(μl) may be rather tiresome. The snag with some otherwise excellent approximations, including this one, is that it may be nearly as much work to determine the approximate as the exact value.

Yours faithfully,

L. E. COWARD

2 Toronto Street
Toronto 1
Ontario

The Joint Editors
The Journal of the Institute of Actuaries Students' Society

12 *December* 1951

Annuities with a guaranteed term

Sirs,

The article by Mr Hymans under the above title (*J.S.S.* **10**, 230) prompted me to calculate specimen values of the additions to be made to the value of a continuous annuity in order to arrive at the value of a continuous annuity payable for various minimum guaranteed periods. Examples of the values of $\bar{a}_{\overline{n}|}-\bar{a}_{x:\overline{n}|}$ based on the $a(m)$ ultimate table are given below:

Term	Age	Rate of Interest per annum					
		$2\frac{1}{2}$%	3%	$3\frac{1}{2}$%	4%	$4\frac{1}{2}$%	5%
5	60	·255	·250	·247	·243	·238	·235
	65	·370	·363	·357	·351	·345	·340
	70	·533	·523	·514	·506	·498	·490
	75	·811	·797	·784	·772	·759	·747
10	60	1·026	·992	·961	·931	·901	·873
	65	1·453	1·407	1·360	1·318	1·277	1·238
	70	2·066	2·000	1·936	1·877	1·818	1·762

It will be seen that the ratio of the cost of the guarantee at one rate of interest to the corresponding cost at another rate of interest is practically independent of age; e.g. the cost on a 5 per cent basis can be obtained almost exactly by reducing the corresponding cost on a $2\frac{1}{2}$ per cent basis by 8 per cent if the guaranteed term is 5 years or by 15 per cent if the guaranteed term is 10 years. Furthermore, the above figures, together with other specimen values not reproduced here for considerations of space, show that these ratios vary approximately linearly with the term. From calculations made on both the $a(m)$ and $a(f)$ ultimate tables it would appear that, with sufficient accuracy for most practical puposes, an increase in the rate of interest of $\frac{1}{2}$ per cent would reduce the cost by say ·32 per cent for each year of the term and a decrease in the rate of interest of $\frac{1}{2}$ per cent would increase the cost by say ·33 per cent for each year of the term. For a change of $\frac{1}{4}$ per cent in the rate of interest a deduction, or addition, of ·16 per cent for each year of the term would seem satisfactory. If, however, the costs are available at two rates of interest, the cost at an intermediate rate could be found with greater accuracy by geometric interpolation.

Given Mr Hymans's problem, the above method could be used by valuing the pensions grouped in a double entry table according to age and unexpired term by $a_{\overline{n}|} - a_{x:\overline{n}|}$. The age x may be found by the Z method, the Z being chosen according to the spread of ages and terms—for example, 1·08 for males would probably give acceptable accuracy. The total cost of the guarantee for each value of n is thus determined for one particular rate of interest. The sum of these total costs weighted according to the term should also be calculated and then the result of say an increase of $\frac{1}{4}\%$ in the rate of interest could be found by deducting ·16% of this sum from the total cost of the guarantees at the original rate of interest.

The results of a similar investigation using the A 1924–29 table showed that similar relationships held and examples of the value of $a_{\overline{n}|} - a_{x:\overline{n}|}$ are given on p. 56.

It would seem that a reasonable adjustment for an increase (or decrease) of $\frac{1}{4}$ per cent in the rate of interest would be to deduct $\frac{1}{6}$th per cent (or add ·175 per cent) for each year of the term.

Thus on the rare occasions on which Family Income Benefits are valued accurately the effect of a change in the rate of interest can readily be obtained.

Since $a_{x:\overline{n}|} = a_{\overline{n}|} - (a_{\overline{n}|} - a_{x:\overline{n}|})$ the method can also be used to

Term	Age	Rate of Interest per annum				
		2%	$2\frac{1}{4}$%	$2\frac{1}{2}$%	$2\frac{3}{4}$%	3%
10	20	·112	·110	·108	·106	·104
	30	·128	·125	·123	·121	·119
	40	·223	·219	·215	·211	·208
	50	·473	·464	·456	·448	·440
20	20	·385	·373	·361	·350	·338
	30	·514	·498	·481	·465	·450
	40	·968	·936	·904	·873	·844
	50	2·168	2·095	2·023	1·955	1·889
30	20	·836	·797	·759	·723	·689
	30	1·281	1·217	1·156	1·099	1·045
	40	2·598	2·465	2·339	2·221	2·109
40	20	1·566	1·466	1·371	1·284	1·204
	30	2·720	2·537	2·365	2·206	2·060

estimate the values of temporary annuities at varying rates of interest, provided that the values of the corresponding annuities certain at the required rates of interest are available.

Yours faithfully,

A. C. RICHARDS

60 Meadowcroft Close
Balcombe Road
Horley
Surrey

The Joint Editors
The Journal of the Institute of Actuaries Students' Society

14 *December* 1951

The recent marriage and fertility data of England and Wales

Sirs,

In his paper published in your last Part (**10**, 261) Mr P. R. Cox discusses the notion of a differential fertility of marginal marriages which he (rightly, I think) attributes to me. For the sake of the record I should like to mention that I put this idea forward during the war in the early days of the Royal Commission on Population as a fatal objection to the process then in favour of using nuptiality