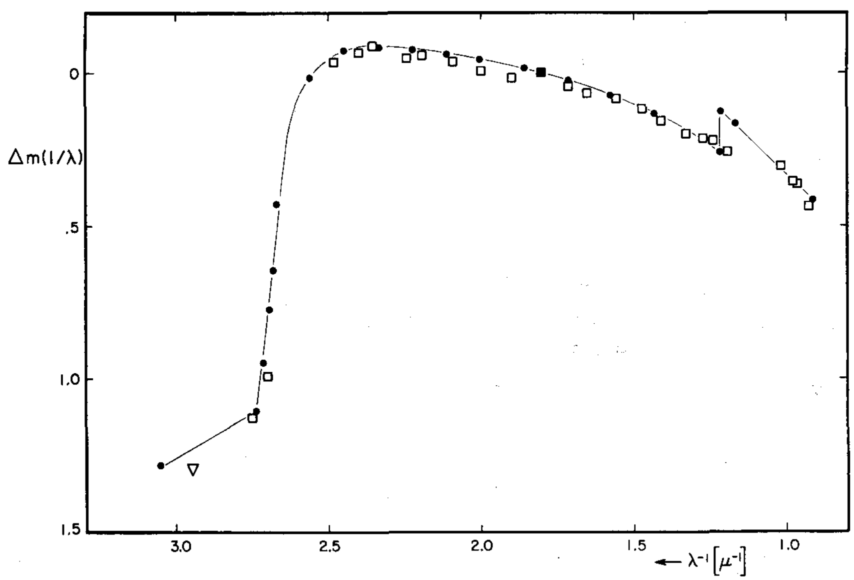

Fig. 2. The energy distribution for a rotating model fitted to Altair ($V_e \sin i$, $\bar{R}$, and L) (filled circles and line) and deblanketed observations of Altair (open squares). The open triangle is derived from the observed point at 3400 Å and the blanketing for 3636 Å. The squares are $0.^{m}025$ high; formal probable errors in the comparison between Altair and Vega are $0.^{m}005$ to $0.^{m}015$, but errors in the absolute calibration of Vega and in the assumed blanketing will increase these somewhat.

were made with the scanner operating in a rapid-scan, multiple pass mode incorporated recently by M. MacFarlane. The infrared observations were made earlier. The scans were reduced relative to Vega; the absolute calibration of Hayes (1967) corrected for the change in the gold point temperature (Labs and Neckel, 1968) was used to place the data on an absolute scale relative to 5556 Å. Blanketing corrections for

$\lambda \geqslant 3636$ Å were taken from the table in Oke and Conti (1966) interpolated to $(B-V) = 0.22$. The resulting continuum energy distribution is shown in Figure 2 by the open squares. The open triangle at $\lambda^{-1} = 2.94$ (3400 Å) was obtained by assuming the blanketing at 3400 Å to be same as at 3636 Å, probably an underestimate.

The energy distribution from the fitted rotating model calculated *without regard* to the observed energy distribution is shown in Figure 2 by the filled circles and the smooth curve. The agreement between theoretical and observed energy distributions appears good. This might indicate that our assumptions are adequate for calculated models to represent the continuum observations. However, in view of the large number of assumptions concerning the specific test case, further models will be constructed to determine the sensitivity of the energy distribution fit to these assumptions and choices.

Another possibility for improvement (suggested by Dr. Collins at this Colloquium, this volume, p. 85), would be to calculate the 'average radius' with limb darkening and gravity darkening included. However, what really should be done is to determine the monochromatic intensity profile projected onto 'diameters' ranging from polar to equatorial for the wavelength used by Hanbury Brown and determine the radius from the Fourier transform of this profile in the same manner as the observational procedure. This would in addition show the degree of sensitivity of the radius so determined to the shape of the star; one might also be able to check the possibility of detecting (as mentioned at this Colloquium by Dr. Mark) a very elongated object, such as the rotating models constructed by Mark (1968), by use of interferometric observations.

Acknowledgements

I should like to thank Dr. Frank Edmonds and Dr. Terence Deeming for reading a preliminary version of this paper, and Kelly MacFarlane and Catherine Hilburn for their excellent work on the figures. The computing facilities and time provided by the University of Texas Computer Center are gratefully acknowledged. Special thanks are due the Director and staff of McDonald Observatory for making it possible to obtain these observations, and to M. MacFarlane and D. C. Wells III for assistance at the telescope and in the photometric reductions.

Note added in proof: Sanderson, Smith and Hazlehurst (1970, *Astrophys. J.* **159**, Part II, L69) offer an explanation of the discrepant results of Roxburgh *et al.*, on the variation of R_p and L with rotation.

References

Collins, G. W., II: 1965, *Astrophys. J.* **142**, 265.
Eggen, O. J.: 1962, *Roy. Obs. Bull.*, No. 51.
Faulkner, J., Roxburgh, I. W., and Strittmatter, P. A.: 1968, *Astrophys. J.* **151**, 203.
Hanbury Brown, R., Davis, J., Allen, L. R., and Rome, J. M.: 1967, *Monthly Notices Roy. Astron. Soc.* **137**, 393.
Hayes, D. S.: 1967, unpublished dissertation, University of California, Los Angeles.
Jenkins, L. F.: 1963, *General Catalogue of Trigonometric Stellar Parallaxes*, Yale University Observatory, New Haven.

Kelsall, T. and Strömgren, B.: 1966, *Vistas in Astronomy* **8**, 159.
Labs, D. and Neckel, H.: 1968, *Z. Astrophys.* **69**, 1.
Mark, J. W.-K.: 1968: *Astrophys. J.* **154**, 627.
Oke, J. B. and Conti, P. S.: 1966, *Astrophys. J.* **143**, 134.
Roxburgh, I. W., Griffith, J. S., and Sweet, P. A.: 1965, *Z. Astrophys.* **61**, 203.
Slettebak, A.: 1966, *Astrophys. J.* **145**, 126.
Stoeckly, T. R.: 1968, *Monthly Notices Roy. Astron. Soc.* **140**, 121.
Strom, S. E. and Avrett, E. H.: 1964, *Astrophys. J.* **140**, 1381.

Discussion

Collins: I would suggest that instead of using geometrically determined $\bar{r}$ to compare with an interferometer measurement, that an intensity-weighted mean be used so as to include effects of limb-darkening and gravity-darkening.

Jordahl: The results given by the interferometer group are given in two forms, the first being for the case of no limb darkening, the second for the case where limb-darkening is derived from model atmosphere data for the appropriate spectral type and wavelength, and the values so obtained are used to correct the derived radius. I am using the second value. The effect of gravity darkening for the case most nearly like Altair appears to be relatively small even for a model near break-up, according to Hanbury Brown.

Faber: Have the published radii measured by interferometry been derived by incorporating some correction for limb darkening?

Collins: Yes, the values of the radius derived from interferometric measurements do depend on the intensity distribution over the apparent disk. This is true for both the intensity interferometer and the phase interferometer. Thus radii must be corrected for both limb and perhaps 'gravity darkening'.

Mark: Concerning Dr. Collins' comment on gravity darkening effects on stellar interferometry, the Hanbury Brown group (Johnston, I. D., and Warbing, N. C.: 1969, Preprint CSUAC No.177, 'On the Possibility of Observing Interferometrically the Surface Distortion of Rapidly Rotating Stars'), has reported that for solid body rotation, oblateness effects are almost cancelled by gravity darkening. But since differential rotation results in small gravity darkening, it is perhaps still possible to detect oblateness if stars rotate in this way.

Jordahl: One would presumably have to put a model for the gravity darkening into the interferometer data reduction. Hanbury Brown also points out that the details of the intensity distribution are contained in the wings of the Fourier transform of the distribution (which is actually what is measured) and the signal-to-noise ratio in this part essentially precludes obtaining the information. One can of course simply *assume* a certain distribution across the disk, go through the transform, and see if the result is discernably different from the standard form.

Klinglesmith: Both the Lyman lines and carbon continuous opacity must be included. Also, small overabundances of metals will increase the opacity effects.

Jordahl: Certainly. I intended to mention that although the Si and Mg continua seem to drop out around 10000°, the carbon continua should remain to a higher T_e and should be included. The point here is that these are *not minor* opacity sources, *not* that my models are complete.

Kraft: Suppose one maintained the position that all slowly rotating A's are in fact peculiar A's, so that if one sees a sharp-line normal A, it must be a rapid rotator seen pole-on. Vega falls in this class. Yet Vega is fundamental in all basic calibrations of T_e, R, etc. for early A's. Would this not constitute an embarrassment?

Jordahl: It would not bother the use of Vega as a spectrophotometric standard. But, yes, it would be poor to use such a star in the calibration of T_e vs. $(B-V)$ or T_e vs. spectral type. However, (as Dr. Collins pointed out), this assumption very severely limits the possible inclination of Vega, since it has such a low $v \sin i$ ($\leqslant 5$ km/sec) that no broadening is detectable. Vega does have an interferometric radius $\frac{3}{2}$ that of Sirius, but this is more easily explained as an evolutionary effect, than as a result of a star similar to Sirius rotating at or near break-up. To my knowledge, none of several analyses of Vega at very high dispersion have yielded any peculiarity in its spectrum that might be attributable to rotation.