BULL. AUSTRAL. MATH. SOC. 73G05
VOL. 7 (1972), 463-465.

Some partial solutions of finite elasticity

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Elastic deformations beyond the range of the classical infinitesimal theory of elasticity are governed by highly non-linear partial differential equations and in general there exists no clear statement of these equations which does not involve stress components. Orthodox methods of solution are not usually applicable and consequently only a few exact solutions are known. In this thesis for some special classes of deformations and for some special materials we obtain substantial new reductions of the equilibrium equations which involve the deformation only. From these reduced equilibrium equations a number of new exact partial solutions are derived for the neo-hookean and Mooney materials.

For plane and axially symmetric deformations of isotropic incompressible elastic materials simplified stress-strain relations can be obtained. Using these relations together with the equilibrium equations it is possible to deduce relatively simple equations for the deformation. These equations provide both a simple and direct means of studying the semi-inverse solutions of finite elasticity. For the general deformation similar reduced equilibrium equations can be derived for the neo-hookean and extreme-Mooney materials.

In this thesis a number of deformations are studied using these reduced equilibrium equations. Exact partial solutions expressible in terms of Bessel functions are obtained for the following deformations:

(i) plane straightening and stretching of a sector of a Mooney circular-cylindrical tube,

Received 19 September 1972. Thesis submitted to the University of Queensland, June 1972. Degree approved, September 1972. Supervisor: Dr V.G. Hart.

(ii) axially symmetric radial stretching of a circular neo-hookean disc containing a concentric circular hole,

(iii) three-dimensional flattening, straightening, stretching and shearing of a neo-hookean spherical shell.

In addition from each of these solutions a further exact solution can be deduced for the inverse deformation.

Each of the above deformations are rendered as approximate solutions of mixed boundary value problems. These solutions are approximate in the sense that the pointwise vanishing of the stress vector on a free surface is assumed to be replaceable by the vanishing of force and moment resultants. Thus for example for the deformation (i) we can obtain approximate solutions to the bending of a sector of a circular-cylindrical tube by end couples alone.

The deformations described so far are heavily dependent upon the use of the neo-hookean and Mooney forms of the strain-energy function. In contrast to these deformations we consider in the final chapter of the thesis classes of deformations which are "well-defined" for all isotropic materials, not necessarily elastic but which satisfy the principle of material indifference. For these deformations we can always be sure of finding non-trivial solutions of the equilibrium equations no matter what form the response coefficients may take.

The most general plane deformation of this latter type describes the mapping of a plane region bounded by two logarithmic spirals into a similar shaped region. New closed form exact solutions are given for special cases of this deformation for the Mooney material.

The reduced equilibrium equations and the partial solutions form the main contribution of the thesis. However in addition to this work certain other related results have been established.

Firstly for Ericksen's problem we have shown that there can be no further spherically symmetric controllable deformations other than those already known.

Secondly in seeking some reduction of the equilibrium equations for the general deformation it has been observed that if both the deformation and its inverse are employed then the equilibrium equations can be given in a reciprocal form. From these reciprocal equations we can deduce the above mentioned reduced equilibrium equations for the neo-Hookean and extreme-Mooney materials as well as the standard reciprocal theorems of finite elasticity.

Thirdly, in the context of second-order effects we have used these reciprocal theorems to obtain new formulae for the second-order pressure function. Results of this kind emphasize the importance of studying the symmetry properties of the governing equations.

Finally we establish a completely new symmetry property for plane deformations of isotropic compressible hyperelastic materials. We show that the stress and deformation fields of a given material are respectively deformation and stress fields for another material. Moreover we give an explicit formula connecting the strain-energy functions of the two materials.

https://doi.org/10.1017/S0004972700045391 Published online by Cambridge University Press